**Randomised Load Balancing on Networks**

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**Discrete Load Balancing on Networks**

- **Initially:** Each node has any number of tokens
- **Goal:** Each node has \( \approx \) average number of tokens

For large-scale distributed networks, nodes should operate locally without global knowledge on the load distribution or topology.

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**Communication Models**

- **Diffusion**
- **Matching Model**

In Diffusion, nodes balance their load concurrently with all neighbours, whereas in the Matching Model nodes average their load with matched neighbours only.

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**Questions**

1. How to achieve a small discrepancy as quickly as possible?
2. How well can we approximate the continuous case where load is divisible?

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**Discrepancy Bounds**

**Deterministic Rounding (Always Rounding Down)**

For any regular graph, the discrepancy is \( O(\frac{\lambda K}{\sqrt{n}}) \) after \( O(\frac{\lambda K}{\sqrt{n}} \log(1 - \frac{K}{n})) \) rounds.

**Randomised Rounding**

For any regular graph, the discrepancy is constant w.h.p. after \( O(\frac{\lambda K}{\sqrt{n}}) \) rounds.

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**Iterative Protocol (Matching Model)**

- For every round \( t = 1, 2, \ldots \)
  - Generate a random matching
  - Average load across edges of the matching

There are several distributed protocols for generating proper random matchings.

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**Approximating the Continuous Case**

- \( \Theta(\log n) \) for expanders
- \( O(\sqrt{\log n}) \) for any graph
- \( O(\log \log n) \) for grids

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**References**


