# Modelling contrast matching across luminance levels

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## Abstract

The study investigates the modelling of contrast matching functions (SCMF) across various luminance levels, addressing the nonlinear behaviour of the human visual system in perceiving suprathreshold contrasts. Using a comprehensive dataset of contrast matching experiments involving younger and older observers, the research tests existing models and proposes a new hybrid model. The additive model by Kulikowski (1976) and the multiplicative model inspired by Peli et al. (1996) are evaluated against the dataset, revealing their limitations in predicting contrast matching across a broad luminance range. A novel model combining additive and multiplicative elements is introduced, accounting for threshold ratios and differences, and optimised using regression analysis. The proposed model demonstrates superior prediction accuracy, particularly for achromatic contrasts at extreme luminance levels, and holds potential applications in image processing, particularly for high dynamic range (HDR) content adaptation across different luminance conditions.

## Introduction

Suprathreshold contrast vision refers to the ability of the human visual system to perceive differences in luminance or colour when presented with stimuli above the detection threshold. There is extensive literature available on contrast sensitivity or the threshold contrast vision of the human visual system. However, most of our daily visual experiences do not occur at this borderline of visibility; they occur well above it, in what is known as suprathreshold contrast vision. Unlike simple detection, suprathreshold sensitivity measures the ability to perceive and characterise (spatial, chromatic, temporal, etc.) contrasts when stimuli are well above the level at which they can be just barely seen. This level of vision encompasses the vast majority of our visual experiences, where the contrast between objects and their background is significantly greater than the minimum detectable levels. Suprathreshold contrast vision is integral to tasks that require the discrimination of details within the visual scene, such as texture segregation, edge detection, and pattern recognition, which are essential for complex visual tasks like reading, face recognition, and navigating through our environment.

Suprathreshold contrast vision is typically measured and characterised using pair-wise contrast matching (the contrast of a test stimulus is adjusted until it matches the reference stimulus), contrast discrimination (the stimulus with higher or lower contrast is identified) or contrast magnitude estimation (the perceived contrast is estimated on a given numerical scale) experiments. In a previous work, a dataset of contrast matching was presented, which described the non-linear behaviour of the visual system when matching across two luminance levels and the joint effect of spatial frequency and luminance levels [1]. This dataset can be used to further our understanding of contrast perception and model the contrast appearance at higher contrast levels for real-world stimuli as it focuses not just on achromatic contrast but chromatic contrasts as well.

In this work, the focus is on computational modelling of suprathreshold contrast which involves developing a mathematical framework that can predict the perceived contrast based on the physical parameters of the stimulus and the context in which it is viewed. A robust suprathreshold contrast matching function (SCMF) model needs to account for the complexities of human contrast perception, which is influenced by factors such as luminance, spatial frequency, chromatic modulation, temporal properties, etc. The visual system's sensitivity at the threshold level can be extrapolated to predict contrast perception at higher levels but the relationship is likely non-linear. This paper is a step towards proposing a unified contrast vision model as it aims to establish a relationship between the mathematical models of contrast sensitivity from literature and spatiochromatic contrast vision at suprathreshold levels.

In this work, the contrast matching data from [1] is used to: (i) test the additive model of suprathreshold contrast introduced by Kulikowski (1976) [2], (ii) test the multiplicative model inspired by work from Peli, Arend & Labianca (1996) [3], (iii) fit values of matched contrast as a linear function of reference contrast and evaluate the statistical significance of the model, and (iv) propose a new model that combines the additive and multiplicative models with model parameters as functions of contrast sensitivity. In addition, new contrast matching data from older observers (with the same methodology and stimuli used in [1]) is used to validate the proposed model. We quantify contrast in terms of *cone contrast* in this work, following the same definition as used in [1, 4, 5, 6].

### Suprathreshold contrast matching dataset

The dataset used in this study originates from a series of contrast-matching experiments conducted with both younger (n=22, mean age: 28 years) and older (n=20, mean age: 65) observer groups. The younger observers' data has previously been reported in [5]. The experiments measured the perceived contrast of a test stimulus on a high-dynamic-range (HDR) screen compared to a reference stimulus presented on a standard dynamic range (SDR) screen at a fixed luminance. The stimuli consisted of Gabor patches at three spatial frequencies (0.5, 2, and 4 cpd) and three chromatic directions (achromatic, red-green, and limeviolet). The test stimulus luminances varied across five levels (0.02, 0.2, 2, 20, and 2000 cd/m<sup>2</sup>) on the HDR display, while the reference stimulus was set at 200 cd/m<sup>2</sup> on the SDR display.

Observers adjusted the test stimulus contrast under different luminance conditions to match the perceived contrast of the reference stimulus across high, medium, and low suprathreshold contrast levels. Each contrast-matching condition was measured 3 to 5 times per observer, with adjustments made until the test and reference stimuli were perceived as having equal contrast. More details of the experiment are presented in [5, 6].

## Additive SCMF model (Kulikowski's)

Kulikowski (1976)'s model [2] is based on the contrast sensitivity assumption in the linear contrast domain. It postulates that the perceived contrast of the reference and the test suprathreshold contrasts is equal to the corresponding physical contrast that is reduced by the threshold contrast (minimum required contrast to perceive that stimulus).

$$C_1 - C_1^t = C_2 - C_2^t \tag{1}$$

where,  $C_1$  and  $C_2$  are the suprathreshold contrasts of the two stimuli at two different luminance levels, and  $C_1^t$  and  $C_2^t$  are the contrast detection thresholds at the corresponding luminance levels. For suprathreshold levels that are high enough, the difference introduced by the threshold values is negligible and thus the perception of the two matching stimuli is equivalent. This was shown to be true when contrast was matched across different luminance levels. However, the range of luminance tested in Kulikowski's work is less than 2 log units, while the contrast matching data from [1] spans 6 log units. Whether Kulikowski's constancy model holds for this wide range of luminance is tested in this section.

The Eq. (1) can be rearranged to predict the matched contrast of the test stimuli to the reference contrast:

$$C_{test} = C_{ref} + \Delta C^t , \quad \Longrightarrow \Delta C^t = C^t_{test} - C^t_{ref}.$$
(2)

The threshold contrasts  $C^t$  are predicted by a contrast sensitivity model that can predict the contrast threshold for Gabor





Figure 1. Kulikowski's SCMF model predictions for younger observers. The coloured data points with  $\pm$ SEM error bars are the measured data points for different suprathreshold contrast levels. The corresponding dashed lines are the model predictions. The grey dashed lines show the threshold contrasts predicted from the age-dependent CSF model [7]. The red text denotes the RMSE error in decibels (dB) for each colour direction and spatial frequency across all luminances and reference levels.

patches with specific spatial frequency, colour modulation, mean luminance, size and for different observers' age in [6]. Eq. (2) is essentially an equation of the straight line with a slope of 1 and the intercept as the difference between the threshold contrasts for reference and test stimuli. The predictions from Kulikowski's model for younger and older observers are shown in Figures 1-2. The model predictions faithfully follow the shape of the trends from the measured data points at higher luminances. For matching at lower test luminances, the model underpredicts the required test contrast needed for equivalent perception of the two contrasts. The prediction of contrast when the test stimulus is at a higher luminance level  $(2,000 \text{ cd/m}^2)$  than the reference  $(200 \text{ cd/m}^2)$  is also underpredicted for achromatic stimuli. The CSF data presented in [4], showed a decrease in achromatic sensitivity at very high light levels. This feature is also present in achromatic contrast matching data but is not well-predicted by the model. The model also shows higher prediction errors for higher frequency yellow-violet stimuli for both older and younger observers.



Figure 2. Kulikowski's SCMF model predictions for older observers. The plots' description and the legend are the same as in Figure 1.

## Multiplicative model (Peli's)

The contrast matching studies presented in Peli et al. (1991) [8] and [3] do not explicitly introduce a mathematical SCMF model, but they have shown their data following the contrast matching predictions in log contrast space. The contrast matching data presented in this work also spans only 2 log units of luminance. In a model inspired by Peli, Arend & Labianca (1996) [3]'s work, the test and reference contrast are related in the log space and Eq. (2) becomes:

$$\log_{10}(C_{test}) = \log_{10}(C_{ref}) + \log_{10}(C_{test}^{t}) - \log_{10}(C_{ref}^{t}),$$
(3a)

$$\implies C_{test} = r^t C_{ref}, \quad \implies r^t = \frac{C_{test}}{C_{ref}^t}.$$
 (3b)

Similar to the additive model, the threshold contrasts  $C^{t}$  is predicted from an age-dependent contrast sensitivity model [6]. Eq. (3) models the matched test contrast as linearly proportional to the reference contrast with the ratio of the test and reference threshold contrasts as the gradient of this relationship. The predictions from Peli's model for younger and older observers are shown in Figures 3-4. The lines of matching contrast for different suprathreshold levels (high, medium, and low) are predicted with a constant offset in the log contrast axis. The shape of these curves along luminance follows the trend characteristic to the DeVries-Rose to Weber region transition curves. The model predictions are well-aligned with the higher required test contrast for lower luminance levels for lower suprathreshold contrasts (purple curves in Figures 3-4), but overpredict the test contrast for medium to high suprathreshold contrasts. This model is also able to predict the increase in test contrast for high-luminance matches. The predictions from Kulikowski's and Peli's models are suited for different ranges of the stimuli parameter space and thus, combining the strengths of both models is needed to predict the full range of stimuli tested in this work.



Figure 3. Peli's SCMF model predictions for younger observers. The plots' description and the legend are the same as in Figure 1.



Luminance (cd/m<sup>2</sup>) of test stimulus

Figure 4. Peli's SCMF model predictions for older observers. The plots' description and the legend are the same as in Figure 1.

#### Empirical modelling of SCMF

Neither the additive ([2]) nor the multiplicative ([3]) models from the literature could fully explain the contrast matching trends across luminance levels from the measured contrast matching data. Figure 5 shows the matched test contrasts with respect to the reference contrasts for the mean data from the younger observer group. The relationship between the two contrasts is linear for each combination of spatial frequency and luminance level for all three colour directions but with different slopes and offsets, similar to the results in Biondini and De Mattiello (1985) [9]. To investigate this relationship, a straight line was fitted to the matching curves and the best-fitted values of slope and intercept were estimated for each observer in the younger group.



Figure 5. Linear relationship of contrast matching data for younger observers. The linear function between the matched test and the reference contrast data is shown for different spatial frequencies and luminance levels. From the data, it is clear that the two contrast values are linearly related and the slope and intercept of these matches depend on the spatial frequency and the luminance of the stimuli. The dashed lines represent the reference unity slope.

The polyfit function in MATLAB was used to fit the following equation for individual observers:

$$C_{test} = \delta C_{ref} + \alpha, \tag{4}$$

where  $\delta$  is the slope and  $\alpha$  is the intercept of the linear function between test and reference contrast. In the statistical tests shown in the Appendix, no effect of age group was shown except for the yellow-violet colour direction. The effect of age on yellowviolet contrast matching can be explained by the higher loss of yellow-violet contrast sensitivity in older observers [5]. Thus, a model that depends on the threshold contrasts of reference and test stimuli, should be able to adequately compensate for these differences. To test this, the data from only the younger group was used in this contrast matching modelling analysis and the results were validated on the data from the older observer group.

With the per-observer fitted slopes and intercept values, a multiple linear regression analysis was performed within each colour group to find the statistical significance of the effect of spatial frequency, luminance difference, the difference in threshold contrasts ( $\Delta C^t$ ), and the ratio of the threshold contrasts ( $r^t$ ) on the fitted values of slope and intercept. The data spanned three spatial frequencies, six luminance levels, and three colour directions. To determine how these factors affected the slope and intercept of contrast matching, a linear mixed effect model (LMEM) was fitted to data from each of the three colour directions. The model included the main effects of the four independent variables as well as their two-way and three-way interaction terms. The best-fitting linear mixed-effect model was determined by a backward procedure of removing the factors that did not contribute significantly. In other words, none of the remaining effects or interactions can be removed without reducing the variance explained. The statistical analyses were performed in R using packages lmerTest, caret, performance, and see [10, 11, 12, 13, 14, 15, 16]. The threshold ratio variable was found to be significant on average both as the main effect and in interaction terms for the matching slope values. Similarly, the effect of threshold difference and its interaction terms were significant for the intercept values. Following this analysis, a model is proposed that combines the strengths of both the additive and the multiplicative models of contrast matching to better predict the matching data across different luminance levels.

## Luminance-adaptive SCMF

A new model of contrast matching was proposed where the test contrast is a power function of the reference contrast, taking inspiration from Stevens' power law which stipulates that the strength of perception is a power function of the physical intensity of the stimulus [17]. In the case of contrast matching, the magnitude of the test contrast represents the response or the perception and the reference contrast represents the intensity of the stimulus. The multiplier ( $\delta(.)$ ) is a function of the threshold ratio ( $r^t$  in Eq. (3)), and the intercept ( $\alpha(.)$ ) is a function of the threshold differences ( $\Delta C^t$  in Eq. (2)):

$$C_{test} = \delta(r^t)(C_{ref})^{\gamma} + \alpha(\Delta C^t), \qquad (5a)$$

$$\delta(r^t) = \delta_m r^t + \delta_i, \tag{5b}$$

$$\alpha(\Delta C^t) = \alpha_m \Delta C^t + \alpha_i, \qquad (5c)$$

where  $\gamma$ ,  $\delta_m$ ,  $\delta_i$ ,  $\alpha_m$ , and  $\alpha_i$  are the parameters of the model with different values for each colour direction.  $\gamma$  represents the value of the exponent to correct for the non-linearity between the two matched contrasts. The  $\delta(.)$  function represents a scaling factor of the reference contrast as a function of the threshold ratio with  $\delta_m$ , and  $\delta_i$  as the slopes and the intercepts of the linear relationship respectively. Finally, the  $\alpha(.)$  function scales the contribution of an additional term — the difference in thresholds - with  $\alpha_m$ , and  $\alpha_i$  as the slopes and the intercepts of the linear relationship respectively.

The parameter values were optimised using fminsearch in MATLAB with the RMSE error, between the measured test contrast values from the data and the predicted values from Eq. (5), as the objective function. The data used for optimisation was the mean contrast matching data from the younger observer group. The optimised values of the parameters are listed in Table 1 and the predicted test contrasts are shown in Figures 6-7.

Table 1: Optimised values of parameters for the proposed CMF model

Color direction	$\delta_m$	$\delta_i$	$\alpha_m$	$\alpha_i$	γ
Achromatic	-0.0116	2.6194	1.9299	-2.0204	0.0940
Red – Green	0.0300	0.1490	0.2525	-0.0876	0.1367
Yellow – Violet	0.1164	8.4239	-0.4673	-8.1108	0.0211



Luminance (cd/m<sup>2</sup>) of test stimulus

Figure 6. Proposed SCMF model predictions for younger observers. The plots' description and the legend are the same as in Figure 1.



Figure 7. Proposed SCMF model predictions for older observers. The plots' description and the legend are the same as in Figure 1.

#### Disussion

The numerical errors from predictions of all the tested models are shown in Table 2 for both younger and older observers. The predictions from Kulikowski's and Peli's models were only dependent on the thresholds predicted by the CSF model and no parameters were optimised. For these models from the literature, the prediction errors for older observers were smaller compared to younger observers. It should be noted that for older observers, more low luminance data points were removed as the observers were not able to match the test contrast within the display gamut and this is not accounted for in the mean RMSE value.

Table 2:	Summary	of CMF	models
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Model	Summary	Eqs.	Mean RMSE (dB)		
Kulikowski	No optimisation	Eq. 2	Younger group	4.4331	
			Older group	3.0990	
Peli	No optimisation	Eq. 3	Younger group	6.8856	
			Older group	3.8968	
Proposed	Optimisation with 15 free parameters listed in Table 1	Eq. 5	Younger group	2.2075	
			Older group	2.7768	

In the proposed SCMF model, 5 free parameters for each of the three colour directions were optimised and the model was trained for younger observers only. The proposed model has the lowest prediction error among the three tested models for younger observers. This is not surprising as the model parameters are fitted to the training set. Qualitatively, the shape of the matching curves predicted by the proposed model follows the measurements quite closely as shown in Figures 6-7. The elevated test contrast for both very low and very high luminance matching was predicted well especially for achromatic contrasts. The mean error in predictions for the unseen older observer data is comparable but slightly larger than that from Kulikowski's model. This could also be due to the model feature where the values of the predicted test contrasts are capped at the corresponding threshold values, while naive models like Kulikowski's allow for test contrast to be predicted lower than the threshold, or in the sub-threshold region.

## Conclusions

The proposed model shows a promising direction to unify suprathreshold and threshold contrast models for very large dynamic ranges. Future works could aim to measure similar datasets for higher spatial and temporal frequencies as well to test the validity of the hybrid additive + multiplicative model. Currently, the three colour directions are treated independently of each other and as correctly pointed out by Switkes and Crognale (1999) [18], the quantitative measurement of qualitative chromatic contrast perception can be quite tricky. This could be a very interesting venue for future research where matching of contrast appearance across different colour directions could be characterised and lead to useful applications such as chroma-subsampling.

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## Appendix: SCMF statistical tests

Table 3: Data transforms and pre-processing

Variable         Unit / Category         Type         Transform / Contrasts coding           Test contrast         Cone contrast         Continuous         Box car transformed to rem heteroscedasticity           Contrast level         Low / Medium / High         Categorical         Low vs medium (0.5, 0.5, Medium vs high (0, -0.5, 0.5, Medium vs meter (cd/m <sup>2</sup> )           Age group         Transform / Contrasts         Continuous         Base 10 log           Subjects         Anonymous observer ID         Categorical         Simple coding (-0.5, 0.5). In cept = overall mean			1	1
Test contrast     Cone contrast     Continuous     Box car transformed to rem heteroscedasticity       Contrast level     Low / Medium / High     Categorical     Low vs medium (-0.5, 0.5, Medium vs high (0, -0.5, 0.5, Medium vs medium content)       Luminance     Candela per square     Continuous     Base 10 log       meter (cdm <sup>2</sup> )     meter (cdm <sup>2</sup> )     Gategorical     Simple coding (-0.5, 0.5). In cept = overall mean       Subjects     Anonymous observer ID     Categorical     Simple coding (-0.5, 0.5). In cept = overall mean	Variable	Unit / Category	Туре	Transform / Contrasts coding
Contrast level         Low / Medium / High         Categorical         Low vs medium (-0.5. 0.5, Medium vs high (0, -0.5, 0.5           Spatial         Cycles per visual degree         Continuous         Base 2 log           frequency         (cpd)         Base 10 log           Luminace         Candela per square         Continuous         Base 10 log           meter (cdm <sup>2</sup> )         Age group         Younger / Older         Categorical         Simple coding (-0.5, 0.5). In cept = overall mean           Subjects         Anonymous observer ID         Categorical         Simple coding (-0.5, 0.5). In cept = overall mean	Test contrast	Cone contrast	Continuous	Box car transformed to remove heteroscedasticity
Spatial         Cycles per visual degree         Continuous         Base 2 log           frequency         (cpd)           Luminance         Candela per square         Continuous         Base 10 log           meter (cd/m <sup>2</sup> )         meter (cd/m <sup>2</sup> )         Gategorical         Simple coding (-0.5, 0.5). Ir cept = overall mean           Subjects         Anonymous observer ID         Categorical         Simple coding (-0.5, 0.5).	Contrast level	Low / Medium / High	Categorical	Low vs medium (-0.5, 0.5, 0). Medium vs high (0, -0.5, 0.5)
Luminance         Candela per square         Continuous         Base 10 log           meter (cd/m <sup>2</sup> )         Age group         Younger / Older         Categorical         Simple coding (-0.5, 0.5). In cept = overall mean           Subjects         Anonymous observer ID         Categorical         Categorical	Spatial frequency	Cycles per visual degree ( cpd)	Continuous	Base 2 log
Age group         Younger / Ölder         Categorical         Simple coding (-0.5, 0.5). Ir cept = overall mean           Subjects         Anonymous observer ID         Categorical         Categorical	Luminance	Candela per square meter ( cd/m <sup>2</sup> )	Continuous	Base 10 log
Subjects Anonymous observer ID Categorical	Age group	Younger / Older	Categorical	Simple coding (-0.5, 0.5). Inter- cept = overall mean
	Subjects	Anonymous observer ID	Categorical	•

## Achromatic contrast matching

Best Model: Test contrast ~ Frequency + Luminance + Contrast level + (1 + Frequency + Luminance — Subjects) + Frequency : Luminance + Luminance : Contrast level

Table 4: Estimated model fixed effects. p-values estimated via t-tests using the Satterthwaite approximations to degrees of freedom

Effect	Estimate	Std. Error	df	t-value	$\Pr(> t )$	
Intercept	-1.133	0.043	30.4	-26.3	<2.00E-16	***
Frequency	0.1	0.017	46	5.9	0	***
Luminance	-0.08	0.018	32.2	-4.4	0.0001	***
Contrast level (low vs med)	0.52	0.034	1509	15.2	<2.00E-16	***
Contrast level (med vs high)	0.572	0.035	1509	16.2	<2.00E-16	***
Frequency : Luminance	-0.034	0.005	1529	-6.4	0	***
Luminance : Contrast level (low vs med)	0.153	0.018	1509	8.6	<2.00E-16	***
Luminance : Contrast level (med vs high)	0.154	0.018	1508	8.4	<2.00E-16	***

Signif. codes: '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05

## Red-green contrast matching

Best Model: Test contrast ~ Frequency + Luminance + Contrast level + (1 + Frequency + Luminance - Subjects) + Frequency : Luminance + Frequency : Contrast level + Luminance : Contrast level

Table 5: Estimated model fixed effects. p-values estimated via t-tests using the Satterthwaite approximations to degrees of freedom

Effect	Estimate	Std. Error	df	t-value	$\Pr(> t )$	
Intercept	-3.107	0.096	30.7	-32.5	<2.00E-16	***
Frequency	0.203	0.038	41.8	5.4	0	***
Luminance	-0.939	0.04	32.9	-23.6	<2.00E-16	***
Contrast level (low vs med)	1.06	0.077	1515	13.9	<2.00E-16	***
Contrast level (med vs high)	1.065	0.079	1516	13.4	<2.00E-16	***
Frequency : Luminance	-0.068	0.012	1542	-5.8	0	***
Frequency : Contrast level (low vs med)	-0.162	0.049	1514	-3.3	0.0011	**
Frequency : Contrast level (med vs high)	-0.158	0.049	1513	-3.2	0.0015	**
Luminance : Contrast level (low vs med)	0.376	0.039	1514	9.7	<2.00E-16	***
Luminance : Contrast level (med vs high)	0.351	0.04	1514	8.8	<2.00E-16	***

Signif. codes: '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05

#### Yellow-violet contrast matching

Best Model: Test contrast  $\sim$  Age group + Frequency + Luminance + Contrast level + (1 + Frequency + Luminance — Subjects) + Age group : Frequency + Age group : Luminance + Frequency : Luminance + Luminance : Contrast level

Table 6: Estimated model fixed effects. p-values estimated via t-tests using the Satterthwaite approximations to degrees of freedom

Effect	Estimate	Std. Error	df	t-value	$\Pr(> t )$	
Intercept	-0.6	0.047	30.4	-12.7	0	***
Frequency	-0.101	0.019	44.4	-5.3	0	***
Luminance	-0.231	0.016	33.6	-14.7	0	***
Contrast level (low vs med)	0.406	0.033	1196	12.4	<2.00E-16	***
Contrast level (med vs high)	0.449	0.037	1198	12.2	<2.00E-16	***
Frequency : Luminance	-0.013	0.005	1220	-2.6	0.0088	**
Luminance : Contrast level (low vs med)	0.069	0.016	1194	4.4	0	***
Luminance : Contrast level (med vs high)	0.061	0.017	1196	3.5	0.0004	***
Age group	0.253	0.094	29.9	2.7	0.0116	*
Age group : Frequency	0.083	0.034	29.3	2.4	0.0228	*
Age group : Luminance	-0.067	0.031	31.9	-2.2	0.0385	*

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