Why bouncing droplets are a pretty good model for quantum mechanics

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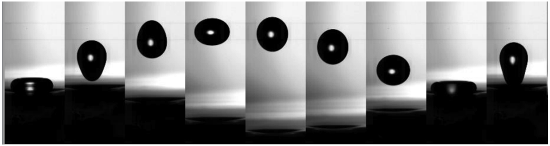
Bouncing droplets

1. Introduction
2. Basic characteristics of the experiment
3. Inverse square force
4. Walker
5. Predicted analogue of the magnetic field
6. Quantum mechanical behaviour
7. Transverse waves in a fluid
8. Further work and conclusion

Bouncing droplet experiments



Experiments intrigued the world over the last 10 years

- Analogue of phenomena in quantum mechanics

Growing body of papers in *Nature, JFM, Phys. Rev. Lett.*

We offer a consistent explanation

- Experimental analogue of the electromagnetic force
- Analogue of Schrödinger's equation

http://www.youtube.com/watch?v=W9yWv5dqSKk

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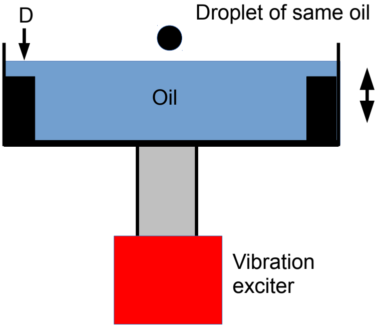
Modern design of the apparatus



Air can't squeeze out from underneath the droplet quickly enough

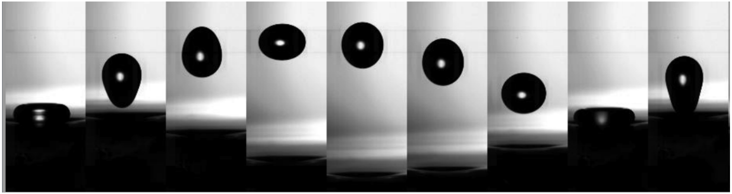
- lubricates horizontal motion

Shallow region *D* is a recent innovation to absorb energy

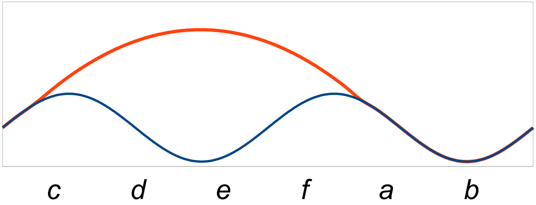
- Can now use low viscosity oils (until droplet starts to evaporate)
- Viscosity relatively unimportant for the phenomena of interest

The bouncing motion

Simple bouncing



Most phenomena of interest at double period



Numerical simulation at $a/g = 3.5\cos(\omega t)$

Monochromatic wave field near a droplet

A propagating wave is a sum of two standing waves

$$\cos(kx - \omega t) \;=\; \underset{A}{\cos(kx)\cos(\omega t)} \;+\; \underset{B}{\sin(kx)\sin(\omega t)}$$

The vertical shaking amplifies *A* and reduces its wave speed

- *A* interacts with the droplet and determines its motion

The vertical shaking drains energy out of *B* and increases its speed

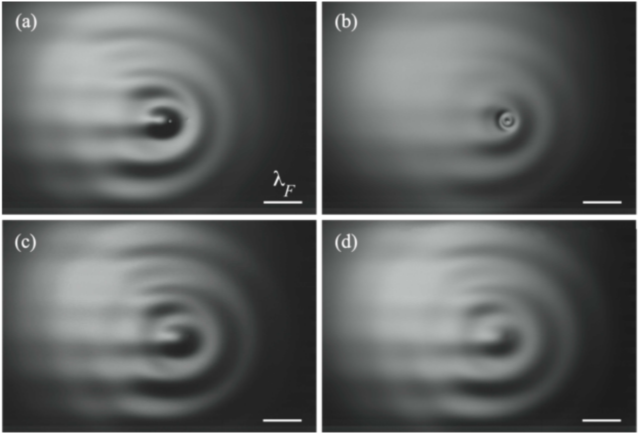
- *B* can contribute to the waves further from the droplet

Standard to neglect *B*

- Standing Bessel function solution to the wave equation

$$h \;=\; \cos(\omega t)\, J_0\left(\frac{\omega r}{c}\right)$$

Photograph of the wave field (‘ghost droplet’)



Droplet coalesces and ‘dies’ in (b)

- Energy can’t escape (band gap)
- Standing Bessel function solution to wave equation

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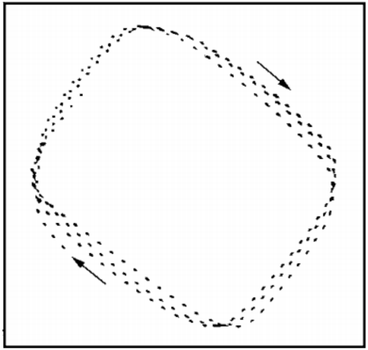
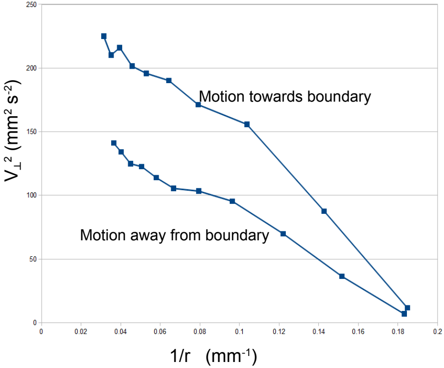
Deflection from boundary of dish

We blew up a stroboscopic photo (Protière 2006) and measured it

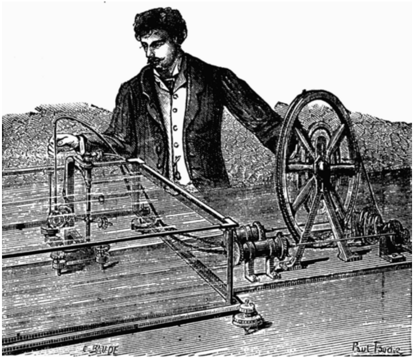




Force is inverse square near the boundary, $\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 - K/r$
Angle of incidence $\neq$ angle of reflection

Inverse square force

Carl Bjerknes predicted the inverse square force in 1875 and demonstrated it experimentally in 1880.



Pistons create pressure waves

Bubbles pulsate

Measured an inverse square force

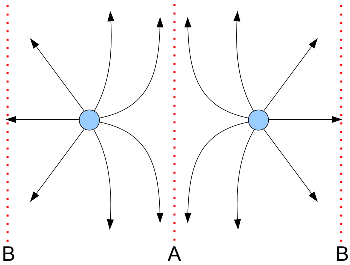
- In-phase pulsations attract
- Antiphase pulsations repel

Degassing oils

The secondary Bjerknes force is used for degassing oils by applying ultrasonic vibration

Bubbles pulsate in phase with one another, attract and merge

Origin of the secondary Bjerknes force



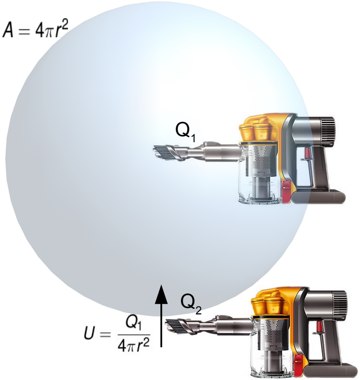
In-phase pulsations

- Greater average flow speed near *A*
- Reduced Bernoulli pressure
- Force of attraction

Droplets repelled from boundary because image droplet is antiphase.

Magnitude of the secondary Bjerknes force (1)

Secondary Bjerknes force is average over a cycle



Flow speed from 1

$$U = \frac{Q_1}{4\pi r^2}$$

Momentum ingested by 2

$$\frac{dp}{dt} = \rho U Q_2 = -\rho \frac{Q_1 Q_2}{4\pi r^2}$$

Magnitude of the secondary Bjerknes force (2)

Our calculation for the resonant case where maximum speed $\sim c$

Secondary Bjerknes force

$$
\begin{aligned}
F &= \alpha \frac{\text{ƀ} c}{r^2} \\
\alpha &\sim 1 \\
\text{ƀ} &= \frac{mc^2}{\omega}
\end{aligned}
$$

Compare force between electrons

$$
\begin{aligned}
F &= \alpha \frac{\hbar c}{r^2} \\
\alpha &\approx \frac{1}{137.036} \\
\hbar &= \frac{mc^2}{\omega}
\end{aligned}
$$

The fine structure constant of the secondary Bjerknes force is two orders of magnitude larger than for an electron

ƀ is an analogue of Planck's reduced constant

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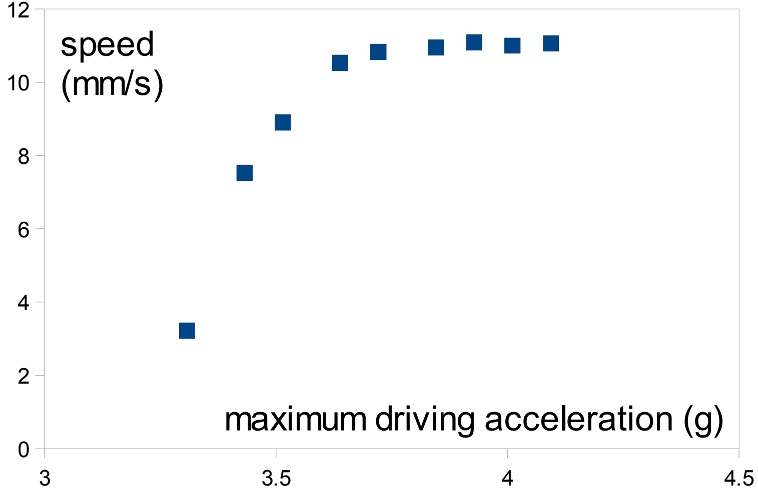
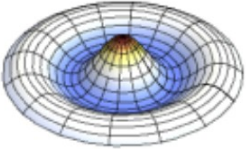
Walker speed



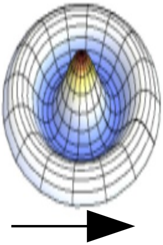
Two ways to calculate the field of a walker

Conventional approach



Each bounce excites a standing Bessel function solution to the wave equation, which decays slowly due to absorption at the boundary and band gap effects. Simulate in a computer.

Symmetry approach

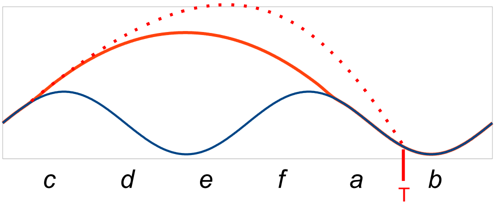


Bessel function $f(x, y, t)$ obeys wave equation. The wave equation is symmetric under Lorentz transformation, so that $f(x', y', t')$ is another solution, where

$$
\begin{aligned}
x' &= \gamma(x - vt) \\
t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\
\gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{aligned}
$$

A second-order scale symmetry is also involved. Decay is slow because of parametric effects.

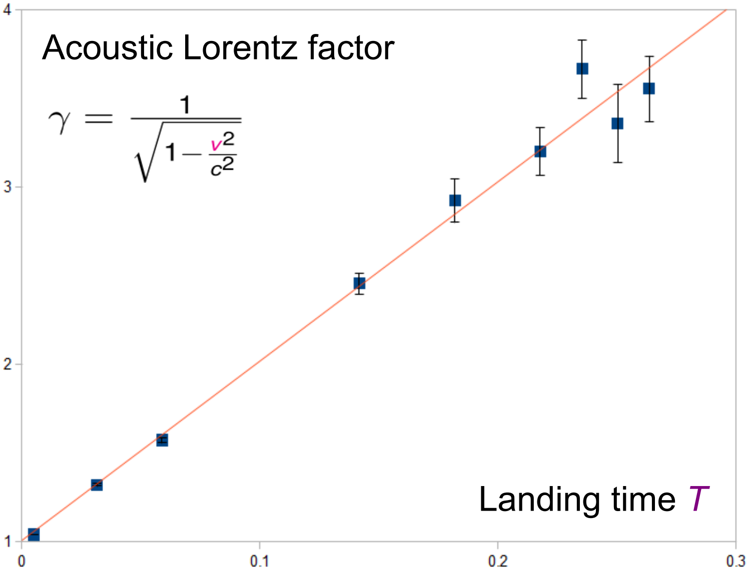
Increase forcing acceleration



Greater amplitude (dotted)

- The droplet lands later in the cycle
- The walker velocity increases

We re-plot the original (2005) experimental results



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Prediction

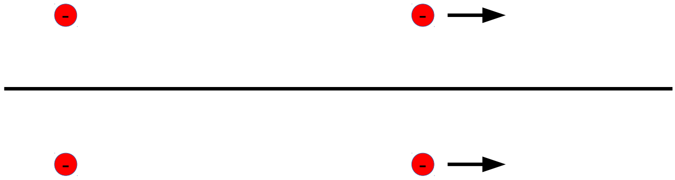
The wave equation is symmetric under Lorentz transformation

The experimental measurements suggest the wave field has the same symmetry

So we predict the inverse square secondary Bjerknes force must also be symmetric under acoustic Lorentz transformation

Borrow calculation from electromagnetism

"Inverse square force + Lorentz symmetry = Maxwell's equations"



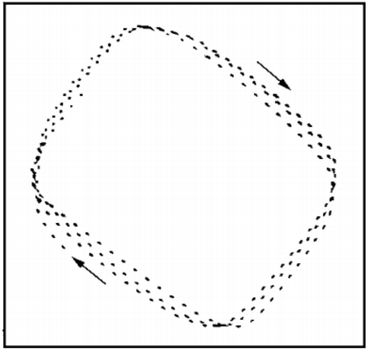
Coulomb force repels

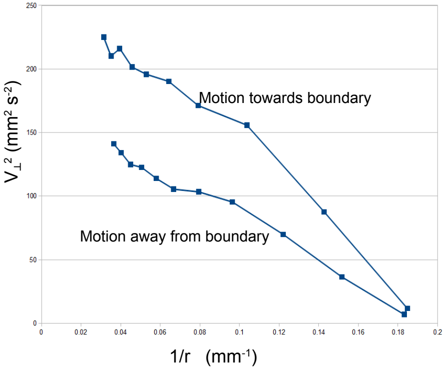
Coulomb force repels
Magnetic force attracts

In this geometry, magnetic force = $\frac{v^2}{c^2} \times$ Coulomb force

So total force reduced by factor $1 - \frac{v^2}{c^2}$

Experimental test

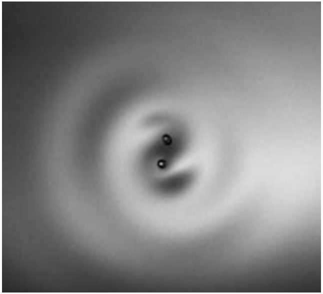




Droplet moves faster parallel to the boundary after reflection
And we see a reduced force corresponding to $v \sim 0.5c$
Consistent with our prediction of an analogue of the magnetic force

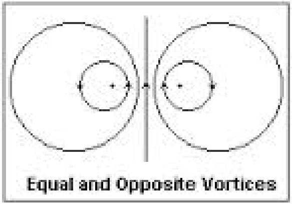
Prediction for a rotating droplet pair

Rotating droplet pair



Interaction with image in the boundary

- Static forces cancel (droplets are antiphase)
- Magnetic-like attraction remains
- Predict fine structure constant $\sim$ 1/20
- Couder observed droplet pair ‘hopscotch’



Visualising the mechanism

- Flow field – Bessel function J_1
- Rotates around the centre
- Attracted to image in boundary, like two vortices

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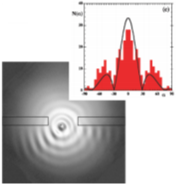
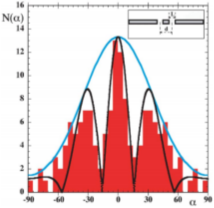
Experimental results

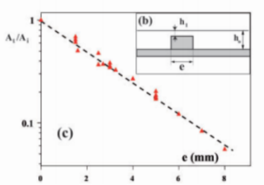
Single slit



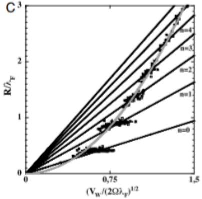
Two-slit



Tunnelling



Quantised orbits



Y Couder, E Fort 'Single-Particle Diffraction and Interference at a Macroscopic Scale' PRL 97 154101 (2006)
A Eddi, E Fort, F Moisi, Y Couder 'Unpredictable tunneling of a classical wave-particle association' PRL 102, 240401 (2009)
E Fort et al 'Path-memory induced quantization of classical orbits' PNAS 107 41 17515-17520 (2010)

Moving wave field

Factorise the field of a droplet

Stationary droplet
$$\begin{aligned} h &= \psi\,\chi \\ \psi &= R\cos(-\omega_o t) \\ \chi &= J_0\left(\frac{\omega_o r}{c}\right) \end{aligned}$$

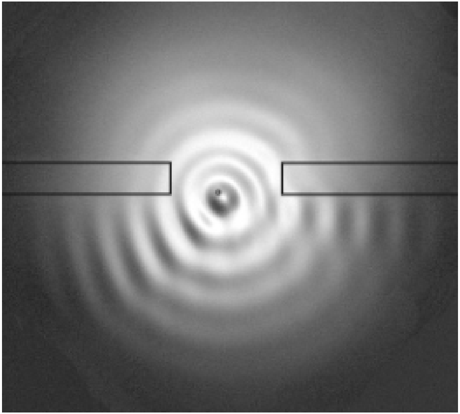
Lorentz transform
$$\begin{aligned} \psi &= R\cos(-\omega_o t') \\ &= R\cos(kx - \omega t) \end{aligned}$$

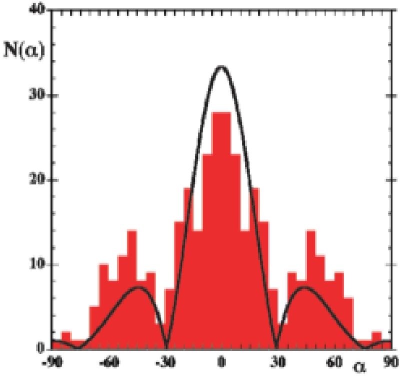
where
$$\begin{aligned} k &= \frac{\gamma\omega_o}{c^2}\, v_x \\ \omega &= \gamma\omega_o \end{aligned}$$

Wavelength (de Broglie!)
$$\begin{aligned} \lambda &= \frac{2\pi}{k} = \frac{b}{p} \\ b &= 2\pi\frac{mc^2}{\omega} \end{aligned}$$

b is the same analogue of the Planck constant which we saw in the inverse square force

Single-slit diffraction





ψ modulates the amplitude of the wave field
Can be likened to the modulation of a carrier wave
Wavelength visible in photograph, matches diffraction pattern

Klein-Gordon and Schödinger equations

$\psi = R\cos(\omega_o t)$ obeys

$$\frac{\partial^2\psi}{\partial t^2} = -\omega_o^2\psi$$

But the motion has Lorentz symmetry
so we need a Lorentz covariant equation, which is

$$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi = \omega_o^2\psi$$

Klein-Gordon equation

Schrödinger equation is a low-velocity approximation to this equation

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Motivation

We predicted that the secondary Bjerknes force obeys Maxwell's equations with an acoustic value for *c*

- Experiments on the magnetic analogue support this prediciton

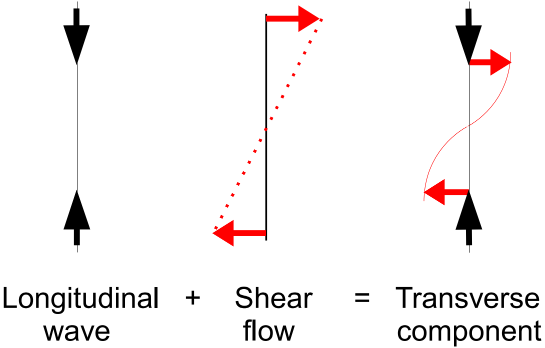
Maxwell's equations have solutions in the form of polarised propagating waves

Have such waves ever been observed in a fluid?

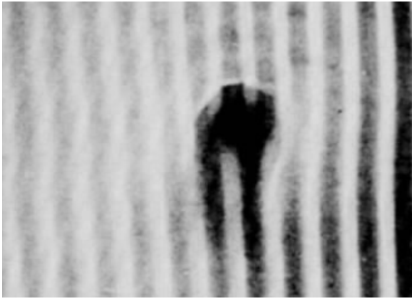
1957 - Lev Landau predicted transverse waves in superfluid helium, leading to many experiments since (but complicated maths)

Prior to 1957, many scientists thought them impossible in a fluid.

Here is a much simpler model of transverse sound:



Transverse water waves (Michael Berry 1980)



- Observed water waves going past a vortex
- Waves induce flows in $\pm x$ direction
- Interact with the shear flow
- Component in the $\pm y$ direction
- Analogue of Aharonov-Bohm effect

Called a 'phase vortex'. Associated features:

- More wavelengths above the centre than below it (waves travel faster with the flow)
- Berry showed it has angular momentum

Same phenomenon (but less photogenic) in sound waves near a vortex in 3D

Polarised propagating waves

What happens if we modulate the amplitude of the waves in Berry's experiment?

- Works just like radio modulation
- Modulation has real and imaginary components and obeys the wave equation
- Angular momentum oscillates at the same frequency
- Behaves like a polarised propagating wave!

CHSH measurements

Properties of a polarised wave in a fluid

Observable quantity	Wave	Pre-existing shear flows
Frequency	X	
Wavelengh	X	
Polarisation		X

Just like the signal in a radio transmission

- polarisation is a property of the carrier wave, not the signal

CHSH measurements

- In 1969, John Clauser, Michael Horne, Abner Shimony and Richard Holt assumed the contrary for light waves
- Assumed all properties ‘carried by and localised within’ a photon
- Measurements showed their assumptions were false

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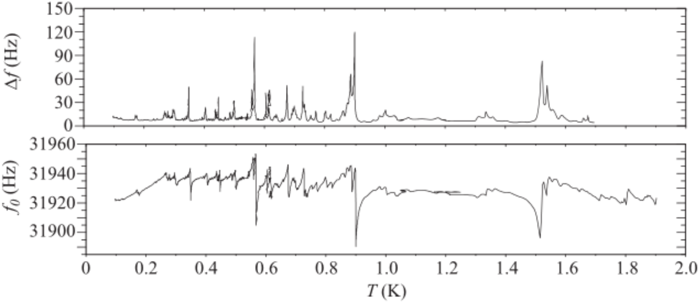
Further work

Similar results in three dimensions in superfluid helium

- Isotropic model of transverse sound based on Euler's equation
- Analogues of Maxwell and Schrödinger both apply to rotons

Possible explanation for absorption spectra measured in second sound



Open problem for the last 10 years: why do bouncing droplets behave like quantum mechanical particles?

We've found an elegant explanation!

- Lorentz covariant oscillating phenomena
- Experimental analogue of the electromagnetic interaction
- Analogue of Schrödinger's equation too

Our model points to fascinating further problems in superfluids and even the foundations of quantum mechanics

See arxiv 1401.4356 or email us for draft of superfluid work