Violation of Bell's inequality in fluid mechanics

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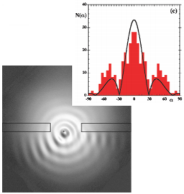
Warwick, June 2013

Motivation

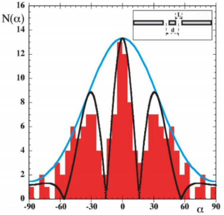
To understand the quantum-like behaviour
observed in collective phenomena in fluid mechanics

Bouncing droplet on a vibrating bath of silicone oil

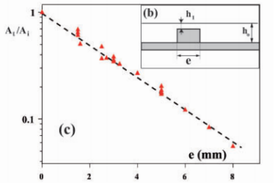
Single slit



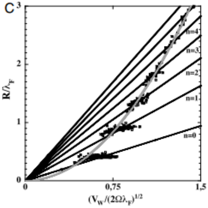
Two-slit



Tunnelling



Quantised orbits



Y Couder, E Fort 'Single-Particle Diffraction and Interference at a Macroscopic Scale' PRL 97 154101 (2006)
A Eddi, E Fort, F Moisi, Y Couder 'Unpredictable tunneling of a classical wave-particle association' PRL 102, 240401 (2009)
E Fort et al 'Path-memory induced quantization of classical orbits' PNAS 107 41 17515-17520 (2010)
http://www.youtube.com/watch?v=B9AKCJjtKa4

Agenda

How can a classical fluid system display this quantum-like behaviour?

1. The Bell tests rule out classical models
2. The motion isn't Lorentz covariant
3. There aren't corresponding phenomena in fluids in 3 dimensions
4. Classical systems don't have spin-half symmetry

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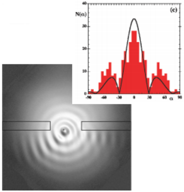

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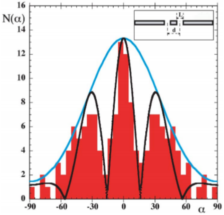

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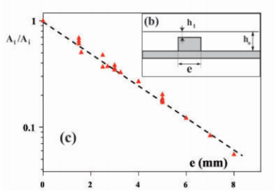

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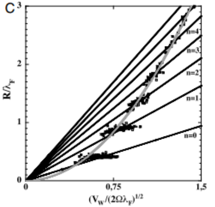
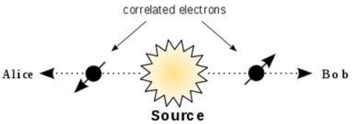


What do the Bell tests show?

There are two hypotheses about ‘locality’



J. S. Bell. On the Einstein-Podolsky-Rosen paradox
Physics, 1(3):195–200, 1964.

✓ No-signalling hypothesis

Signals cannot travel faster than a maximum speed (eg sound in the air, or light in relativity)

✗ Bell’s hypothesis

“the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past”

✓ consistent with experiment
✗ falsified by the Bell tests

Why does this rule out classical fluid models?

"Now we make the hypothesis, and it seems one at least worth considering.."

J. S. Bell. On the Einstein-Podolsky-Rosen paradox Physics, 1(3):195–200, 1964.

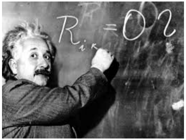
Not considered if it applies in classical fluids

No longer seems worth considering

"I wish to clarify that, on the particular question of whether you can violate the Bell inequality with a classical local-realist model – involving fluid dynamics or anything else – I'm 100% as close-minded as Lubos. Here we're not talking physics but math, and simple math at that. Either your model involves faster-than-light interactions, or you mistake a delocalized phenomenon (like a scissors closing) for a particle, or you mangle the statement of the Bell inequality itself, or there's some other boring problem ..."

Scott Aaronson, MIT 2013 (with permission)

Why did Bell think his hypothesis worth considering?



Einstein's principle of locality

"But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1 which is spatially separated from the former"

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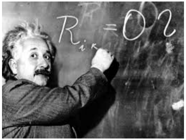

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rel·e·vance [rel-uh-vuhns]
noun
the condition of being relevant, or connected with the matter at hand

Is it relevant to collective fluid phenomena?

The fluid motion is correlated over large distances, so collective phenomena whose centres are far apart might not be "spatially separated" in the way intended.

Example - vortex in a compressible fluid

Fluid speed $u = \frac{C}{r}$

1. Linear operations
   - Insert a rod into the eye
   - Observe forces as rod is moved
2. Rotational operations
   - Couple to the rotational motion
   - Large paddles must be used

T. E. Faber. Fluid dynamics for physicists. Cambridge University press, Cambridge, UK, 1995.

Example - vortex in a compressible fluid

Fluid speed $u = \frac{C}{r}$

Kinetic energy
$E = \int \frac{1}{2}\varrho u^2 .2\pi r dr \approx \pi\varrho C^2 \log r$

Angular momentum
$L = \int \varrho u r.2\pi r dr \approx \pi\varrho C r^2$

Approximate (Bernoulli terms reduce density at small r)

1. Linear operations
   - Insert a rod into the eye
   - Observe forces as rod is moved
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E and L reside at large distance

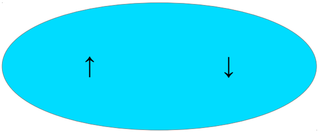
- correlated out to large distance

Boundary condition: E and L finite

- not satisfied at large r

T. E. Faber. Fluid dynamics for physicists. Cambridge University press, Cambridge, UK, 1995.

Boundary condition - vortices created in pairs

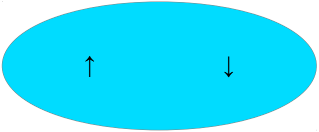


(schematic)
Most energy is in shaded region

Circulations precisely opposed

- No net angular momentum *L*
- Energy *E* is finite
  - Fluid velocities reinforce between the centres
  - but opposed at large distance

Boundary condition - vortices created in pairs



(schematic)
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Circulations precisely opposed

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- Energy E is finite
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Intertwined whatever the separation

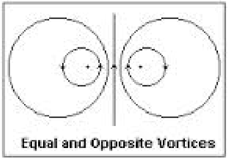
- since E and L must be finite

Physical picture:

- Vortices are large compared to the distance between the cores

or from scale-free symmetry of Euler's equation
If $\rho(\mathbf{x}, t)$ is a solution, then so is $\rho(a\mathbf{x}, at)$

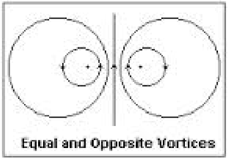
Rotational operations



You can't affect the rotation of one system without affecting the other

- If you could alter only one system, then E and L would be unbounded (boundary condition not met)
- To couple to the rotation, a fluid mechanic might imagine inserting a horizontal wall (or using large paddles) – both systems are affected.

Rotational operations



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Bell's hypothesis

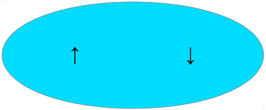
"the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past"

✓ Linear operations (rods)

✗ rotational operations (horizontal walls/paddles)

R. Brady and R. Anderson. Violation of Bell's inequality in fluid mechanics. ArXiv 1305.6822 (2013)

For philosophers



schematic

“Your conclusion contradicts Einstein’s principle of locality”

- Einstein’s principle is about systems which are spatially separated. The rotational motion is not spatially separated.

“Special relativity forbids violating Bell’s inequality in a classical system.”

- Special relativity is about measuring events. Events do not couple to the rotational motion because they are too small.

“If you suddenly perturb one system, the other can’t react instantaneously”

- Either the perturbation overlaps both systems, or it is too small to affect the rotational motion significantly.

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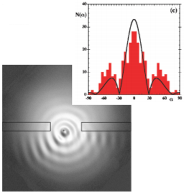

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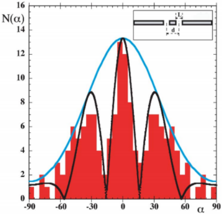

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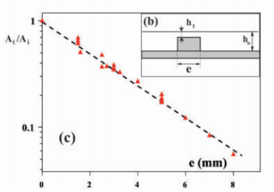
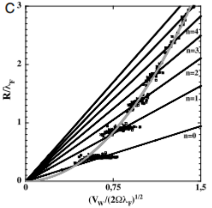
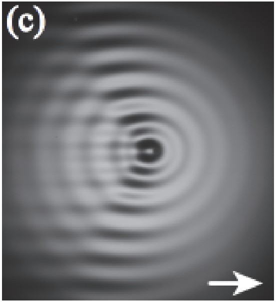


Quantised orbits



Experimental measurement



A Eddi et al 'Information stored in Faraday waves: the origin of a path memory' J Fluid Mech. 674 433-463 (2011)

Increase amplitude of vibration A

- Droplet bounces higher
- Frequency reduces below driving frequency
- Velocity v increases $v \approx c' \sqrt{(A - A_o)/A}$

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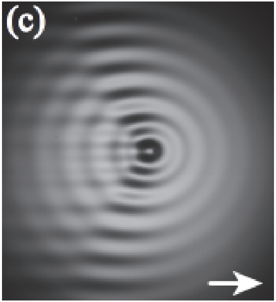

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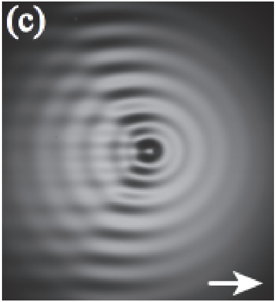

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Perturbed in this experiment

- Characteristic speed c' reduced near the droplet (eg by its mass)
- Perturbation evident in the wake

Explanation

Solutions obey the wave equation to first order

- $\frac{\partial^2 h}{\partial t^2} - c^2 \nabla^2 h = 0$ where h is the wave height

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Wave equation invariant under Lorentz transformation $x \to x'$, $t \to t'$

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Extended in the field of analogue gravity

- ‘Acoustic metric’ for irrotational motion of a compressible inviscid fluid is analogous to the metric in general relativity
- Deviations from Lorentz covariance average to zero
  (related to d’Alambert’s paradox 1752 – no drag on a solid object if flow is irrotational)

C. Barceló, S. Liberati, and M. Visser. Analogue gravity. Living Reviews in Relativity, 14(3), 2011.
R. Brady. The irrotational motion of a compressible inviscid fluid. ArXiv 1301.7540, 2013.

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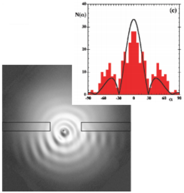

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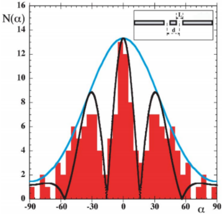

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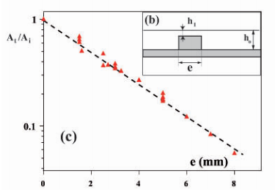

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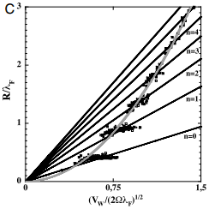

Quasiparticles

Quasiparticles in semiconductors (eg holes) – basis of electronics

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Toplogical dynamics

- Quasiparticles in an abstract fluid
- Research in biology, physics and string theory
- In general, wind up tightly due to tension

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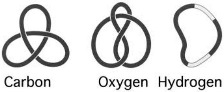

Lord Kelvin's vortex atoms 1867

Vortex loops

- Pinned to the medium by the circulation

click to watch

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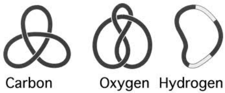

Lord Kelvin's vortex atoms 1867

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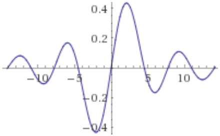
We will examine

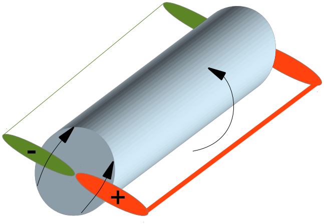
- Loops with no circulation
- irrotational and Lorentz covariant

Irrotational vortex ('eddy')

$J_1(r)$



Cylindrical Bessel function



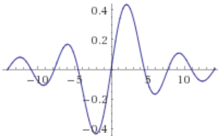
click to watch
Near-field, schematic only

$$\Delta\rho = A\cos(\omega_o t - m\theta)\ J_m(k_r r)$$

Solution to the wave equation

Irrotational vortex (‘eddy’)

$J_1(r)$



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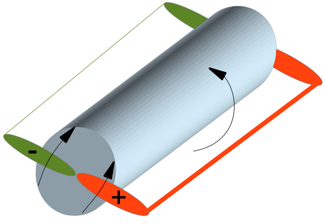

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Near-field, schematic only

$$\Delta\rho = A\cos(\omega_o t - m\theta)\, J_m(k_r r)$$

Solution to the wave equation

Sound in a compressible fluid

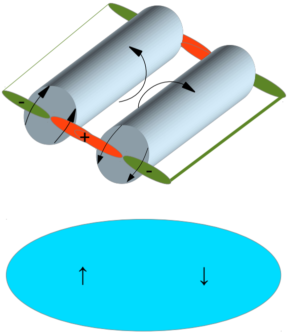
Euler’s equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} \;=\; -\frac{1}{\rho}\nabla P$$

Reduces to wave eqn at low amplitude

$$\frac{\partial^2 \rho}{\partial t^2} - c^2\nabla^2\rho \;=\; 0 \quad \left(c^2 = \frac{\partial P}{\partial \rho}\right)$$

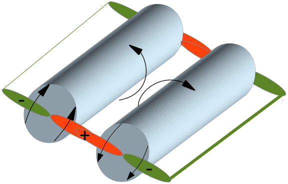
T. E. Faber. Fluid dynamics for physicists. CUP 1995.

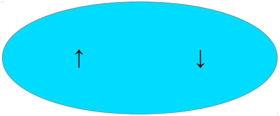


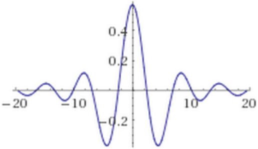
Eddies are created in opposed pairs

- No angular momentum

Boundary condition – similar to vortices







$J_1(r+\pi) - J_1(r-\pi)$

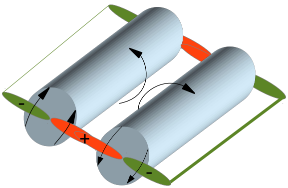
Eddies are created in opposed pairs

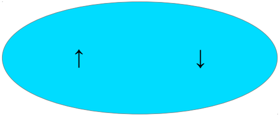
- No angular momentum

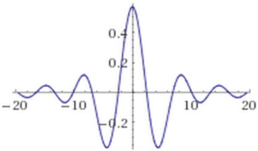
Waves opposed at large distance

- Boundary condition
- E bounded in the plane

Boundary condition – similar to vortices







$$J_1(r+\pi) - J_1(r-\pi)$$

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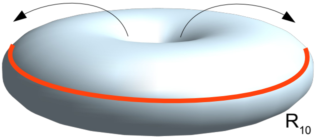
4 eddies

- E also bounded on mirror plane due to cancellation at large distance

More eddies (if needed)

- Bragg mirror boundary condition
- ‘Outgoing waves reflected back’

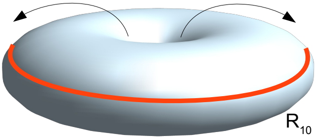
Irrotational quasiparticles – similar to dolphin air rings



click to watch

Curve eddy into a ring

Irrotational quasiparticles – similar to dolphin air rings



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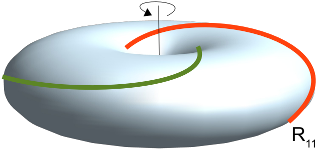
Curve eddy into a ring

add a twist
(chiral quasiparticle)



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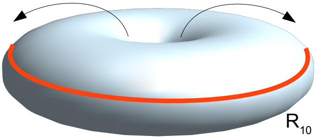

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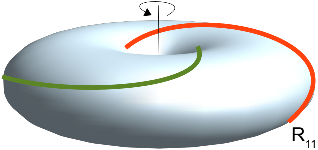

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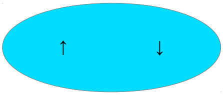
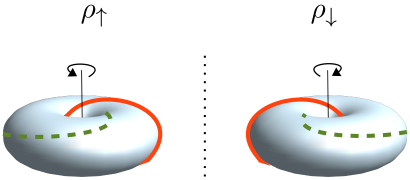
Chiral quasiparticles created in pairs with no angular momentum





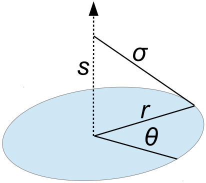
For mathematicians

More formal description than bending the z axis of the eddy

For mathematicians

More formal description than bending the z axis of the eddy

A cylindrical Bessel function is a sum of spherical Bessel functions



$J_1(r)$ cylindrical Bessel function
$j_1(r)$ spherical Bessel function

$$\begin{aligned}\Delta\rho &= A \cos(\omega_o t + \theta)\, J_1(k_r r) \\ &= A' \int_{-\infty}^{\infty} \cos(\omega_o t + \theta)\, j_1(k_r \sigma) ds\end{aligned}$$

Integrand (at fixed s) obeys wave equation
So does the integral

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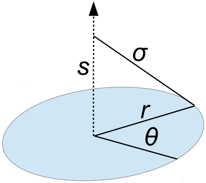

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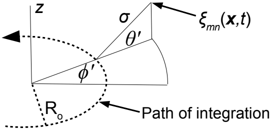


Quasiparticle – integrate on a circular path

$$\Delta\rho_{mn} = A \int \cos(\omega_o t + \theta' - n\phi')\, j_m(k_r \sigma)\, R_o d\phi'$$

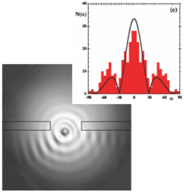
$n = +1$ ($\rho_\uparrow$) – angular momentum in $+z$ direction
$n = -1$ ($\rho_\downarrow$) – angular momentum in $-z$ direction

Agenda

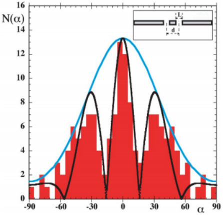
How can a classical fluid system display this quantum-like behaviour?

1. ~~The Bell tests rule out classical models~~
2. ~~The motion isn't Lorentz covariant~~
3. ~~There aren't corresponding phenomena in fluids in 3 dimensions~~
4. Classical systems don't have spin-half symmetry

Single slit



Two-slit



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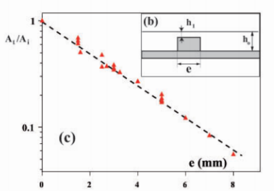
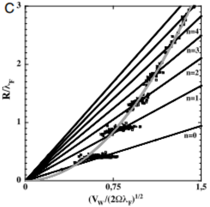


Quantised orbits



Two-valued degree of freedom

Ordinary propagating waves can be superposed

Linear terms don't interact

Quadratic interactions average to zero (unless resonantly coupled)

$$h_1 = A_1 \cos(\omega_1 t - k_1 x)$$
$$h_2 = A_2 \cos(\omega_2 t - k_2 x)$$
$$E \propto \int (h_1 + h_2)^2 dx^3$$
$$E \propto A_1^2 + A_2^2$$

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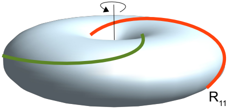

Two-valued degree of freedom rotating in ↑ and ↓ directions

$$\Delta\rho = A_1 \rho_\uparrow + A_2 \rho_\downarrow$$
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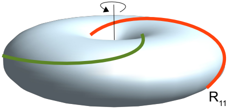


Two-valued degree of freedom
rotating in $\uparrow$ and $\downarrow$ directions

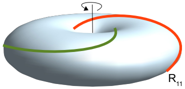
$\Delta\rho = A_1 \rho_\uparrow + A_2 \rho_\downarrow$
$E \propto A_1^2 + A_2^2$

Continuum of degenerate states of constant energy parametrised by ϑ

$\Delta\rho = (\cos\vartheta)\rho_\uparrow + (\sin\vartheta)\rho_\downarrow$ $\quad$ E constant since $\cos^2\vartheta + \sin^2\vartheta = 1$

Rotational symmetry

Degenerate states of constant *E* parametrised by ϑ



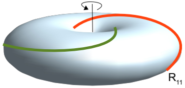
$$\Delta\rho = (\cos\vartheta)\rho_\uparrow + (\sin\vartheta)\rho_\downarrow$$

ϑ	$\Delta\rho$	Net angular momentum in *z* direction
0	$\rho_\uparrow$	$\uparrow$
$\pi/4$	$(\rho_\uparrow + \rho_\downarrow)/\sqrt{2}$	–
$\pi/2$	$\rho_\downarrow$	$\downarrow$
$3\pi/4$	$(-\rho_\uparrow + \rho_\downarrow)/\sqrt{2}$	–
π	$-\rho_\uparrow$	$\uparrow$

- One period of the angular momentum – ρ reverses sign
- Spin-half formalism is very convenient for describing this (completely classical) system

Rotational symmetry

Degenerate states of constant E parametrised by $\frac{1}{2}\theta$



$$\Delta\rho = (\cos \tfrac{1}{2}\theta)\rho_\uparrow + (\sin \tfrac{1}{2}\theta)\rho_\downarrow$$

$\frac{1}{2}\theta$	$\Delta\rho$	Net angular momentum in z direction
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$\frac{1}{2}\theta$ convention used in Bloch formalism

For mathematicians (1)

Continue the excess density $\Delta\rho$ into the complex plane

- $\Delta\rho_{mn} = \Re(\xi_{mn})$ where $\Re$ means real part
- $\xi_{mn} = A \int e^{-i(\omega_o t + m\theta' - n\phi')} j_m(k_r\sigma) R_o d\phi'$
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α_i – complex number. $|\alpha_i|$ is amplitude of component, $\arg(\alpha_i)$ its phase

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Write the superposition as a vector

- $(\alpha_1, \alpha_2) = e^{i(s-\frac{1}{2}\phi)}(\cos\frac{1}{2}\theta, e^{i\phi}\sin\frac{1}{2}\theta)$

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The normalised net angular momentum in the z direction is

- $\sigma_z = \frac{\boldsymbol{\alpha}^*.\,\widehat{\sigma}_z\boldsymbol{\alpha}}{\boldsymbol{\alpha}^*.\boldsymbol{\alpha}} = \cos^2\frac{1}{2}\theta - \sin^2\frac{1}{2}\theta = \cos\theta$

where $\widehat{\sigma}_z$ is the Pauli matrix.

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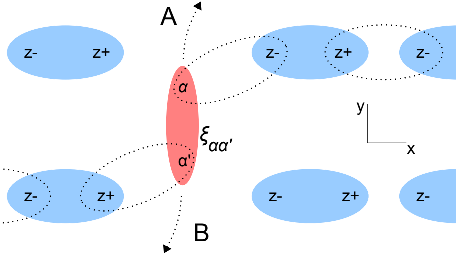
Axis-independent description

- Extend to $\widehat{\sigma}_y$ and $\widehat{\sigma}_x$ in the usual way.

For mathematicians (2)



$\alpha\alpha'$ is a sum of components

- ↑↓ couples as drawn, ↓↑ in opposite horizontal direction
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- Can't get cross-coupling
- Directions are opposed, irrespective of make-up of $\alpha\alpha'$

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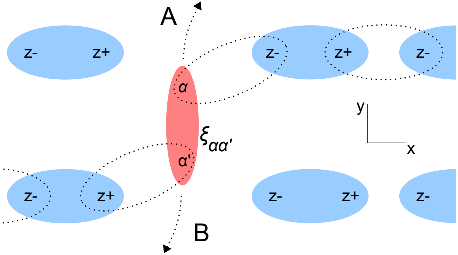

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Extend to arbitrary axes using spin-$\frac{1}{2}$ formalism (see paper for detail)

- If directions are **a** and **b**, correlation $-\mathbf{a}.\mathbf{b}$

R. Brady and R. Anderson. Violation of Bell's inequality in fluid mechanics. ArXiv 1305.6822 (2013)

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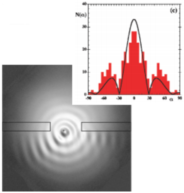

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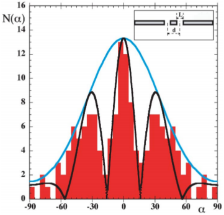

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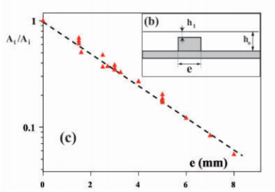

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