

# Modules, Abstract Types, and Distributed Versioning

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## Abstract

In a wide-area distributed system it is often impractical to synchronise software updates, so one must deal with many coexisting versions. We study static typing support for modular wide-area programming, modelling separate compilation/linking and execution of programs that interact along typed channels. Interaction may involve communication of values of abstract types; we provide the developer with fine-grain versioning control of these types to support interoperation of old and new code. The system makes use of a second-class module system with singleton kinds; we give a novel operational semantics for separate compilation/linking and execution and prove soundness.

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# 1 Introduction

**Background** Module systems provide an important tool for structuring large programs, both to express their conceptual structure and to support separate compilation and linking. They have been much studied – most relevant to this paper is a line of work on ML-style modules [MTH90], with *structures* (collections of named types, values and substructures) and *functors*, which are parameterised structures. A key issue is the treatment of sharing equality for abstract types. The original ML static semantics involved explicit generation of new type names; the translucent sums/manifest types of Harper and Lillibridge [HL94] and Leroy [Ler94] showed that more type-theoretic treatments were possible and could be expressed using the machinery of singleton kinds. These works treat separate compilation and linking either implicitly or as applications of higher-order functors. Cardelli [Car97] gives a more explicit model of linking, though without abstract types or parameterised modules. Various aspects of modularity and linking have been investigated more recently, e.g. in [BA99, Dro00, Dug00, FF98, GM99, HWC00].

**Problem** The works cited above focus on development of single sequential programs. In this paper we address issues arising from wide-area distributed programming. There are two key differences:

1. We must deal with whole programs that interact with each other at arbitrary interfaces, not simply programs that interact with the outside world at the fixed types of library functions.
2. In wide-area distributed systems it is often impractical to synchronise software updates, so we must deal with many coexisting versions of programs that interact with each other.

Further, in a wide-area system it becomes particularly important to detect errors early in the software development process; it is therefore worth using static typing as far as possible – and we would like to see how far. The language described here does not involve any run-time checks (save for the implicit equality testing of channel names that is intrinsic to channel-based interaction).

**Outline** In the following two sections we develop a model language, equipped with static and dynamic semantics, that supports:

1. Separate compilation/linking and execution for modular programs that interact along typed channels.
2. Interaction along channels carrying abstract types.
3. Version control of those abstract types, for interoperation of old and new code.

It consists of a core expression language, a module system, and an imperative command-line language. The core language is taken to be an asynchronous  $\pi$ -calculus, providing a concise form of typed interaction between whole programs. The module system is based on a standard second-class system, with first-order functors and using singleton kinds. It is extended with new channel creation and with a novel type coercion for versioning. The command-line language allows compiling and linking (here merged into a single step) of a module to an object file, and executing such files with ‘main’ components. The semantics requires careful treatment of new name creation, both for channel names and type names – in brief, abstract types must be compiled to manifest types with new-bound type names; these are used by the versioning coercion. New type names arise in module reduction steps such as

$$\begin{array}{l} [T, e] \text{ as } \exists X :: \text{Type}. T' \\ \longrightarrow \\ \text{New } Y :: \text{EQ}(T) \text{ in} \\ [Y, e] \text{ as } \exists X :: \text{EQ}(Y). T' \end{array}$$

Here the first line is an existential package with a hidden representation type  $T$  and an operation part  $e$ ; the reduction creates a new name  $Y$  for the representation type, recording it in both the term and

signature parts of a manifestly-typed package. The scope of the new type name must extrude to cover both compiled object code (as stored in files) and the running system.

The versioning coercion allows the developer, when building a module expression  $m$  and assigning the result to a filename  $B$ , to declare that abstract types provided by  $m$  should be made compatible (if possible) with the previous version of the module stored in  $A$ . This is written  $B := (m \text{ with! } A)$ .

We prove subject reduction for both build-time and run-time semantics, show the absence of run-time errors, and (in a simple case) relate monolithic and separately-compiled programs, giving a tight link between the singleton-kind system and the type name generation occurring during module reduction. Section 4 concludes with discussion of related work and future directions. The full definitions of the typing judgements, build-time semantics and run-time semantics for the language are given in Appendix A. Details of the soundness proofs are in Appendix B. This is a full version of the paper [Sew01].

The original motivation for this paper arose from work with Pierce, Unyapoth and Wojciechowski on NOMADIC PICT [SWP98, WS00, Woj00, US01], a distributed programming language designed to express infrastructure algorithms for location-independent communication between mobile computations. NOMADIC PICT is based on a distributed process calculus (Nomadic  $\pi$ ) following the concurrent PICT language of Pierce and Turner [PT00], based on the  $\pi$ -calculus [MPW92]. Our work on the language, and on distributed infrastructures expressed in it, has shown a clear need for the support outlined above. The problems are more general, however – similar issues would arise in many other settings where programs interact and have long execution lifetimes (as compared with the development cycle). The exact form of interaction is more-or-less orthogonal to the typing issues. NOMADIC PICT has asynchronous message passing to named channels at located agents, but typing would be similar for e.g. Distributed Join-style communication [FGL<sup>+</sup>96], interaction via persistent references, or RPC mechanisms. For simplicity we here adopt standard  $\pi$ -calculus communication, omitting explicit distribution from the core language. This is not a realistic form of wide-area interaction, but it would be straightforward to extend the system with the Nomadic  $\pi$  distribution and mobility primitives, thereby allowing modularisation of our distributed infrastructures.

We are not here dealing with problems of *numbered versions* (some of which will be familiar to those who regularly encounter DLLs), but regard what we do as a necessary preliminary for a satisfactory treatment. We also do not address dynamic linking [GM99, HWC00, Dro00, Dug00] except in the limited sense that a program might be replaced by another that interacts on an overlapping set of channels, nor do we consider hot code upgrade [AWWV95, HN00], in which new code is given access to the datastructures of that being replaced. These are clearly sometimes required, but inevitably lead to the possibility of late link-time or upgrade-time errors; we are exploring how far one can go without them.

## 2 Informal Discussion

This section discusses the issues and our solutions informally, leaving rigorous development to §3.

### 2.1 Interaction along typed channels

We begin by discussing separate compilation/linking for programs that interact by message passing along channels carrying values of simple types. The development is quite straightforward, but it is interesting to see how it must diverge from the single-program case (e.g. as in the flat modules of [Car97]), and it is a necessary preliminary for addressing abstract types later.

To introduce our core language, consider the monolithic  $\pi$ -process

```
new c : int chan in (c!27 | c?x>(c!(x+1)))
```

$C \stackrel{def}{=} $ <pre> module   export c : int chan begin   newval c : int chan end </pre>	$MAIN1 \stackrel{def}{=} $ <pre> module   import c : int chan   export main : proc begin   val main : proc = c!27 end </pre>	$MAIN2 \stackrel{def}{=} $ <pre> module   import c : int chan   export main : proc begin   val main : proc = c?x&gt;(c!(x+1)) end </pre>
---	---	---

Figure 1: Flat modules with `newval`

---

This declares a new channel `c` of type `int chan` (for messages of type `int`); it has an output `c!27` of `27` along `c` in parallel with an input `c?x>(c!(x+1))` that receives an integer on `c`, binding it to `x`, and then outputs `x+1`.

In a distributed system this process might be split, with the output `c!27` in one program and the input `c?x>(c!(x+1))` in another program that will be executed concurrently, perhaps on a different machine. This is expressed in Figure 1 in the simplest flat module system of [Car97], extended with a `newval` declaration for declaring new channels. The whole system would be built and run by compiling `C`, `MAIN1` and `MAIN2` to give some `C'`, `MAIN1'` and `MAIN2'`, linking `C'` with each of the other two, and executing the two resulting complete programs. To ensure statically that communications on `c` are well-typed, even if some other program declares `c` to be of type (say) `(int*int*int) chan`, a new internal channel identifier must be generated when `C` is compiled. This can be represented by taking `C'` to be

```

module
  export c : int chan
begin
  val c : int chan = z
end

```

where `z` is fresh (in a sense made precise below). At this level of abstraction compilation is simply intra-module type checking and new channel name generation. Linking `MAIN1'` and `MAIN2'` with this will give outputs and inputs on `z`.

Unfortunately, to keep soundness we can only generate a new channel name when the type it will carry is known. To see this, suppose one compiled

 $D \stackrel{def}{=}$   

```

module
  import type t
  export val c : t chan
begin
  newval c : t chan
end

```

to give

 $D' \stackrel{def}{=}$   

```

module
  import type t
  export val c : t chan
begin
  val c : t chan = z2
end

```

```

C :=      struct
          newval c : int chan
        end
MAIN1 := (functor(U: sig val c:int chan end) struct main = (U.c)!27 end) C
MAIN2 := (functor(U: sig val c:int chan end) struct main = (U.c)?x>((U.c)!(x+1)) end) C

```

Figure 2: A sequence of build commands with newval

---

with fresh  $z_2$ . One could then successfully build two programs that attempt to interact on  $z_2$  but have a run-time error, as follows. Take

$T_3 \stackrel{def}{=} \begin{array}{l} \text{module} \\ \quad \text{export type t} \\ \text{begin} \\ \quad \text{type t=int*int*int} \\ \text{end} \end{array}$	$T_1 \stackrel{def}{=} \begin{array}{l} \text{module} \\ \quad \text{export type t} \\ \text{begin} \\ \quad \text{type t = int} \\ \text{end} \end{array}$
---	---

and  $MAIN_3$  to be as  $MAIN_1$  but with an output of a triple. Write  $T_3'$ ,  $T_1'$  and  $MAIN_3'$  for the results of compiling these, then consider interaction between the two programs built by linking  $T_3'$ ,  $D'$ ,  $MAIN_3'$  and linking  $T_1'$ ,  $D'$ ,  $MAIN_2'$ .

To avoid dealing with compilation and linking of such modules that declare new channels depending on unresolved imported types, we force a bottom-up build order. We take module expressions  $m$  including structures, functors and functor application. Compiling and linking are merged into ‘build-time’ evaluation of such module expressions and assignment of the resulting module values to filenames. A module being built can refer only to filenames of previously-built modules; it is typechecked with respect to their stored signatures (in examples we sometimes functorise to make signature constraints explicit). Figure 2 recasts the example into a sequence of three assignments, to filenames  $C$ ,  $MAIN_1$  and  $MAIN_2$ . We pun filenames and free module identifiers (both written in an upright font). Module expressions are reduced call-by-value, with a structure

$$C_2 \stackrel{def}{=} \begin{array}{l} \text{struct} \\ \quad \text{newval c : int chan} \\ \text{end} \end{array}$$

reducing to a value

```

struct
  val c : int chan = z
end

```

for a fresh  $z$ .

Now, in what sense must this  $z$  be fresh? It should be distinct from all channels generated earlier, either at build-time or at run-time, across the whole distributed system. In an implementation a globally unique bitstring must be generated. We represent this using name binding and scope extrusion, as in the  $\pi$ -calculus. To a first approximation,  $C_2$  above reduces to

```

New z : int chan in
  struct
    val c : int chan = z          (*)
  end

```

In more detail, we idealise the state of the whole system, including both the running computations and the module values stored in the various developers' filesystems produced by earlier builds, as a triple

$N, F, e$

which should be read as `New N in (F, e)`. Here  $N$  is a type environment of new bindings such as `z : int chan`, the  $F$  models the union of all filesystems<sup>1</sup> as a finite list of pairs of module ids and module values (with no repeated ids –  $F$  will often be regarded as a partial function), and  $e$  is the running process expression. We take these triples up to alpha-renaming of the bindings in  $N$ . Now the effect of developers' command-line build and load commands, and computation steps, can all be regarded as changes of system state. Executing a build command

```
C := struct newval c : int chan end
```

(say) in the state  $N, F, e$  results in the new state  $N', F', e$ , where  $z \notin \text{dom}(N)$  and

$$\begin{aligned} N' &= N, z : \text{int chan} \\ F' &= F \oplus (C \mapsto \text{struct val } c : \text{int chan} = z \text{ end}) \end{aligned}$$

The new value of  $C$  in  $F'$  is obtained by module reduction. Note that for simplicity we are working as much as possible at the source language level, taking a single set of names rather than separate sets of program identifiers and internal channel ids. The  $N$  binding ensures that code written after a build command cannot mention any of the newly generated names. Note also that  $N$  is kept to allow proof of soundness; in an implementation it can be discarded.

For the load command, if  $U$  is a filename referring to a module value `struct val main : proc = e' end` then executing the command

```
run U
```

in the state  $N, F, e$  results in the new state  $N, F, e | e'$  in which  $e'$  is put in parallel with the existing running computation, allowing them to interact.

The computation reductions of a state  $N, F, e$  are simply those of its process part  $e$ .

## 2.2 Interaction with Abstract Types

We now address systems that interact by communicating elements of abstract types. Similar new-name machinery will be required to ensure soundness, though now for type names rather than channel names. We first recall some aspects of ML-style type abstraction, particularly with translucent sums/manifest types in signatures [HL94, Ler94, Ler96, Ler00, Lil97] (for brevity the examples will all be degenerate, without any operation parts or field dependencies). The module

```
A = struct type t = int end
    : sig type t          end
```

provides a type  $A.t$  with representation `int`. It has an explicit signature in which no information about the representation is visible, though, so from the outside the type is entirely abstract. In contrast, the module

```
C = struct type t = int end
    : sig type t = int end
```

---

<sup>1</sup>The semantics uses a disjoint union of all filesystems, implicitly extruding the `New N` of (\*) to the outside, to reduce notational clutter. It would be straightforward to give an equivalent model with explicit extrusion that would keep distinct filesystems.

```

T      := struct type t=int  val x=27  val i=λz.z  end : TSIG
C      := (functor(T:TSIG) struct newval c : T.t chan end) T
MAIN1 := (functor(T:TSIG, C:CSIG(T.t)) struct main = (C.c)!(T.x) end) T C
run MAIN1
wait 6 months
T      := struct type t=int*int  val x=(27,3) val i=λ(z,w).z end : TSIG
MAIN2 := (functor(T:TSIG, C:CSIG(T.t)) struct main = (C.c)?y> ... (T.i)y ... end) T C
run MAIN2

```

where we use abbreviations

$$\text{TSIG} \stackrel{\text{def}}{=} \text{sig type } t \text{ val } x:t \text{ val } i:t \rightarrow \text{int end} \quad \text{CSIG}(X) \stackrel{\text{def}}{=} \text{sig val } c : X \text{ chan end}$$

Figure 3: Attempted communication and use of elements of a changed abstract type

---

reveals that its representation type is `int`; the equality `C.t=int` may be used in typechecking the rest of the program. Such manifest types are particularly important in functor signatures. For example

```

functor(U : sig type t end)
  struct type t2=U.t end
  : sig type t2=U.t end

```

allows code using a structure, say `D`, created by applying this anonymous functor to `A`, to depend on the fact that `D.t2` and `A.t` are equal types.

Translucent sums and manifest types were motivated partly by the need to refine SML modules to provide enough type equality information in signatures for separate compilation, partly by concerns of higher-order functors and first-class modules that we do not discuss here, and partly by a desire to move from the generative SML static semantics to a more flexible type-theoretic style. Intuitively, instead of the SML semantics' use of new type names to distinguish between otherwise-identical abstract types, they use the module identifiers (or, more generally, paths) that occur in the source program.

In our distributed setting, we must reintroduce type name generation, albeit in a more controlled form. The broken example in Figure 3 shows why. Here we have two programs, `MAIN1` and `MAIN2`, communicating elements of an abstract type from module `T`, on a channel from `C`. Unfortunately `T` is re-built, with a changed representation type, between the builds of `MAIN1` and `MAIN2`. There will be a run-time error. To prevent this we should detect a build-time error when typechecking the functor application in the build of `MAIN2`, as the values of `C` and `T` there are not compatible. To make this lack of compatibility evident, we generate new type names when evaluating module expressions, reducing an abstract type to a type which is manifestly equal to a fresh type name. For example,

```

struct type t = int end      (*)
  : sig type t      end

```

reduces to

```

New X = int in
  struct type t = X end
  : sig type t = X end

```

and the body

```

struct type t = X end
  : sig type t = X end

```

of this will be a module value whereas `(*)` will not. Nonetheless, we keep the type-theoretic style as much as possible – type checking of a module expression in any individual build command will not involve new name generation. We need `t=X` both in the structure, so that it is correctly propagated during module



substitution (when a functor is applied to this structure), and in the signature, so that there is enough type sharing.

The system of §3 uses singleton kinds, due independently to Harper and Leroy, for expressing manifest type declarations in signatures. There are kinds `Type`, of all types, and `EQ(T)` for any type `T`, of all types provably equal to `T`. In the example signatures above `type t` becomes `type t::Type`, and `type t=t'` becomes `type t::EQ(t')`.

## 2.3 Abstract Type Versioning

The semantics outlined in the previous subsection guarantees soundness but, because every rebuild gives incompatible executables, it would be unusably rigid in practice. For example, one might initially build compatible `MAIN1` and `MAIN2` with the first `T` command and the `C`, `MAIN1` and `MAIN2` commands of Figure 3. Rebuilding `MAIN1` from the same source code, i.e. with the `C`, first `T` and `MAIN1` commands again, would give a version that could not interact with copies of `MAIN2` running elsewhere in the network. `MAIN1` and `MAIN2` would have different versions of both the type `T.t` and the channel `C.c`.

The developer therefore needs a mechanism for forcing a rebuild of a module that provides an abstract type `t` or a channel `c` to produce *the same* abstract type or channel as before. We address only the (more interesting) type part of the problem; the channel part can be dealt with using similar mechanisms. There are four cases:

1. The source code of the module (and all that it depends upon) is syntactically unchanged.
2. The source code is changed, but the representation type of `t` is unchanged and the new code has the same important invariants as the old.
3. The representation type is unchanged but the new code has changed invariants.
4. The representation type is changed.

Naively, one might allow a rebuild to produce the same type only in case 1. This could be entirely automatically checked, but would be too inflexible – we must allow for changed comments, performance improvements, minor bug fixes, and even some changes in functionality. On the other hand, in cases 3 and 4 a rebuild should certainly produce a new type. Distinguishing between cases 2 and 3 clearly cannot be done automatically, so the developer must provide some annotation for the new code, asserting that it has not changed any important invariant of the old abstract type, and hence that values produced by the old and new code will be compatible. Such an annotation should force an automatic check that the representation type is unchanged.

We envisage that large programs will require development environment support for managing these annotations, allowing defaults such as ‘always generate a new type’, or ‘always produce types compatible with the previous build unless the code is syntactically changed’, or ‘always produce compatible types unless the representation type has changed’ – all to be overridden locally as needed. There is an important pragmatic question here, of what expressiveness is really necessary or useful, but we do not investigate it in this paper. Instead, we introduce a minimal form of annotation, that might be generated by the development environment from higher-level defaults, and show how it can be given a sound semantics.

In particular, we allow module expressions containing coercions, e.g. as in the right hand side of the build command

`A := m with! U` (\*)

Suppose `m` is statically type-checkable with signature `sig type t::Type val x:tops end`, and that `U` is a filename that refers to a previously-built module of similar shape. Executing command `(*)` will evaluate `m` to a struct, say

```

struct type t=trep  val x=e      end
: sig  type t::Type  val x:tops  end

```

check that its representation type `trep` is compatible with that of `U`, and finally reduce it to a module value containing the same type name as `U`. In more detail, if the state in which `(*)` is executed has an `N` component with `X::EQ(trep2)`, and an `F` component mapping `U` to

```

struct type t=X      val x=e2  end
: sig  type t::EQ(X)  val x:tops2  end

```

then executing `(*)` will check `trep` and `trep2` are equal and result in a state with `A` mapped to

```

struct type t=X      val x=e      end
: sig  type t::EQ(X)  val x:tops  end

```

The notion of type equality required is complicated by the fact that the representation types may themselves involve other abstract types. It is made precise in §3, where it is also shown that it is not necessary to keep the whole development history in order to check it, but only modules actually referred to in coercions.

Note that the coercion must involve a build-time check, during evaluation of module expressions, as `m` must be reduced to a structure to make its representation type available.

### 3 Formal Development

In this section the previous informal discussion is made precise by giving a language of interacting processes, modules, and commands. It is equipped with build-time and run-time semantics, which are proven sound. The full syntax is given in Figure 4. We begin by discussing the choice of constructs.

**Commands** There are two command-line commands. Executing the build command `U:=m` type-checks and evaluates the module expression `m` and assigns the resulting module value to filename `U`. Executing `run U`, where `U` is a filename referring to a previously-built main module, loads the process part of that module in parallel with the rest of the running system. The grammar includes also `tau` – computation steps of the running system may be interleaved with build and load commands. We have simplified the system (in an unimportant way) by not representing source code stored in files, and (more significantly) by not dealing with separate builds of module interfaces.

**Modules** We take as simple a module language as possible: second class, with only first-order functors, without substructures, and with structures that have single type and term parts, not general dependent records. We do include singleton kinds, though – both to allow non-trivial sharing between functor arguments and results, and so that the type equality check needed for `with!` can be expressed. The structure

```
[T,e] as [X::K,T2]
```

is similar to our informal

```

struct type t=T  val x=e  end
: sig  type t::K  val x:T2  end

```

It consists of a pair of a type `T` (of kind `K`) and a term `e` (loosely, of type  $\{T/X\}T2$ ). The kind `K` here might be `Type`, making this a fully abstract structure with representation type `T` hidden, or `EQ(T')` for any `T'=T`, revealing the representation type. The type and term parts of a structure variable `U` can be projected out by `U.Type` and `U.term` (if this can be given a type). The functor

```
λU:SS.m:S
```

<b>Commands</b>		
Com ::= U := m		build
run U		load
tau		execute a step
<b>Structure signatures</b>		
SS ::= [X::K,T]		structure sig
<b>Module signatures</b>		
S ::= SS		
$\Pi U:SS.S$		functor sig
<b>Kinds</b>		
K ::= Type		kind of all types
EQ(T)		kind of types equal to T
<b>Types</b>		
T ::= X		variable
U.Type		type part of a structure
[T .. T]   int		tuples and integers
T chan   proc		channels and processes
<b>Module expressions</b>		
m ::= U		variable
[T,e] as SS		structure
$\lambda U:SS.m:S$		functor
m m		application
m:SS		seal
new x:T in m		new channel
m with! m		version coercion
<b>Core expressions</b>		
e ::= x		variable
U.term		term part of a structure
$\underline{n}$		integer
[e .. e]		tuple
new x:T in e		new channel
0   e e		nil and parallel processes
e!e   e?pat>e'		output and input processes
<b>Core patterns</b>		
pat ::= x   [pat .. pat]		
<b>Typing environments</b>		
E ::= empty   E,x:T   E,X::K   E,U:S		

U, X and x range over module, type and term variables respectively. Module variables are also used as filenames. The binding is as follows: in a structure sig X binds in T; in a functor sig U in S; in a functor U in m and S; in a new (module or core) x in m or e; and in an input the variables of pat in e'. We work up to alpha conversion.

Figure 4: Syntax

is a dependent function taking structures of signature  $SS$  and returning modules of signature  $S$ , which may mention  $U$ . Functors  $\lambda U:SS1.([T,e] \text{ as } SS2):SS2$  will often be written  $\lambda U:SS1.[T,e] \text{ as } SS2$ , eliding one copy of  $SS2$  (the explicit annotation is in the syntax just so the filesystem only has to contain module values).

To these standard constructs we add a module-level new channel declaration and our `with!` coercion. The module-level `new` allows uses of the `newval` of §2.1 such as `struct ... newval c : int chan end` to be expressed as

```
new c:int chan in ([..,c] as [..,int chan])
```

(this is a little awkward, however – a channel carrying an abstract type must be declared in a different structure than the type). The module expression `m with! m2`, where `m` and `m2` both have structure signatures, attempts to evaluate `m` and coerce it to provide an abstract type compatible with `m2`.

**Processes** The core language is simply an asynchronous  $\pi$ -calculus with tuples, allowing communication on newly-generable channels between parallel outputs and inputs.

**System States** The state of the whole system is (exactly as in §2.1) a triple

$N, F, e$

where  $N$  is a type environment of channel and type bindings (such as `z : int chan` and `X::EQ(T)`),  $F$  models the union of all developers' filesystems as a finite list of pairs of module ids and module values (with no repeated ids), and  $e$  is the running process expression. We take state triples up to alpha-renaming of the variables in  $\text{dom}(N)$ . Module values are simply

```
mval ::= [T1,e] as [X::EQ(T1'),T2]
        λU:SS.m:S'
```

We will never need to deal with `mvals` that have free module variables. Note that module variables and filenames are punned.

## 3.1 Typing

The type system for modules and processes is largely standard, with judgements:

$E \vdash K$	$E \vdash S \text{ sig}$
$E \vdash K <:: K'$	$E \vdash S < S' \text{ sig}$
$E \vdash K == K'$	$E \vdash S == S' \text{ sig}$
$E \vdash T::K$	$E \vdash m:S$
$E \vdash T == T' :: K$	$E \vdash \text{ok}$
$E \vdash \text{pat} : T \triangleright E'$	
$E \vdash e:T$	

It includes subkinding, with  $\text{EQ}(T) <:: \text{Type}$  for wellformed types  $T$ , a subsignature relation based on this that allows concrete type information to be forgotten, and a `self-type` rule for manipulating type equalities (expressed with singleton kinds) in signatures. The typing rules are given in Appendix A.1.

### 3.1.1 Sharing

We first review the standard aspects of the type system – the use of singleton kinds and subkinding to express ML-style sharing; for further explanation we refer the reader to [HL94, Ler94, Ler96, Ler00, Lil97]. The examples use alternate notation  $T * T'$  and  $(e, e')$  for binary products  $[T, T']$  and pairs  $[e, e']$ . One can write structures that are either abstract or concrete:

$$\begin{array}{l}
A \stackrel{def}{=} [\text{int}, \underline{6}] \text{ as } [X::\text{Type}, X] \quad \vdash A : [X::\text{Type}, X] \\
C \stackrel{def}{=} [\text{int}, \underline{6}] \text{ as } [X::\text{EQ}(\text{int}), X] \quad \vdash C : [X::\text{EQ}(\text{int}), X]
\end{array}$$

$A$  here provides an abstract type and a single value of that type;  $C$  is a pair of type `int` and value `6` of type `int`. To use a structure it must first be bound to a variable – the language allows projections `U.Type` and `U.term` of the type and term parts only of a structure variable  $U$ , not of an arbitrary module expression. If  $U$  has an abstract signature, eg in the type environment  $U: [X::\text{Type}, X]$ , then we know only

$$\begin{array}{l}
U: [X::\text{Type}, X] \vdash U.Type :: \text{Type} \\
U: [X::\text{Type}, X] \vdash U.term : U.Type
\end{array}$$

This suffices for typechecking a functor *Fopaque* that builds a new abstract type:

$$\begin{array}{l}
Fopaque \stackrel{def}{=} \lambda U: [X::\text{Type}, X]. [U.Type * U.Type, (U.term, U.term)] \text{ as } [Y::\text{Type}, Y] \\
\vdash Fopaque : \Pi U: [X::\text{Type}, X]. [Y::\text{Type}, Y] \\
\vdash Fopaque A : [Y::\text{Type}, Y]
\end{array}$$

*Fopaque* can also be applied to  $C$ , using the subsignature relation to ignore the manifest type.

$$\begin{array}{l}
\vdash \text{Type} <:: \text{EQ}(\text{int}) \\
\vdash [X::\text{Type}, X] < [X::\text{EQ}(\text{int}), X] \text{ sig} \\
\vdash C : [X::\text{Type}, X] \\
\vdash Fopaque C : [Y::\text{Type}, Y]
\end{array}$$

A more interesting variant of *Fopaque* involves type sharing between argument and result with a dependent functor signature, revealing that the type part of its result is the product of the type part of its argument.

$$\begin{array}{l}
Ftrans \stackrel{def}{=} \lambda U: [X::\text{Type}, X]. [U.Type * U.Type, (U.term, U.term)] \text{ as } [Y::\text{EQ}(U.Type * U.Type), Y] \\
\vdash Ftrans : \Pi U: [X::\text{Type}, X]. [Y::\text{EQ}(U.Type * U.Type), Y]
\end{array}$$

*Ftrans* might be applied to a structure variable of an abstract signature:

$$U' : [X::\text{Type}, X] \vdash Ftrans U' : [Y::\text{EQ}(U'.Type * U'.Type), Y]$$

or to a structure variable of a manifest signature (here assuming  $\vdash T::\text{Type}$ ):

$$U' : [X::\text{EQ}(T), X] \vdash Ftrans U' : [Y::\text{EQ}(T * T), Y]$$

To derive this one can first use the subsignature relation to make the argument signature of *Ftrans* match the manifest signature of  $U'$

$$\begin{array}{l}
\vdash \text{EQ}(T) <:: \text{Type} \\
\vdash [X::\text{Type}, X] < [X::\text{EQ}(T), X] \text{ sig} \\
\vdash \Pi U: [X::\text{Type}, X]. [Y::\text{EQ}(U.Type * U.Type), Y] \\
\quad < \Pi U: [X::\text{EQ}(T), X]. [Y::\text{EQ}(U.Type * U.Type), Y] \text{ sig} \\
\vdash Ftrans : \Pi U: [X::\text{EQ}(T), X]. [Y::\text{EQ}(U.Type * U.Type), Y]
\end{array}$$

then make use of the type equality  $U.Type == T$  in the result signature of *Ftrans*

$$\begin{array}{l}
U: [X::\text{EQ}(T), X] \vdash U.Type == T :: \text{Type} \\
U: [X::\text{EQ}(T), X] \vdash [Y::\text{EQ}(U.Type * U.Type), Y] == [Y::\text{EQ}(T * T), Y] \text{ sig} \\
\vdash \Pi U: [X::\text{EQ}(T), X]. [Y::\text{EQ}(U.Type * U.Type), Y] \\
\quad < \Pi U: [X::\text{EQ}(T), X]. [Y::\text{EQ}(T * T), Y] \text{ sig} \\
\vdash Ftrans : \Pi U: [X::\text{EQ}(T), X]. [Y::\text{EQ}(T * T), Y]
\end{array}$$

One can express binary functors that require their two arguments to have equal type parts:

$$Fb \stackrel{def}{=} \lambda U1 : [X :: \text{Type}, X]. \lambda U2 : [Y :: \text{EQ}(U1.\text{Type}), Y]. (U1.\text{Type}, U2.\text{term}) \text{ as } [Z :: \text{Type}, Z]$$

$$\vdash Fb : \prod U1 : [X :: \text{Type}, X]. \prod U2 : [Y :: \text{EQ}(U1.\text{Type}), Y]. [Z :: \text{Type}, Z]$$

so

$$\vdash (\lambda U : [X :: \text{Type}, X]. Fb U U) A : [Z :: \text{Type}, Z]$$

$$\vdash (\lambda U : [X :: \text{Type}, X]. Fb U (Ftrans U)) A : [Z :: \text{Type}, Z]$$

$$\not\vdash Fb A A : S$$

$$\not\vdash (\lambda U : [X :: \text{Type}, X]. Fb U (Fopaque U)) A : S$$

We show in more detail how the type system allows variables of abstract and concrete structure signatures to be used:

Suppose  $E=E1, U : [X :: \text{Type}, T]$  (and  $E \vdash \text{ok}$  and  $X$  not in  $\text{dom } E$ )  
 By  $m.\text{var}$  and  $T.\text{Proj}$   $E \vdash U.\text{Type} :: \text{Type}$

By  $T=.refl$  and  $T.T=$   $E \vdash U.\text{Type} :: \text{EQ}(U.\text{Type})$   
 By  $m.\text{self-type}$   $E \vdash U : [X :: \text{EQ}(U.\text{Type}), T]$   
 By the  $T=$  rules  $E, X :: \text{EQ}(U.\text{Type}) \vdash T == \{U.\text{Type}/X\}T :: \text{Type}$   
 By  $S=.Struct$   $E \vdash [X :: \text{EQ}(U.\text{Type}), T] == [X :: \text{EQ}(U.\text{Type}), \{U.\text{Type}/X\}T] \text{ sig}$   
 By  $S<$  and  $m.S<$   $E \vdash U : [X :: \text{EQ}(U.\text{Type}), \{U.\text{Type}/X\}T]$   
 By  $e.\text{proj}$   $E \vdash U.\text{term} : \{U.\text{Type}/X\}T$

Suppose  $E=E1, U : [X :: \text{EQ}(T0), T]$   
 By  $m.\text{var}$  and  $T.\text{Proj}$   $E \vdash U.\text{Type} :: \text{EQ}(T0)$   
 By  $T=.EQ$   $E \vdash U.\text{Type} == T0 :: \text{Type}$

By the  $T=$  rules  $E, X :: \text{EQ}(T0) \vdash T == \{T0/X\}T :: \text{Type}$   
 By  $S=.Struct$   $E \vdash [X :: \text{EQ}(T0), T] == [X :: \text{EQ}(T0), \{T0/X\}T] \text{ sig}$   
 By  $S<$ ,  $m.\text{var}$  and  $m.S<$   $E \vdash U : [X :: \text{EQ}(T0), \{T0/X\}T]$   
 By  $e.\text{proj}$   $E \vdash U.\text{term} : \{T0/X\}T$

More useful module examples quickly become rather verbose, especially with our cut-down term language. As a minimal example, one can express an abstract type of pairs that provides a once-only channel interface as follows (writing  $T \rightarrow T'$  for  $[T, T' \text{ chan}] \text{ chan}$ ).

```
new mkpair : int*int → int*int,
  fst      : int*int → int,
  snd      : int*int → int
in
  [int*int, (mkpair, fst, snd, ( mkpair?((x,y), result) → result!(x,y)
                               | fst?((x,y), result) → result!x
                               | snd?((x,y), result) → result!y) )]
as
  [X::Type, (int*int→X) * (X→int) * (X→int) * proc ]
```

### 3.1.2 New, with!, and system states

Turning now to the new aspects, the `new` and `with!` module constructs have rules

$$\frac{E, x : T \text{ chan} \vdash m : SS}{E \vdash \text{new } x : T \text{ chan in } m : SS} \text{m.new}$$

$$\begin{array}{lll}
N \vdash (\lambda U:SS1.m:S2) \text{ mval} & \longrightarrow \{ \text{mval}/U \} m & \text{m.red.beta} \\
N \vdash [T1, e] \text{ as } [X::EQ(T1'), T2] : SS & \longrightarrow [T1, e] \text{ as } SS & \text{m.red.seal} \\
N \vdash \text{new } x:T \text{ in } m & \longrightarrow \text{New } x:T \text{ in } m & \text{m.red.new} \\
N \vdash [T1, e] \text{ as } [X::Type, T2] & \longrightarrow \text{New } Y::EQ(T1) \text{ in } [Y, e] \text{ as } [X::EQ(Y), T2] & \text{m.red.abstype} \\
& \text{(for } Y \notin \text{dom}(N)) & \\
\\
m = [T1, e] \text{ as } [X::Type, T2] & & \\
m' = [Z, e'] \text{ as } [Y::EQ(Z), T2'] & & \\
N = N1, Z::EQ(TZ), N2 & & \\
\text{typify}(N) \vdash T1==TZ::Type & & \\
\hline
N \vdash m \text{ with! } m' \longrightarrow [Z, e] \text{ as } [X::EQ(Z), T2] & \text{m.red.with!} & 
\end{array}$$

Figure 5: Module reduction axioms

$$\begin{array}{l}
E \vdash m : [X::K, T2] \\
E \vdash m' : [Y::EQ(T'), T2'] \\
\hline
E \vdash (m \text{ with! } m') : [X::EQ(T'), T2] \quad \text{m.with}
\end{array}$$

The first allows new channel declaration in a structure. The second allows  $m$  to be coerced to have the same type part as  $m'$  – statically it always succeeds (if  $m'$  is a value), leaving the build-time check to determine if the representation types are in fact compatible. Typing for a system state  $N, F, e$  requires that (1) all module values in the filesystem (which may have free channel or type names created earlier) have the signature they claim; (2) the process  $e$  is a well-typed `proc`; and (3)  $N$  has only channel and type names.

$$\begin{array}{l}
\forall U \in \text{dom}(F). N \vdash F(U) : \text{sig}(F(U)) \\
N \vdash e : \text{proc} \\
N \text{ atomic} \\
N \text{ has no module bindings} \\
\hline
\vdash N, F, e \text{ ok} \quad \text{system}
\end{array}$$

Here `sig` is the function that extracts the signature of a module value, defined by

$$\begin{array}{ll}
\text{sig}( [T, e] \text{ as } [X::K, T'] ) & = [X::K, T'] \\
\text{sig}( \lambda U:SS.m:S' ) & = \Pi U:SS.S'
\end{array}$$

and a type environment  $E$  is *atomic* if for all term variable bindings  $x:T$  in  $E$  there exists  $T2$  such that  $T=T2$  `chan`.

## 3.2 Build and Run-Time Semantics

The language semantics is expressed with transitions

$$N, F, e \xrightarrow{\text{Com}} N', F', e'$$

labelled by commands  $U:=m$ , `run`  $U$ , and `tau`, expressing how the system state can change. The run-time semantics is straightforward, with transitions labelled by `tau` arising from  $\pi$ -calculus reductions  $e \longrightarrow e'$ . It is defined in Appendix A.3. The build-time semantics, for transitions labelled  $U:=m$ , is novel. It involves an auxiliary one-step reduction relation for module expressions, discussed in §3.3, written

$$N \vdash m \longrightarrow \text{New } N' \text{ in } m'$$

for  $m$  reducing to  $m'$  with new channel or type names  $N'$ . Note that the `New  $N'$`  here is part of the judgement, not module syntax. Multistep reductions will be written with the double arrow  $\Longrightarrow$ . In addition, we identify various error cases. Run-time errors are simply mismatched communications, e.g.  `$x!3 \mid x?[y \ z]\Rightarrow e$` . Build-time errors arise when trying  `$U:=m$`  for a badly-typed  $m$ , or when the dynamic check of a `with!` coercion within  $m$  fails. Load-time errors arise when trying `run  $U$`  where  $U$  is not the filename of a structure containing a process. We omit details of the error cases, but give the main transition rules:

$$\frac{\text{typify}(N), \text{env}(F) \vdash m : S \quad N \vdash F(m) \Longrightarrow \text{New } N' \text{ in mval} \quad \text{dom}(N) \text{ disjoint from } \text{fv}(m) \text{ and } \text{dom}(N')}{N, F, e \xrightarrow{U:=m} (N, N'), F \oplus (U \mapsto \text{mval}), e} \text{ build}$$

$$\frac{F(U) = [T, e'] \text{ as } [X::K, \text{proc}]}{N, F, e \xrightarrow{\text{run } U} N, F, e|e'} \text{ load}$$

$$\frac{e \longrightarrow e'}{N, F, e \xrightarrow{\text{tau}} N, F, e'} \text{ compute}$$

We discuss the key aspects of the `build` rule. Firstly,  $m$  is typechecked. It may mention previously-built modules in  $\text{dom}(F)$ , so this should be with respect to their signatures. These signatures may involve type variables in  $\text{dom}(N)$  previously generated for abstract types, but the representations of those types should not be visible for typechecking. We write  $\text{env}(F)$  for the type environment mapping each  $U \in \text{dom}(F)$  to  $\text{sig}(F(U))$ , and  $\text{typify}(N)$  for the type environment mapping each  $X \in \text{dom}(N)$  to  $\text{Type}$ . Secondly,  $F(m) - m$  with all filenames replaced by the module values they refer to - is reduced to `mval`. This may generate new type or channel names, which are propagated to the resulting state, and involves checking any coercions in  $m$ . Finally, the disjointness condition, and the fact that both the  $N$  part of  $N, F, e$  and the  $N'$  part of  $N \vdash F(m) \Longrightarrow \text{New } N' \text{ in mval}$  are treated as binders, ensure that all previously-generated names are alpha-converted away from the free names of  $m$ . The full rules for whole systems can be found in Appendix A.4.

### 3.3 Module reduction

Module reduction is defined by the axioms in Figure 5, with (roughly) evaluation contexts

$$C ::= \_ \mid C \ m2 \mid \text{mval } C \mid C:SS \mid C \ \text{with! } \text{mval} \mid m1 \ \text{with! } C$$

and rule

$$\frac{N \vdash m \longrightarrow \text{New } N' \text{ in } m' \quad \text{dom}(N') \text{ and } \text{fv}(C) \text{ disjoint}}{N \vdash C[m] \longrightarrow \text{New } N' \text{ in } C[m']} \text{ substitution}$$

The substitution in `m.red.beta` is nonstandard - it also reduces any projections from structures that are introduced by the substitution. Note the new type name generation of `m.red.abstype`, and the check in `m.red.with!` that the representation type  $T1$  of  $m$  is provably equal to the representation type  $TZ$  of the module  $m'$  that  $m$  is being coerced to. As in `build`, this must be with respect to  $\text{typify}(N)$ . (In fact, we have glossed over a subtlety - `m.red.abstype` should not be used on the left of a `with!`; we ensure this by splitting the reduction relation into two.) The full rules can be found in Appendix A.2.



The dynamic check does require the representation types of any modules that are referred to to be stored (as part of a struct that is inaccessible except to the `with!` check), but as the equality check is wrt. `typify(N)` no other type equalities from the development history are needed.

### 3.4 Examples

First, some examples of module reduction. For  $C$  as defined in §3.1.1, the structure

$$[\text{int}, \underline{6}] \text{ as } [X::\text{EQ}(\text{int}), X]$$

is already a module value. For  $A$  there is a single reduction, creating a new type id, to a module value:

$$\vdash [\text{int}, \underline{6}] \text{ as } [X::\text{Type}, X] \longrightarrow \text{New } Z::\text{EQ}(\text{int}) \text{ in } [Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X] \text{ by m.red.abstype}$$

Applying *Fopaque* to  $C$ :

$$\begin{aligned} \vdash \text{Fopaque } C &\longrightarrow \{C/U\} [U.\text{Type}*U.\text{Type}, [U.\text{term}, U.\text{term}]] \text{ as } [Y::\text{Type}, Y] && \text{ by m.red.beta} \\ &= [\text{int}*\text{int}, (\underline{6}, \underline{6})] \text{ as } [Y::\text{Type}, Y] \\ &\longrightarrow \text{New } Z::\text{EQ}(\text{int}*\text{int}) \text{ in } [Z, (\underline{6}, \underline{6})] \text{ as } [Y::\text{EQ}(Z), Y] && \text{ by m.red.abstype} \end{aligned}$$

Applying *Fopaque* to  $A$ , first  $A$  is reduced (creating a new type id), then there is a beta step, then another new type id is created.

$$\begin{aligned} \vdash \text{Fopaque } A &\longrightarrow \text{New } Z::\text{EQ}(\text{int}) \text{ in } \text{Fopaque } ([Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X]) \\ &\longrightarrow \text{New } Z::\text{EQ}(\text{int}) \text{ in } \{[Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X]/U\} ([U.\text{Type}*U.\text{Type}, (U.\text{term}, U.\text{term})] \\ &\hspace{15em} \text{as } [Y::\text{Type}, Y]) \\ &= \text{New } Z::\text{EQ}(\text{int}) \text{ in } [Z*Z, (\underline{6}, \underline{6})] \text{ as } [Y::\text{Type}, Y] \\ &\longrightarrow \text{New } Z::\text{EQ}(\text{int}), W::\text{EQ}(Z*Z) \text{ in } [W, (\underline{6}, \underline{6})] \text{ as } [Y::\text{EQ}(W), Y] \end{aligned}$$

Now consider the build commands

$$\begin{aligned} C &:= C \\ A &:= A \end{aligned}$$

(here  $C$  and  $A$  are module variables, used as filenames, and  $C$  and  $A$  are, as above, abbreviations for module expressions). Executing these in the empty state `New empty in ( $\emptyset$ , 0)` gives the state

$$\text{New } Z::\text{EQ}(\text{int}) \text{ in } (\{C \mapsto C, A \mapsto [Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X]\}, 0) \quad (*)$$

Subsequent build commands can then refer to the type and term components of  $A$  and  $C$ , eg in

$$B := [A.\text{Type}*C.\text{Type}, (A.\text{term}, C.\text{term})] \text{ as } [X::\text{EQ}(A.\text{Type}*C.\text{Type}), X]$$

The module expression on the right-hand side of this will be type checked in the environment

$$Z::\text{Type}, C: [X::\text{EQ}(\text{int}), X], A: [X::\text{EQ}(Z), X]$$

Rebuilding  $A$  will produce an abstract type that is different from that of  $A$  – executing  $A' := A$  in the state  $(*)$  above results in

$$\begin{aligned} \text{New } Z::\text{EQ}(\text{int}), Z'::\text{EQ}(\text{int}) \text{ in } (\{C \mapsto C, A \mapsto [Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X], \\ A' \mapsto [Z', \underline{6}] \text{ as } [X::\text{EQ}(Z'), X]\}, 0) \end{aligned}$$

so executing `U := Fb A A'` will give a type checking error. On the other hand, executing `A' := A with! A` in the state  $(*)$  results in

$$\begin{aligned} \text{New } Z::\text{EQ}(\text{int}) \text{ in } (\{C \mapsto C, A \mapsto [Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X], \\ A' \mapsto [Z, \underline{6}] \text{ as } [X::\text{EQ}(Z), X]\}, 0) \end{aligned}$$

and here  $U := \text{Fb } A \ A'$  will succeed. For a more interesting use of `with!`, illustrating the fact that only the representation types must be equal, executing

```
A' := [int, (7,8)] as [X::Type, X*X] with! A
```

in the state `(*)` results in

```
New Z::EQ(int) in ({C→C, A ↦ [Z,6] as [X::EQ(Z),X],
                  A' ↦ [Z,(7,8)] as [X::EQ(Z),X*X]}, 0)
```

Finally, consider a module that provides an abstract type, implementing it with a representation type that involves an abstract type from another module, eg (again in state `(*)`)

```
A2 := [A.Type*A.Type, (A.term,A.term)] as [X::Type,X]
```

resulting in the state

```
New Z::EQ(int),W::EQ(Z*Z) in ({C→C, A ↦ [Z,6] as [X::EQ(Z),X],
                              A2 ↦ [W,(6,6)] as [X::EQ(W),X]}, 0)
```

If one rebuilds `A2`

```
A2' := m with! A2
```

the `m.red.with!` rule checks that the representation type of `m` is equal to `Z*Z`, in an environment  $Z::\text{Type}, W::\text{Type}$ . Simply checking that the representation type of `m` is equal to the underlying representation type `int*int` would be mistaken, as `A` may have been rebuilt, either with the same invariants (and using `with!`) or changed (and not using `with!`).

The examples above are artificial, with useless abstract types, to illustrate the semantics. They should be enough to see how natural examples (which would be unfortunately lengthy) would go, however.

### 3.5 Soundness

This subsection states only the main lemmas and the soundness properties of the semantics. Full details can be found in Appendix B.

**Lemma 5 (Weakening for  $\vdash$ )** If  $E, E'' \vdash J$  and  $E, E' \vdash \text{ok}$  and  $\text{dom } E', \text{dom } E''$  disjoint and if  $J$  is  $\text{pat}:T \triangleright E1$  then  $\text{dom } E', \text{var pat disjoint}$  then  $E, E', E'' \vdash J$ .

**Lemma 15 (Kind, type and sig well-formedness for  $\vdash$ )**

If $E \vdash K <:: K'$	then $E \vdash K$ and $E \vdash K'$
If $E \vdash K == K'$	then $E \vdash K$ and $E \vdash K'$
If $E \vdash T::K$	then $E \vdash K$
If $E \vdash T == T' :: K$	then $E \vdash T::\text{Type}$ and $E \vdash T'::\text{Type}$
If $E \vdash S < S' \text{ sig}$	then $E \vdash S \text{ sig}$ and $E \vdash S' \text{ sig}$
If $E \vdash S == S' \text{ sig}$	then $E \vdash S \text{ sig}$ and $E \vdash S' \text{ sig}$
If $E \vdash \text{pat} : T \triangleright E'$	then $E \vdash T::\text{Type}$ and $E, E' \vdash \text{ok}$
If $E \vdash e:T$	then $E \vdash T::\text{Type}$
If $E \vdash m:S$	then $E \vdash S \text{ sig}$

The proof of this involves an alternative characterisation of the kind, type, signature and type environment formation judgements, removing the dependencies on (in)equality judgements.

**Lemma 16 (Strengthening of  $\vdash$  by term bindings)** If  $E, E', E'' \vdash J$  and  $E'$  has only term bindings and  $\text{dom } E' \text{ not in } J$  then  $E, E'' \vdash J$ .

**Lemma 17 (Substitution – type)** If  $E, Z :: KZ, E' \vdash J$  and  $E \vdash TZ :: \text{Type}$  and  $KZ$  is either  $\text{Type}$  or  $\text{EQ}(TZ)$  then  $E, \{TZ/Z\}E' \vdash \{TZ/Z\}J$ .

**Lemma 19 (Permutation)** If  $E_1, E_2, E_3, E_4 \vdash J$  and  $\text{dom } E_2$  not in  $E_3$  then  $E_1, E_3, E_2, E_4 \vdash J$ .

**Lemma 20 (Narrowing)** If  $E, Z :: KZ', E' \vdash J$  and  $E \vdash KZ < :: KZ'$  then  $E, Z :: KZ, E' \vdash J$ .

**Lemma 25 (Cancellation – type equality)** If  $E \vdash T == T' :: \text{Type}$  and both  $T$  and  $T'$  are of the form  $[T \dots T] \mid \text{chan } T \mid \text{proc} \mid \text{int}$  (not necessarily the same) then either  
 1)  $T = [T_1 \dots T_n]$  and  $T' = [T_1' \dots T_n']$  and for  $i=1..n$   $E \vdash T_i == T_i' :: \text{Type}$ ,  
 2)  $T = \text{chan } T_1$  and  $T' = \text{chan } T_1'$  and  $E \vdash T_1 == T_1' :: \text{Type}$ ,  
 3)  $T = \text{proc} = T'$ , or  
 4)  $T = \text{int} = T'$ .

The proof of this involves an alternative characterisation of the type equality judgement, reducing it to simple equational reasoning.

**Lemma 30 (Substitution – module)**

If  $E, U :: S, E' \vdash J$  and  $E \vdash \text{mval} :: S$  then  $E, \{\text{mval}/U\}E' \vdash \{\text{mval}/U\}J$ .

**Lemma 31 (Unique decomposition)** If  $N$  has no module bindings and  $N \vdash m :: S$  then either  $m$  is an  $\text{mval}$  xor there is a unique  $j \in \{1, 2\}$ ,  $C_j$ , rule  $r \in m.\text{red}.*$  and instance  $lr$  of the redex of  $r$  such that  $m = C_j[lr]$ .

**Lemma 35 (Module subject reduction)**

If  $N \vdash m :: S$  and  $N \vdash m \implies \text{New } N' \text{ in } m'$  and  $N, N'$  disjoint then  $N, N' \vdash m' :: S$ .  
 Moreover if  $N$  atomic and  $N$  has no module bindings then  $N, N'$  is likewise.

**Lemma 36 (Well-formed term substitutions)** If  $E$  atomic and has no module bindings,  $E \vdash e :: T$  and  $E \vdash \text{pat} :: T \triangleright E'$  then  $\{e/\text{pat}\}'$  is defined and  $E \vdash \{e/\text{pat}\}' :: E'$ .

**Lemma 37 (Substitution – term)**

If  $E, E', E'' \vdash e :: T$  and  $E \vdash s :: E'$ , where  $s$  is a term substitution, then  $E, E'' \vdash se :: T$ .

**Lemma 38 (Type soundness of structural congruence)**

If  $E \vdash e :: \text{proc}$  and  $e == e'$  then  $E \vdash e' :: \text{proc}$ .

**Lemma 39 (Process subject reduction)**

If  $E$  is atomic and has no module bindings,  $E \vdash e :: \text{proc}$  and  $e \longrightarrow e'$  then  $E \vdash e' :: \text{proc}$ .

**Lemma 40 (Process soundness)**

If  $E$  is atomic and has no module bindings and  $E \vdash e :: \text{proc}$  then not  $(e \xrightarrow{\text{err}})$ .

**Theorem 1** If  $\vdash N, F, e$  ok and  $N, F, e \xrightarrow{\text{Com}} N', F', e'$  then  $\vdash N', F', e'$  ok.

**Theorem 2** If  $\vdash N, F, e$  ok then there is no transition  $N, F, e \xrightarrow{\text{tau}} \text{err}(\text{runtime error})$ .

There may of course be build- or load-time errors.

### 3.6 Relating separately-compiled and monolithic programs

A desirable property of systems for separate compilation is that splitting a program into separate compilation units is guaranteed not to change its type-correctness or behaviour. For the language of this paper, a simple version of the property is:

**Theorem 3** *If  $m_1$  and  $m_2$  are structure expressions*

$$\begin{aligned} m_1 &= [T_1, e_1] \text{ as } [X : K_1, T_1'] \\ m_2 &= [T_2, e_2] \text{ as } [X : K_2, T_2'] \end{aligned}$$

and

$$m_2' = (\lambda U_1 : SS_1.m_2 : SS_2) [T_1, e_1] \text{ as } [X : EQ(T_1), T_1']$$

where  $SS_i$  is the signature of  $m_i$ , then

$$(\exists N, F, e. \text{ empty}, \emptyset, 0 \xrightarrow{U_1 := m_1} \xrightarrow{U_2 := m_2} N, F, e)$$

iff

$$(\exists N, F, e. \text{ empty}, \emptyset, 0 \xrightarrow{U_2 := m_2'} N, F, e)$$

Note that one cannot take  $m_2'$  to be simply  $(\lambda U_1 : SS_1.m_2 : SS_2) m_1$ , as if  $K_1 = \text{Type}$  and  $U_1$  is used in  $T_2'$  this cannot generally be given a useful signature. Instead, the abstraction is enforced solely by the argument signature of the functor. The proof of the theorem is in Appendix B.11. The interesting case of the proof is the left-to-right direction for  $K_1 = \text{Type}$  – taking also  $K_2 \neq \text{Type}$ , the execution of  $U_1 := m_1$  involves new type name generation whereas the execution of  $U_2 := m_2'$  does not. We therefore have a tight link between the generative view of abstract types, used in the separately-compiled version, and the singleton-kind view, in typechecking  $m_2'$ .

Intuitively, the two resulting module values  $F(U_2)$  are syntactically the same modulo certain type equalities, but we do not make this precise here. One might also generalise the result, giving translations between arbitrary module expressions and sequences of structure and functor build commands. This could then be contrasted to the result of Leroy [Ler96] relating the expressiveness of manifest-type-based and stamp-based static semantics for ML-style modules. The latter involves new stamp generation during type-checking (elaboration) of a module expression.

## 4 Conclusion

In summary, we have provided a solid basis for programming wide-area systems involving interaction at abstract types. It required new constructs – the versioning coercion and build-time channel generation – and novel operational semantics for module reduction and for build/load/compute-time system state changes. We illustrated these, demonstrating how they can be set up coherently, by giving a model language of processes, modules and commands, equipped with build- and run-time semantics, and proving soundness.

The work is a necessary preliminary for more refined treatments of numbered versions, and for extending traditional distributed systems programming with communication of values of abstract types.

**Further Related Work** We argued in §2.3 that developers must – in a limited way – be able to break the abstraction boundary of an abstract type. The `with!` coercion does this, allowing new types to be made compatible with old, provided (a) their immediate representation types are equal, and (b) the developer asserts no important invariants have changed. The closest primitive in previous work seems to be the

partial revelations of opaque types in Modula 3 [CDG<sup>+</sup>89], allowing any opaque type to be made concrete (to a specified subtype) within a scope. Turning to formal models, Cardelli [Car97] discusses linking and separate compilation in detail, but without module type components. The MTAL system of Glew and Morrisett [GM99] models linking for typed assembly language. It incorporates abstract types, but has a flat namespace and does not deal with differing versions. There is therefore no need for explicit generation of type names. Other work on separate compilation, notably [BA99, Dro00, Dug00, FF98, HWC00], focusses largely on name space and hierarchy issues, and on dynamic linking.

**Future Directions** Firstly, it would be interesting to generalise the results of §3.6, as indicated there, to arbitrary programs.

The idea of the module reduction semantics may have other applications. In particular, it should permit a typed operational semantics for Cardelli and Leroy’s dot-notation calculi [CL90]. Adding term-level annotations delimiting subterms that originated from abstract types, building on [ZGM99], may allow nice syntactic statements of abstractness properties.

One might hope to address first class modules [HL94, Lil97, Rus98]. Typed semantics for dot-notation would go some way towards this, but note that term-level execution here may involve distributed communication, which may only make sense with respect to a local machine state. It is therefore desirable to be able to stratify execution (in the extreme case of the language of this paper no core reduction is done at build time, trivially preventing such distributed side-effects).

As for extensions, for usability the system would have to be extended to general dependent records and substructures, not simply binary translucent sums. Dealing with named interfaces would allow more direct treatment of traditional IDLs, extended with abstract types. There are many pragmatic issues of what development environment support is required to make the `with!` coercion and its channel-name analogue usable; they will have to be investigated by experiment. Finally, to address interface evolution and numbered versions, the system should be extended with subtyping, with polarity on the types used for interaction (here these are the channel types, for which polarities have been studied eg in [PS96, Ode95]), and with a type-level representation of a partial order.

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## A Full Language Definition

This appendix contains the full typing rules, build-time semantics and run-time semantics of the language.

### A.1 Typing

$\boxed{E \vdash K}$

$$\frac{E \vdash \text{ok}}{E \vdash \text{Type}} \text{K.Type} \quad \frac{E \vdash T :: \text{Type}}{E \vdash \text{EQ}(T)} \text{K.EQ}$$

$\boxed{E \vdash K <:: K'}$

$$\frac{E \vdash T :: \text{Type}}{E \vdash \text{EQ}(T) <:: \text{Type}} \text{K<.EQ} \quad \frac{E \vdash K == K'}{E \vdash K <:: K'} \text{K<.K=}$$

$\boxed{E \vdash K == K'}$

$$\frac{E \vdash \text{ok}}{E \vdash \text{Type} == \text{Type}} \text{K=.Type} \quad \frac{E \vdash T == T' :: \text{Type}}{E \vdash \text{EQ}(T) == \text{EQ}(T')} \text{K=.T=}$$

$\boxed{E \vdash T :: K}$

$$\frac{E \vdash T :: K}{E \vdash T :: K'} \text{T.K<} \quad \frac{E \vdash T == T' :: \text{Type}}{E \vdash T :: \text{EQ}(T')} \text{T.T=} \quad \frac{E \vdash U: [X::K, T]}{E \vdash U.Type :: K} \text{T.Proj}$$

$$\frac{E \vdash \text{ok}}{E \vdash [T_1 \dots T_n] :: \text{Type}} \text{T.rec} \quad \frac{E \vdash T :: \text{Type}}{E \vdash T \text{ chan} :: \text{Type}} \text{T.chan} \quad \frac{E \vdash \text{ok}}{E \vdash \text{proc} :: \text{Type}} \text{T.proc} \quad \frac{E \vdash \text{ok}}{E \vdash \text{int} :: \text{Type}} \text{T.int}$$

$$\frac{E, X::K, E' \vdash \text{ok}}{E, X::K, E' \vdash X::K} \text{T.var}$$

$\boxed{E \vdash T == T' :: K}$

$$\frac{E \vdash T :: \text{EQ}(T')}{E \vdash T == T' :: \text{Type}} \text{T=.EQ}$$

and standard rules  $T=.refl, T=.sym, T=.tran, T=.cong.rec, T=.cong.chan$  for equivalence and congruence

$\boxed{E \vdash S \text{ sig}}$

$$\frac{E, X::K \vdash T::\text{Type}}{E \vdash [X::K, T] \text{ sig}} \text{S.Struct} \quad \frac{E \vdash SS \text{ sig} \quad E, U:SS \vdash S' \text{ sig}}{E \vdash \Pi U:SS.S' \text{ sig}} \text{S.Fun}$$

$E \vdash S < S' \text{ sig}$

$$\frac{E \vdash K <:: K' \quad E, X::K \vdash T == T' :: \text{Type}}{E \vdash [X::K, T] < [X::K', T'] \text{ sig}} S<.Struct$$

$$\frac{E \vdash SS1' < SS1 \text{ sig} \quad E, U:SS1' \vdash S2 < S2' \text{ sig}}{E \vdash \Pi U:SS1.S2 < \Pi U:SS1'.S2' \text{ sig}} S<.Fun$$

$$\frac{E \vdash S == S' \text{ sig}}{E \vdash S < S' \text{ sig}} S<.S= \quad \frac{E \vdash S < S' \text{ sig} \quad E \vdash S' < S'' \text{ sig}}{E \vdash S < S'' \text{ sig}} S<.tran$$

$E \vdash S == S' \text{ sig}$

$$\frac{E \vdash K == K' \quad E, X::K \vdash T == T' :: \text{Type}}{E \vdash [X::K, T] == [X::K', T'] \text{ sig}} S=.Struct$$

$$\frac{E \vdash SS1 == SS1' \text{ sig} \quad E, U:SS1' \vdash S2 == S2' \text{ sig}}{E \vdash \Pi U:SS1.S2 == \Pi U:SS1'.S2' \text{ sig}} S=.Fun$$

and standard rules  $S=.refl$ ,  $S=.sym$ ,  $S=.tran$  for equivalence

$E \vdash \text{ok}$

E.empty	E.term	E.type	E.mod
	$E \vdash T :: \text{Type}$	$E \vdash K$	$E \vdash S \text{ sig}$
	$x \text{ not in dom } E$	$X \text{ not in dom } E$	$U \text{ not in dom } E$
$\text{empty} \vdash \text{ok}$	$E, x:T \vdash \text{ok}$	$E, X::K \vdash \text{ok}$	$E, U:S \vdash \text{ok}$

$E \vdash \text{pat} : T \triangleright E'$

$$\frac{E \vdash T :: \text{Type} \quad x \text{ not in dom } E \quad E \vdash \text{ok} \quad E \vdash \text{pati} : T_i \triangleright E_i \quad i=1..n \quad \text{pati all disjoint}}{E \vdash x : T \triangleright x:T \text{ pat.var} \quad E \vdash [\text{pat}_1.. \text{pat}_n] : [T_1..T_n] \triangleright E_1, \dots, E_n \text{ pat.rec}}$$

$E \vdash e:T$

$$\frac{E, x:T, E' \vdash \text{ok} \quad E \vdash e:T \quad E \vdash T == T' :: \text{Type} \quad E \vdash \text{ok} \quad E \vdash e_i : T_i \quad i=1..n}{E, x:T, E' \vdash x:T \text{ e.var} \quad E \vdash e:T' \text{ e.T=} \quad E \vdash [e_1 \dots e_n] : [T_1 \dots T_n] \text{ e.rec}}$$

$$\frac{E \vdash U : [X::K, T] \quad E \vdash T :: \text{Type}}{E \vdash U \text{ term} : T \text{ e.proj}} \quad \frac{E, x: T \text{ chan} \vdash e : \text{proc} \quad E \vdash \text{ok}}{E \vdash \text{new } x : T \text{ chan in } e : \text{proc} \text{ e.new}} \quad \frac{E \vdash \text{ok}}{E \vdash 0 : \text{proc} \text{ e.nil}}$$

$$\frac{E \vdash e_i : \text{proc} \quad i=1,2 \quad E \vdash e_1 : T \text{ chan} \quad E \vdash \text{pat} : T \triangleright E' \quad E, E' \vdash e_2 : \text{proc}}{E \vdash e_1 | e_2 : \text{proc} \text{ e.par} \quad E \vdash e_1 ! e_2 : \text{proc} \text{ e.out} \quad E \vdash e_1 ? \text{pat} \triangleright e_2 : \text{proc} \text{ e.in}}$$

$$\frac{E \vdash \text{ok}}{E \vdash \underline{n} : \text{int} \text{ e.int}}$$

$\boxed{E \vdash m : S}$

$$\frac{E, U : S, E' \vdash \text{ok}}{E, U : S, E' \vdash U : S} \text{m.var} \quad \frac{E \vdash m : S \quad E \vdash S < S' \text{ sig}}{E \vdash m : S'} \text{m.S<} \quad \frac{E, x : T \text{ chan } \vdash m : SS}{E \vdash \text{new } x : T \text{ chan in } m : SS} \text{m.new}$$

$$\frac{E \vdash T :: K \quad E, X :: K \vdash T' :: \text{Type} \quad E, X :: \text{EQ}(T) \vdash e : T' \quad X \text{ not free in } e}{E \vdash [T, e] \text{ as } [X :: K, T'] : [X :: K, T']} \text{m.struct} \quad \frac{E \vdash U : [X :: K, T] \quad E \vdash U.\text{Type} :: K'}{E \vdash U : [X :: K', T]} \text{m.self-type}$$

$$\frac{E, U : SS1 \vdash m : S2}{E \vdash \lambda U : SS1. m : S2 : \text{IIU} : SS1.S2} \text{m.fun} \quad \frac{E \vdash m : \text{IIU} : SS1.S2 \quad E \vdash m' : SS1 \quad U \text{ not free in } S2}{E \vdash m \ m' : S2} \text{m.app}$$

$$\frac{E \vdash m : SS}{E \vdash (m : SS) : SS} \text{m.seal} \quad \frac{E \vdash m : [X :: K, T2] \quad E \vdash m' : [Y :: \text{EQ}(T'), T2']}{E \vdash (m \text{ with! } m') : [X :: \text{EQ}(T'), T2]} \text{m.with}$$

$\boxed{\vdash N, F, e \text{ ok}}$

$$\frac{\text{forall } U \text{ in } \text{dom}(F). N \vdash F(U) : \text{sig}(F(U)) \quad N \vdash e : \text{proc} \quad N \text{ atomic} \quad N \text{ has no module bindings}}{\vdash N, F, e \text{ ok}} \text{system}$$

## A.2 Module Reduction

$\boxed{N \vdash m \longrightarrow_i \text{New } N' \text{ in } m' \quad i=1,2}$

$\boxed{N \vdash m \xrightarrow{\text{err}} s}$

$\boxed{N \vdash m \Longrightarrow \text{New } N' \text{ in } m'}$

$\boxed{N \vdash m \xrightarrow{\text{err}} s}$

Here the (1) reductions do not do outermost new type creation, whereas the (2) reductions do. Note that the New here is part of the judgement, not module syntax. We work up to alpha conversion of  $N'$  binding in  $m'$ . We elide 'New empty in'.

$$\begin{array}{lll} N \vdash (\lambda U : SS1. m : S2) \text{ mval} & \longrightarrow_1 \{ \text{mval} / U \} m & \text{m.red.beta} \\ N \vdash [T1, e] \text{ as } [X :: \text{EQ}(T1'), T2] : SS & \longrightarrow_1 [T1, e] \text{ as } SS & \text{m.red.seal} \\ N \vdash \text{new } x : T \text{ in } m & \longrightarrow_1 \text{New } x : T \quad \text{in } m & \text{m.red.new} \\ N \vdash [T1, e] \text{ as } [X :: \text{Type}, T2] & \longrightarrow_2 \text{New } Y :: \text{EQ}(T1) \text{ in } [Y, e] \text{ as } [X :: \text{EQ}(Y), T2] & \text{m.red.abstype} \\ & (\text{for } Y \notin \text{dom}(N)) & \end{array}$$

$$\frac{m = [T1, e] \text{ as } [X :: \text{Type}, T2] \quad m' = [Z, e'] \text{ as } [Y :: \text{EQ}(Z), T2'] \quad N = N1, Z :: \text{EQ}(TZ), N2 \quad \text{typify}(N) \vdash T1 = TZ :: \text{Type}}{N \vdash m \text{ with! } m' \longrightarrow_1 [Z, e] \text{ as } [X :: \text{EQ}(Z), T2]} \text{m.red.with!}$$



$$\frac{\begin{array}{l} m = [T1, e] \text{ as } [X::\text{Type}, T2] \\ m' = [Z, e'] \text{ as } [Y::\text{EQ}(Z), T2'] \\ N(Z) = \text{EQ}(TZ) \\ \text{not}(\text{typify}(N)) \vdash T1 = TZ::\text{Type} \end{array}}{N \vdash m \text{ with! } m' \xrightarrow{\text{err}} \text{with! failed} \text{ — representation types not equal}} \quad \text{m.red.err.with!1}$$

$$\frac{\begin{array}{l} m = [T1, e] \text{ as } [X::\text{Type}, T2] \\ m' = [T1', e'] \text{ as } [Y::\text{EQ}(T1''), T2'] \\ \text{not exists } Z, TZ \text{ such that } T1' = T1'' = Z \text{ and } N(Z) = \text{EQ}(TZ) \end{array}}{N \vdash m \text{ with! } m' \xrightarrow{\text{err}} \text{with! failed} \text{ — can only with! to an abstract type}} \quad \text{m.red.err.with!2}$$

$$\frac{\begin{array}{l} m = [T1, e] \text{ as } [X::\text{EQ}(T1'''), T2] \\ m' = [T1', e'] \text{ as } [Y::\text{EQ}(T1'''), T2'] \end{array}}{N \vdash m \text{ with! } m' \xrightarrow{\text{err}} \text{with! failed} \text{ — can only apply with! to an abstract}} \quad \text{m.red.err.with!3}$$

We omit definitions of errors for bogus applications etc.

Evaluation contexts:

$$\begin{array}{l} A1 ::= \_ m2 \quad | \quad \text{mval } \_ \quad | \quad \_ \text{ with! mval } \quad | \quad m1 \text{ with! } \_ \quad | \quad \_::\text{SS} \\ A2 ::= \_ m2 \quad | \quad \text{mval } \_ \quad | \quad \_ \text{ with! } \_ \quad | \quad m1 \text{ with! } \_ \quad | \quad \_::\text{SS} \\ \\ C1 ::= \_ \quad | \quad A1[C1] \quad \quad \quad \text{(for the (1) reductions)} \\ C2h ::= A2 \quad | \quad A1[C2h] \\ C2 ::= \_ \quad | \quad C2h \quad \quad \quad \text{(for the (2) reductions)} \end{array}$$

$$\frac{}{N \vdash m \Longrightarrow m} \quad \text{m.wred.refl} \quad \frac{N \vdash m \longrightarrow_i \text{New } N' \text{ in } m' \quad \text{dom}(N') \text{ and } \text{fv}(Ci) \text{ disjoint}}{N \vdash Ci[m] \Longrightarrow \text{New } N' \text{ in } Ci[m']} \quad \text{m.wred.red}$$

$$\frac{\begin{array}{l} N \vdash m \Longrightarrow \text{New } N' \text{ in } m' \\ N, N' \vdash m' \Longrightarrow \text{New } N'' \text{ in } m'' \\ N, N' \text{ and } N'' \text{ have disjoint domains} \end{array}}{N \vdash m \Longrightarrow \text{New } N', N'' \text{ in } m''} \quad \text{m.wred.tran}$$

$$\frac{N \vdash m \xrightarrow{\text{err}} s}{N \vdash C1[m] \xrightarrow{\text{err}} s} \quad \text{m.werr.err} \quad \frac{\begin{array}{l} N \vdash m \Longrightarrow \text{New } N' \text{ in } m' \\ N, N' \vdash m \xrightarrow{\text{err}} s \\ N \text{ and } N' \text{ have disjoint domains} \end{array}}{N \vdash m \xrightarrow{\text{err}} s} \quad \text{m.werr.tran}$$

### A.3 Process reduction

$$\boxed{e \longrightarrow e'} \quad \boxed{e \xrightarrow{\text{err}}}$$

Define a partial function  $\{ \_ / \_ \}'$  taking a core expression and pattern (in which all variables are distinct) and giving a substitution:

$$\begin{array}{l} \{e/x\}' = \{e/x\} \\ \{[e1 \dots en]/[pat1 \dots patn]\}' = \{e1/pat1\}' \cup \dots \cup \{en/patn\}' \\ \{e/pat\}' \quad \quad \quad \text{undefined otherwise} \end{array}$$

Define a structural equivalence  $==$  over core expressions to be the

least relation generated by axioms:

$0|e1 == e1$  sc.id     $e1|e2 == e2|e1$  sc.com     $e1|(e2|e3) == (e1|e2)|e3$  sc.ass  
 $e1| \text{new } x:T \text{ in } e2 == \text{new } x:T \text{ in } e1|e2$  if  $x$  not free in  $e1$     sc.ext

and standard rules sc.cong.par, sc.cong.new, sc.refl, sc.sym, sc.tran.  
 Note it is important not to have congruence rules for tuples, out, or in.

$$\frac{\{e1/pat\}' \text{ defined}}{x!e1 | x?pat \triangleright e2 \longrightarrow \{e1/pat\}' e2} \text{ e.red.comm}$$

$$\frac{e1 \longrightarrow e1'}{e1|e2 \longrightarrow e1'|e2} \text{ e.red.par} \quad \frac{e1 \longrightarrow e1'}{\text{new } x:T \text{ in } e1 \longrightarrow \text{new } x:T \text{ in } e1'} \text{ e.red.res}$$

$$\frac{e2 == e1 \longrightarrow e1' == e2'}{e2 \longrightarrow e2'} \text{ e.red.sc}$$

$$\frac{\{e1/pat\}' \text{ not defined}}{x!e1 | x?pat \triangleright e2 \xrightarrow{\text{err}}} \text{ e.err.comm} \quad \frac{e2 == e1 \xrightarrow{\text{err}}}{e2 \xrightarrow{\text{err}}} \text{ e.err.sc}$$

$$\frac{e1 \xrightarrow{\text{err}}}{e1|e2 \xrightarrow{\text{err}}} \text{ e.err.par} \quad \frac{e1 \xrightarrow{\text{err}}}{\text{new } x:T \text{ in } e1 \xrightarrow{\text{err}}} \text{ e.err.res}$$

## A.4 Whole System

$$\boxed{N, F, e \xrightarrow{\text{Com}} N', F', e'}$$

typify(N), env(F)  $\vdash m : S$   
 $N \vdash F(m) \implies \text{New } N' \text{ in mval}$   
 $\text{dom}(N)$  disjoint from  $\text{fv}(m)$  and  $\text{dom}(N')$   
 $N, F, e \xrightarrow{U:=m} (N, N'), F \oplus (U \mapsto \text{mval}), e$  build

$$\frac{F(U) = [T, e'] \text{ as } [X::K, \text{proc}]}{N, F, e \xrightarrow{\text{run } U} N, F, e|e'} \text{ load} \quad \frac{e \longrightarrow e'}{N, F, e \xrightarrow{\text{tau}} N, F, e'} \text{ compute}$$

$$\boxed{N, F, e \xrightarrow{\text{Com}} \text{err}(s)}$$

not ( typify(N), sigify(F)  $\vdash m : S$  )  
 $\text{dom}(N)$  not free in  $m$   
 $N, F, e \xrightarrow{U:=m} \text{err}(\text{source doesn't typecheck})$  build.err.1

typify(N), sigify(F)  $\vdash m : S$   
 $\text{dom}(N)$  not free in  $m$   
 $N \vdash F(m) \xrightarrow{\text{err}} s$   
 $N, F, e \xrightarrow{U:=m} \text{err}(s)$  build.err.2

$$\frac{e \xrightarrow{\text{err}}}{N, F, e \xrightarrow{\text{tau}} \text{err}(\text{runtime error})} \text{ compute.err}$$

$F(U)$  not of the form  $[T, e'] \text{ as } [X::K, \text{proc}]$   
 $N, F, e \xrightarrow{\text{run } U} \text{err}(\text{cannot run a non-proc})$  run.err.1

$U$  not in  $\text{dom } F$   
 $N, F, e \xrightarrow{\text{run } U} \text{err}(\text{file not found})$  run.err.2

## B Metatheory

This appendix gives the proofs of the results stated in §3.5 and §3.6.

The non-standard substitution  $\{mval/U\}$  of an  $mval$  for a module variable (used in  $m.red.beta$ ) is defined by induction over each syntactic category  $m, S, e, T, K, E$ . If  $mval = [T1, e]$  as  $[X::EQ(T1'), T2]$  simultaneously replace

U.Type by T1  
 U.term by e  
 U by  $[T1, e]$  as  $[X::EQ(T1'), T2]$  in all other contexts

If  $mval = \lambda U:S.m:S'$  then  $\{mval/U\}$  is standard substitution.

The auxiliary functions `typify`, `sig` and `env` are defined as follows.

`typify(empty)=empty`                      `typify(E, X::K)=typify(E), X::Type`  
`typify(E, x:T)=typify(E)`                `typify(E, U:S) =typify(E)`

`sig( [T, e] as [X::K, T'] ) = [X::K, T']`  
`sig(  $\lambda U:SS.m:S'$  ) =  $\Pi U:SS.S'$`

`env(empty)=empty`  
`env(F, (U, mval)) = env(F), U:sig(mval)`

Recall that we are not considering judgements  $E \vdash J$  up to renaming of the variables declared in  $E$  (as we use module variables for filenames). We do, though, use without proof the fact that if  $E \vdash J$  and  $E', J'$  is a bijective renaming of  $E, J$  then  $E' \vdash J'$ . Strictly speaking, we are often doing inductions over renaming-equivalence-classes of derivations, but making that explicit would not add useful rigor. The most routine inductions and cases are omitted throughout.

### B.1 Simple admissible rules, ok-ness and weakening

**Lemma 1** The rules below are admissible.

$$\frac{E \vdash K}{E \vdash K == K} K=.refl \quad \frac{E \vdash K == K'}{E \vdash K' == K} K=.sym \quad \frac{E \vdash K == K' \quad E \vdash K' == K''}{E \vdash K == K''} K=.tran \quad \frac{E \vdash K <:: K' \quad E \vdash K' <:: K''}{E \vdash K <:: K''} K<.tran$$

$$\frac{E \vdash K}{E \vdash K <:: Type} K<.top \quad \frac{E \vdash T :: K}{E \vdash EQ(T) <:: K} (*) \quad \frac{E \vdash U : [X::EQ(T), T']}{E \vdash U.Type == T :: Type} T=.abbrev$$

It is also worth noting that an `m.self-term` rule is admissible.

**Lemma 2** The rule below is admissible.

$$\frac{E \vdash U : [X::K, T] \quad E \vdash U.term : T'}{E \vdash U : [X::K, T']} m.self-term$$

**Proof**

Lemma If  $E \vdash U.term : T'$  then for some  $X, K'', T''$  we have  $E \vdash U : [X::K'', T'']$

and  $E \vdash T' = T'' :: \text{Type}$ .

Proof Simple ind on  $E \vdash e:T$ , with only  $e.\text{proj}$  and  $e.T =$  non-vacuous.

By this Lemma and the second premise we have

$E \vdash U : [X::K'', T'']$                       b  
 $E \vdash T' = T'' :: \text{Type}$ .                      c

By the first premise and  $T.\text{Proj}$              $E \vdash U:\text{Type} :: K$             d  
 By (b,d) and  $m.\text{self-type}$                  $E \vdash U : [X::K, T'']$   
 By (c) by various rules                     $E \vdash [X::K, T''] < [X::K, T'] \text{ sig}$   
 By  $m.S <$                                        $E \vdash U : [X::K, T']$

**Lemma 3** If  $E \vdash S < S' \text{ sig}$  or  $E \vdash S = S' \text{ sig}$  then  $S$  is a struct sig iff  $S'$  is.

**Lemma 4** (ok-ness for  $\vdash$ ) If  $E, E' \vdash J$  then  $E \vdash \text{ok}$ .

**Lemma 5** (Weakening for  $\vdash$ ) If  $E, E'' \vdash J$  and  $E, E' \vdash \text{ok}$  and  $\text{dom } E'$ ,  $\text{dom } E''$  disjoint and if  $J$  is  $\text{pat}:T \triangleright E_1$  then  $\text{dom } E', \text{var pat}$  disjoint then  $E, E', E'' \vdash J$ .

## B.2 Well-formedness

To prove that in any derivable judgement  $E \vdash J$  all kinds, types and signatures in  $J$  are well-formed, we define an auxiliary characterisation of the formation judgements. This makes explicit the fact that formation does not depend on the precise kind or signature assumptions in the type environment, and hence (Lemmas 11, 12) that they can be arbitrarily changed. One can then prove Lemma 15; in particular the cases for  $S < S'$  and  $S = S'$ .

$E' \vdash K$

$\frac{E' \vdash \text{ok}}{E' \vdash \text{Type}} K'.\text{Type}$                        $\frac{E' \vdash T :: \text{Type}}{E' \vdash \text{EQ}(T)} K'.\text{EQ}$

$E' \vdash T :: \text{Type}$

$\frac{E' \vdash \text{ok}}{E' \vdash [T_1 \dots T_n] :: \text{Type}} T'.\text{rec}$                        $\frac{E' \vdash T :: \text{Type}}{E' \vdash T \text{ chan} :: \text{Type}} T'.\text{chan}$                        $\frac{E' \vdash \text{ok}}{E' \vdash \text{proc} :: \text{Type}} T'.\text{proc}$                        $\frac{E' \vdash \text{ok}}{E' \vdash \text{int} :: \text{Type}} T'.\text{int}$

$\frac{E, X::K, E' \vdash \text{ok}}{E, X::K, E' \vdash X::\text{Type}} T'.\text{var}$                        $\frac{E, U:[X::K, T], E' \vdash \text{ok}}{E, U:[X::K, T], E' \vdash U.\text{Type} :: \text{Type}} T'.\text{Proj}$

$E' \vdash S \text{ sig}$

$\frac{E, X::K \vdash T :: \text{Type}}{E' \vdash [X::K, T] \text{ sig}} S'.\text{Struct}$                        $\frac{E' \vdash SS \text{ sig} \quad E, U:SS \vdash S' \text{ sig}}{E' \vdash \text{IIU}:SS.S' \text{ sig}} S'.\text{Fun}$

$E \vdash' \text{ ok}$

$E'.\text{empty}$	$E'.\text{term}$	$E'.\text{type}$	$E'.\text{mod}$
	$E \vdash' T :: \text{Type}$	$E \vdash' K$	$E \vdash' S \text{ sig}$
	$x \text{ not in dom } E$	$X \text{ not in dom } E$	$U \text{ not in dom } E$
$\text{empty} \vdash' \text{ ok}$	$E, x : T \vdash' \text{ ok}$	$E, X :: K \vdash' \text{ ok}$	$E, U : S \vdash' \text{ ok}$

**Lemma 6 (ok-ness for  $\vdash'$ )** If  $E, E' \vdash' J$  then  $E \vdash' \text{ ok}$ .

**Lemma 7 (Weakening for  $\vdash'$ )** If  $E, E'' \vdash' J$  and  $E, E' \vdash \text{ ok}$  and  $\text{dom } E', \text{dom } E''$  disjoint then  $E, E', E'' \vdash' J$ .

**Lemma 8 (Strengthening for  $\vdash'$ )** If  $E, E', E'' \vdash' J$  and  $\text{dom } E'$  not in  $\text{ran } E''$  or in  $J$  then  $E, E'' \vdash' J$ .

**Lemma 9 (Kind well-formedness in environments)**

- a) If  $E, X :: K, E' \vdash \text{ ok}$  then  $E, X :: K, E' \vdash K$ .
- b) If  $E, U : [X :: K, T], E' \vdash \text{ ok}$  then  $E, U : [X :: K, T], E' \vdash K$ .

**Proof**

- a) By 4  $E, X :: K \vdash \text{ ok}$ , which must be derived by  $E.\text{type}$ , so  $E \vdash K$ . By 5  $E, X :: K, E' \vdash K$ .
- b) By 4  $E, U : [X :: K, T] \vdash \text{ ok}$ , which must be derived by  $E.\text{mod}$ , so  $E \vdash [X :: K, T] \text{ sig}$ , which must be derived by  $S.\text{Struct}$ , so (WLG taking  $X$  not in  $\text{dom } E$ )  $E, X :: K \vdash T :: \text{Type}$ . By 4  $E, X :: K \vdash \text{ ok}$ , which must be derived by  $E.\text{type}$ , so  $E \vdash K$ . By 5  $E, U : [X :: K, T], E' \vdash K$ .

**Lemma 10 ( $\vdash'$  implies  $\vdash$ )**

- If  $E \vdash' K$  then  $E \vdash K$
- If  $E \vdash' T :: \text{Type}$  then  $E \vdash T :: \text{Type}$
- If  $E \vdash' S \text{ sig}$  then  $E \vdash S \text{ sig}$
- If  $E \vdash' \text{ ok}$  then  $E \vdash \text{ ok}$

**Proof**

- [ $T'.\text{var}$ ]** By  $\text{ind}$  and  $T.\text{var}$   $E, X :: K, E' \vdash X :: K$ . By 9, 1 and  $T.K < E, X :: K, E' \vdash X :: \text{Type}$ .
- [ $T'.\text{Proj}$ ]** By  $\text{ind}$ ,  $m.\text{var}$ ,  $T.\text{Proj}$ , 9, 1 and  $T.K < E, X :: K, E' \vdash X :: \text{Type}$ .

**Lemma 11 (Preservation of  $\vdash'$  by kind changes)**

If  $E, X :: K, E' \vdash' J$  and  $E \vdash' K'$  then  $E, X :: K', E' \vdash' J$

**Lemma 12 (Preservation of  $\vdash'$  by struct sig changes)**

If  $E, U : SS, E' \vdash' J$  and  $E \vdash' SS' \text{ sig}$  then  $E, U : SS', E' \vdash' J$

**Lemma 13 ( $\vdash$  implies  $\vdash'$ )**

If $E \vdash K$	then $E \vdash' K$
If $E \vdash K <:: K'$	then $E \vdash' K$ and $E \vdash' K'$
If $E \vdash K == K'$	then $E \vdash' K$ and $E \vdash' K'$
If $E \vdash T::K$	then $E \vdash' T::\text{Type}$ and $E \vdash' K$
If $E \vdash T == T' :: K$	then $E \vdash' T::\text{Type}$ and $E \vdash' T'::\text{Type}$
If $E \vdash S \text{ sig}$	then $E \vdash' S \text{ sig}$
If $E \vdash S < S' \text{ sig}$	then $E \vdash' S \text{ sig}$ and $E \vdash' S' \text{ sig}$
If $E \vdash S == S' \text{ sig}$	then $E \vdash' S \text{ sig}$ and $E \vdash' S' \text{ sig}$
If $E \vdash \text{ok}$	then $E \vdash' \text{ok}$
If $E \vdash U:[X::K,T]$	then $E \vdash' K$ and $E=E1,U:SS,E2$ for some $E1,SS,E2$

**Proof**

**[T.Proj]** By ind  $E \vdash' K$  and  $E$  equals some  $E1,U:SS,E2$ . By 6  $E \vdash' \text{ok}$ .

By  $T'.\text{Proj}$   $E \vdash' U.\text{Type} :: \text{Type}$

**[T.var]** Consider  $E=E1,X::K,E2$  and  $E \vdash X::K$ . By ind  $E \vdash' \text{ok}$ . By  $T'.\text{var}$   $E \vdash' X::\text{Type}$ .

By 6  $E1,X::K \vdash' \text{ok}$ , which must be derived by  $E'.\text{type}$ , so  $E \vdash' K$ . By 7  $E,X::K,E \vdash' K$ .

**[S<.Struct]** By ind  $E \vdash' K$  and  $E \vdash' K'$  and  $E,X::K \vdash' T :: \text{Type}$  and  $E,X::K \vdash' T' :: \text{Type}$ .

By  $S'.\text{Struct}$   $E \vdash' [X::K,T] \text{ sig}$ . By 11  $E,X::K \vdash' T' :: \text{Type}$ .

By  $S'.\text{Struct}$   $E \vdash' [X::K',T'] \text{ sig}$ .

**[S<.Fun]** By ind  $E \vdash' SS1' \text{ sig}$  and  $E \vdash' SS1 \text{ sig}$  and  $E,U:SS1 \vdash' S2 \text{ sig}$  and  $E,U:SS1' \vdash' S2' \text{ sig}$ .

By  $S'.\text{Fun}$   $E \vdash' \text{IIU:SS1'.S2'} \text{ sig}$ . By 12  $E,U:SS1 \vdash' S2 \text{ sig}$ .

By  $S'.\text{Fun}$   $E \vdash' \text{IIU:SS1.S2} \text{ sig}$ .

**[m.var]** Consider  $E=E1,U:[X::K,T],E2$  and  $E \vdash U:[X::K,T]$ . By ind  $E \vdash' \text{ok}$

By 6  $E1,U:[X::K,T] \vdash' \text{ok}$ , which must be derived by  $E'.\text{mod}$ , so

$E \vdash' [X::K,T] \text{ sig}$ , which must be derived by  $S'.\text{Struct}$ , so (WLG taking  $X$

not in dom  $E$ )  $E1,X::K \vdash' T::\text{Type}$ . By 6  $E1,X::K \vdash' \text{ok}$ , which must be

derived by  $E'.\text{type}$ , so  $E \vdash' K$ . By 7  $E1,U:[X::K,T],E2 \vdash' K$ .

**[m.S<]** Say  $S'=[X::K',T']$ . By 3  $S$  is a struct sig, say  $[X::K,T]$ .

By ind  $E=E1,U:SS,E2$  for some  $E1,SS,E2$ . By ind  $E \vdash' [X::K',T'] \text{ sig}$

This must be by  $S'.\text{Struct}$ , so (WLG taking  $X$  not in dom  $E$ )  $E,X::K \vdash' T'::\text{Type}$ .

By 6  $E,X::K \vdash' \text{ok}$ , which must be by  $E'.\text{type}$ , so  $E \vdash' K'$ .

**Lemma 14 (Strengthening for  $\vdash$  formation judgements)** For  $J$  one of

$K, T::\text{Type}, S \text{ sig}$  and  $\text{ok}$ , if  $E,E',E'' \vdash J$  and

dom  $E'$  not in ran  $E''$  or in  $J$  then  $E,E'' \vdash J$ .

**Proof** This follows from the result 8 for  $\vdash'$  and Lemmas 13,10.

**Lemma 15 (Kind, type and sig well-formedness for  $\vdash$ )**

If $E \vdash K <:: K'$	then $E \vdash K$ and $E \vdash K'$
If $E \vdash K == K'$	then $E \vdash K$ and $E \vdash K'$
If $E \vdash T::K$	then $E \vdash K$
If $E \vdash T == T' :: K$	then $E \vdash T::\text{Type}$ and $E \vdash T'::\text{Type}$
If $E \vdash S < S' \text{ sig}$	then $E \vdash S \text{ sig}$ and $E \vdash S' \text{ sig}$
If $E \vdash S == S' \text{ sig}$	then $E \vdash S \text{ sig}$ and $E \vdash S' \text{ sig}$
If $E \vdash \text{pat} : T \triangleright E'$	then $E \vdash T::\text{Type}$ and $E,E' \vdash \text{ok}$
If $E \vdash e:T$	then $E \vdash T::\text{Type}$
If $E \vdash m:S$	then $E \vdash S \text{ sig}$

## Proof

$\boxed{E \vdash K <:: K'}$   $\boxed{E \vdash K == K'}$   $\boxed{E \vdash T::K}$   $\boxed{E \vdash T == T' :: K}$   $\boxed{E \vdash S < S' \text{ sig}}$   $\boxed{E \vdash S == S' \text{ sig}}$   
 All immediate from 13 and 10.

$\boxed{E \vdash \text{pat} : T \triangleright E'}$   $\boxed{E \vdash e:T}$  Routine.

$\boxed{E \vdash m:S}$  Induction on derivations:

**[m.var]** By 4  $E, U:S \vdash \text{ok}$ , which must be by  $E.\text{mod}$ , so  $E \vdash S \text{ sig}$ ,  
 so by 5  $E, U:S, E' \vdash S \text{ sig}$ .

**[m.S<]** By the  $S < S'$  part.

**[m.struct]** By  $S.\text{Struct}$ .

**[m.self-type]** By ind  $E \vdash [X::K, T] \text{ sig}$ , so (WLG X)  $E, X::K \vdash T::\text{Type}$ ,  
 so by 13  $E, X::K \vdash T::\text{Type}$ . Also by 13  $E \vdash K'$ . By 11  $E, X::K' \vdash T::\text{Type}$ , so by  
 $S'.\text{Struct}$   $E \vdash [X::K', T] \text{ sig}$  and by 10  $E \vdash [X::K', T] \text{ sig}$ .

**[m.fun]** By ind and  $S.\text{Fun}$ .

**[m.app]** By ind  $E \vdash \text{IIU:SS1.S2 sig}$ , so (WLG U)  $E, U:SS1 \vdash S2 \text{ sig}$ . By 14  $E \vdash S2 \text{ sig}$ .

**[m.seal]** By ind.

**[m.with]** By ind  $E \vdash [X::K, T] \text{ sig}$ , so (WLG X)  $E, X::K \vdash T::\text{Type}$ , so by 13  $E, X::K \vdash T::\text{Type}$ .  
 Also by ind  $E \vdash [Y::\text{EQ}(T'), T2'] \text{ sig}$ , so  $E \vdash \text{EQ}(T')$ .

By 11  $E, X::\text{EQ}(T') \vdash T2::\text{Type}$ , so by  $S'.\text{Struct}$   $E \vdash [X::\text{EQ}(T'), T2] \text{ sig}$   
 and by 10  $E \vdash [X::\text{EQ}(T'), T2] \text{ sig}$ .

**[m.new]** By ind and 14.

## B.3 Strengthening, Permutation, and Narrowing

**Lemma 16 (Strengthening of  $\vdash$  by term bindings)** If  $E, E', E'' \vdash J$  and  $E'$   
 has only term bindings and  $\text{dom } E'$  not in  $J$  then  $E, E'' \vdash J$ .

**Lemma 17 (Substitution – type)** If  $E, Z::KZ, E' \vdash J$  and  $E \vdash TZ::\text{Type}$   
 and  $KZ$  is either  $\text{Type}$  or  $\text{EQ}(TZ)$  then  $E, \{TZ/Z\}E' \vdash \{TZ/Z\}J$ .

**Lemma 18 (Strengthening of  $\vdash$  by type bindings)** If  $E, X::K, E' \vdash J$  and  
 $X$  not in  $E'$  or  $J$  then  $E, E' \vdash J$ .

**Proof** Immediate from 17, taking cases of  $K=\text{EQ}(T)$  or  $(K=\text{Type and } T=\text{proc})$ .

**Lemma 19 (Permutation)** If  $E1, E2, E3, E4 \vdash J$  and  $\text{dom } E2$  not in  $\text{ran } E3$   
 then  $E1, E3, E2, E4 \vdash J$ .

**Proof** The only remotely interesting cases are for  $E \vdash \text{ok}$  – we give just one.

**[E.type]** We have  $E1, E2, E3, E4 = E, X::K$ .

Case  $E4=\text{empty}$  and  $E3=\text{empty}$ . Trivial.

Case  $E4=E4', X::K$ . By ind  $E1, E3, E2, E4' \vdash K$ , so by  $E.\text{type}$   $E1, E3, E2, E4 \vdash \text{ok}$ .

Case  $E4=\text{empty}$  and  $E3=E3', X::K$ , ie  $E1, E2, E3' \vdash K$ .

By ind  $E1, E3', E2 \vdash K$ . By 4  $E1, E3', E2 \vdash \text{ok}$ . By 14  $E1, E3' \vdash K$ ,

so by  $E.\text{type}$   $E1, E3', X::K \vdash \text{ok}$ . By 5  $E1, E3', X::K, E2 \vdash \text{ok}$ .

**Lemma 20 (Narrowing)** If  $E, Z :: KZ', E' \vdash J$  and  $E \vdash KZ < :: KZ'$  then  $E, Z :: KZ, E' \vdash J$ .

**Proof**

**[T.var]** Consider cases of  $X :: K$  before, equal to, or after  $Z :: KZ'$ .

Before and after are by ind. For equal to, by ind and T.var  $E, Z :: KZ, E' \vdash Z :: KZ$ .

By weakening 5  $E, Z :: KZ, E' \vdash KZ < :: KZ'$ . Then use T.K<.

**[E.type]** Consider cases of  $E'$  empty or  $E' = E1', X :: K$

**Lemma 21 (Narrowing and Weakening by typify)** If  $\text{typify}(E) \vdash J$  and  $E \vdash \text{ok}$  then  $E \vdash J$ .

**Proof** We prove by ind on  $E'$  that  $E, \text{typify}(E') \vdash J$  and  $E, E' \vdash \text{ok}$  implies  $E, E' \vdash J$

(in fact using a reversed characterisation of typify).

**[E, (empty)]** Trivial.

**[E, (X :: K, E')]** We have  $E, X :: \text{Type}, \text{typify}(E') \vdash J$  and  $E, X :: K, E' \vdash \text{ok}$

By formation  $E \vdash K$ , so  $E \vdash K < :: \text{Type}$ , so by 20  $E, X :: K, \text{typify}(E') \vdash J$ ,

so by ind (as also  $(E, X :: K), E' \vdash \text{ok}$ )  $E, X :: K, E' \vdash J$ .

**[E, (U :: S, E')]** We have  $E, \text{typify}(E') \vdash J$  and  $E, U :: S, E' \vdash \text{ok}$ .

By formation  $E \vdash S$  sig, so by weakening 5  $E, U :: S, \text{typify}(E') \vdash J$ . By ind  $E, U :: S, E' \vdash J$ .

**[E, (x :: T, E')]** Similar.

## B.4 Type cancellation

To prove a cancellation lemma (25) for type equality we define another auxiliary characterisation, of  $E \vdash T = T' :: \text{Type}$  judgements, in terms of plain equational logic. This removes the need to chase through the kinding, signature and module judgements in the proof of the cancellation lemma.

$$\frac{E, X :: \text{EQ}(T), E' \vdash \text{ok}}{E, X :: \text{EQ}(T), E' \vdash X == T :: \text{Type}} \text{T}' . \text{var}$$

$$\frac{E, U : [X :: \text{EQ}(T1), T2], E' \vdash \text{ok}}{E, U : [X :: \text{EQ}(T1), T2], E' \vdash U . \text{Type} == T1 :: \text{Type}} \text{T}' . \text{abbrev} \quad \frac{E \vdash T :: \text{Type}}{E \vdash T = T :: \text{Type}} \text{T}' . \text{refl}$$

together with rules  $\text{T}' . \text{sym}$ ,  $\text{T}' . \text{tran}$ ,  $\text{T}' . \text{cong.rec}$ ,  $\text{T}' . \text{cong.chan}$

### Lemma 22

If  $E \vdash K < :: K'$  then  $K = \text{Type}$  implies  $K' = \text{Type}$

If  $E \vdash K = K'$  then  $(K = \text{Type} \text{ iff } K' = \text{Type})$

If  $E \vdash [X :: K, T] < [X :: K', T']$  sig then  $K = \text{Type}$  implies  $K' = \text{Type}$

If  $E \vdash [X :: K, T] = [X :: K', T']$  sig then  $(K = \text{Type} \text{ iff } K' = \text{Type})$

### Lemma 23 ( $\vdash$ implies $\vdash'$ – type equality)

If  $E \vdash \text{EQ}(T) < :: \text{EQ}(T')$  then  $E \vdash' T = T' :: \text{Type}$

If  $E \vdash \text{EQ}(T) = \text{EQ}(T')$  then  $E \vdash' T = T' :: \text{Type}$

If  $E \vdash T :: \text{EQ}(T')$  then  $E \vdash' T = T' :: \text{Type}$

If  $E \vdash T = T' :: K$  then  $E \vdash' T = T' :: \text{Type}$

If  $E \vdash [X :: \text{EQ}(T1), T2] < [X :: \text{EQ}(T1'), T2']$  sig then  $E \vdash' T1 = T1' :: \text{Type}$

If  $E \vdash [X :: \text{EQ}(T1), T2] = [X :: \text{EQ}(T1'), T2']$  sig then  $E \vdash' T1 = T1' :: \text{Type}$

If  $E \vdash U : [X :: \text{EQ}(T1), T2]$  then  $E \vdash' U . \text{Type} = T1 :: \text{Type}$



**Proof**

[T.K<] By 22 we can take  $K=EQ(T1)$  and  $K=EQ(T1')$ , then by ind  $E \vdash' T==T1::Type$  and  $E \vdash' T1==T1'::Type$ , so by  $T='.tran$   $E \vdash' T==T1'::Type$ .

[T.var] By 13 and  $T='.var$ .

[T=.refl] By 13 and  $T'=.refl$

[S<.tran] By 22 and ind and  $T'=.tran$ .

[m.var] By 13  $E,U:[X::EQ(T1),T2],E' \vdash'$  ok, then use  $T='.abbrev$ .

[m.S<] By 22 we can take  $S=[X::EQ(T1),T2]$  and  $S'=[X::EQ(T1'),T2']$ .

By ind  $E \vdash' U.Type==T1::Type$  and  $E \vdash' T1==T1'::Type$ , so by  $T='.tran$   $E \vdash' U.Type == T1'::Type$ .

[m.self-type] By ind.

**Lemma 24 ( $\vdash'$  implies  $\vdash$  - type equality)** If  $E \vdash' T==T'::Type$  then  $E \vdash T==T'::Type$ .

**Proof** Induction, using 10.

**Lemma 25 (Cancellation - type equality)** If  $E \vdash T == T' :: Type$ , if both  $T$  and  $T'$  are

of the form  $[T .. T] \mid \text{chan } T \mid \text{proc} \mid \text{int}$  (not necessarily the same) then either

- 1)  $T = [T1..Tn]$  and  $T'=[T1'..Tn']$  and for  $i=1..n$   $E \vdash Ti==Ti'::Type$ ,
- 2)  $T = \text{chan } T1$  and  $T'=\text{chan } T1'$  and  $E \vdash T1==T1'::Type$ ,
- 3)  $T = \text{proc} = T'$ , or
- 4)  $T = \text{int} = T'$ .

**Proof**

We prove the result for  $\vdash'$ , by induction on the sequence of equations (ie bindings  $X::EQ(T)$  or  $U:[X::EQ(T1),..]$ ) in  $E$ . Let  $W$  range over types of the forms  $X$  and  $U.Type$ , and use an abbreviated notation for type environments.

**Base:**  $E$  has no equations. The result is obvious, as provable equality coincides with syntactic identity.

**Ind:**  $E=E1,W=TW,E2$  and  $E2$  has no equations.

If  $T$  and  $T'$  are of the specified form then  $\{TW/W\}T$  and  $\{TW/W\}T'$  are.

We have  $E1,W,E2 \vdash' \{TW/W\}T==\{TW/W\}T'::Type$ .

Consider just one direction of the tuple case, ie  $T = [T1..Tn]$

so  $\{TW/W\} T = [\{TW/W\}T1..\{TW/W\}Tn]$

By the induction hypothesis  $\{TW/W\}T'$  is a tuple of length  $n$ ,

but  $T'$  is not a variable, so for some  $T1'..Tn'$   $T'=[T1'..Tn']$

and  $\{TW/W\}T'=[\{TW/W\}T1'..\{TW/W\}Tn']$ .

By the rest of the induction hypothesis

$E1,W,E2 \vdash' \{TW/W\}Ti == \{TW/W\}Ti' :: Type$  (for  $i=1..n$ )

and by weakening  $E1,W=TW,E2 \vdash' \{TW/W\}Ti == \{TW/W\}Ti' :: Type$ .

Now observe that  $E1,W=TW,E2 \vdash' Ti == \{TW/W\}Ti :: Type$  (and similarly for  $Ti'$ )

so  $E1,W=TW,E2 \vdash' Ti==Ti'$ .

## B.5 Deconstruction lemmas

**Lemma 26 (Deconstruction – sig equality)** If  $E \vdash [X::K, T] == [X::K', T']$  sig and  $X$  not in  $E$  then  $E \vdash K == K'$  and  $E, X::K \vdash T == T' :: \text{Type}$ .

**Proof** Induction, using 20 in  $[S=.tran]$ .

**Lemma 27 (Deconstruction – subsig)** If  $E \vdash [X::K, T] < [X::K', T']$  sig and  $X$  not in  $E$  then  $E \vdash K <:: K'$  and  $E, X::K \vdash T == T' :: \text{Type}$

**Proof** Induction, using 20 in  $[S<.tran]$  and 26 in  $[S<.S=]$ .

**Lemma 28 (Deconstruction – module)**

If  $E \vdash [T1, e]$  as  $[X::K, T2] : [X::K', T2']$  and  $X$  not in  $E$  then  
 $E \vdash T1::K, E, X::K \vdash T2 :: \text{Type}, E, X::EQ(T1) \vdash e : T2, X$  not free in  $e$ ,  
 $E \vdash K <:: K'$  and  $E, X::K \vdash T2 == T2' :: \text{Type}$

If  $E \vdash \lambda U:SS1.m:S2 : \Pi U:SS1'.S2'$  and  $U$  not in  $\text{dom } E$  then  
 $E, U:SS1 \vdash m:S2, E \vdash SS1' < SS1$  sig and  $E, U:SS1' \vdash S2 < S2'$  sig.

If  $E \vdash \text{new } x:T \text{ in } m : SS$  and  $x$  not in  $\text{dom } E$  then  
there exists  $T0$  such that  $T = \text{chan } T0$  and  $E, x:T \vdash m : SS$ .

If  $E \vdash m m' : S$  then there exist  $U, SS1, S2$  such that  
 $E \vdash m : \Pi U:SS1.S2, E \vdash m' : SS1, U$  not in  $S2$  and  $E \vdash S2 < S$  sig.

If  $E \vdash m$  with!  $m' : S$  then there exist  $X, K, T2, Y, T', T2'$  such that  
 $E \vdash m : [X::K, T2], E \vdash m' : [Y::EQ(T'), T2']$  and  $E \vdash [X::EQ(T'), T2] < S$  sig.

If  $E \vdash (m:S) : S'$  then  $E \vdash m:S$  and  $E \vdash S < S'$  sig.

We note some easy consequences of the conclusion of 28:

By  $T.K < E \vdash T1 :: K'$ . By 1  $E \vdash EQ(T1) <:: K$ . By 20  
 $E, X::EQ(T1) \vdash T2 == T2' :: \text{Type}$ . By  $e.T = E, X::EQ(T1) \vdash e : T2'$ .

Hence by  $m.\text{struct}$   $E \vdash [T1, e]$  as  $[X::K, T2] : [X::K, T2]$   
and by  $S<.Struct$   $E \vdash [X::K, T2] < [X::K', T2']$  sig

**Lemma 29 (Deconstruction – term)**

If  $E \vdash e1|e2:T$  then  $E \vdash e1:\text{proc}$  and  $E \vdash e2:\text{proc}$  and  $E \vdash T == \text{proc} :: \text{Type}$ .

If  $E \vdash \text{new } x:T \text{ in } e : T2$  and  $x$  not in  $\text{dom } E$  then  
 $E, x:T \vdash e:T2$  and there exists  $T0$  such that  $T = \text{chan } T0$ .

If  $E$  atomic and has no module bindings and  $E \vdash e:T$  and  $E \vdash T == [T1..Tn] :: \text{Type}$  then  
for some  $e1..en$  we have  $e = [e1 .. en]$  and for  $i=1..n$  we have  $E \vdash ei:Ti$ .

**Proof** We show just the last.

**[e.var]** Vacuous, as  $E$  atomic and by (25)  $\text{not}(E \vdash \text{chan } T == [T1..Tn] :: \text{Type})$

**[e.T=]** By ind hyp.

**[e.rec]** Have  $E \vdash [e1..em] : [T1'..Tm']$  and  $E \vdash \text{ok}$  and for  $i=1..n$   $E \vdash ei:Ti'$  and  
 $E \vdash [T1'..Tm'] == [T1..Tn] :: \text{Type}$ .

By (25)  $m=n$  and for  $i=1..n$   $E \vdash Ti' == Ti :: \text{Type}$ .

By  $e.T =$  for  $i=1..n$   $E \vdash ei:Ti$ .

[e.proj] Vacuous, as E contains no module bindings.  
[e.new][e.nil][e.par][e.out][e.in] Vacuous, as the  
conclusion type is proc in all cases, and by (25) not( $E \vdash \text{proc} = [T1..Tn] :: \text{Type}$ )

## B.6 Module substitution

### Lemma 30 (Substitution – module)

If  $E, U:S, E' \vdash J$  and  $E \vdash \text{mval}:S$  then  $E, \{\text{mval}/U\}E' \vdash \{\text{mval}/U\}J$ .

**Proof** By tedious induction, using 28, 18, 5.

## B.7 Unique Context/Redex Decomposition

This subsection gives the proof that module expressions have unique decompositions as an evaluation context and a redex. The complex structure of evaluation contexts for the different judgements makes it rather finicky, unfortunately.

Consider the judgements  $j$

$$\boxed{N \vdash m \longrightarrow_i \text{New } N' \text{ in } m' \quad i=1,2} \quad \boxed{N \vdash m \xrightarrow{\text{err}} s}$$

and their axioms  $r$

m.red.beta, m.red.seal, m.red.new, m.red.with!  
m.red.abstype  
m.red.err.with!1, m.red.err.with!2, m.red.err.with!3

Let  $C_j$  range over the contexts associated with  $j$  (given by the C1 and C2 grammars).

For an axiom  $r$ , define

MVAL =  $\{m \mid \text{exists } S. N \vdash m:S \text{ and } m \text{ is an mval}\}$   
L( $r$ ) =  $\{m \mid \text{exists } S. N \vdash m:S \text{ and } m \text{ is an instance of the redex of } r\}$

**Lemma 31 (Unique decomposition)** If  $N$  has no module bindings and  $N \vdash m:S$  then either  $m$  is an mval xor there is a unique  $j \in \{1,2\}$ ,  $C_j$ , rule  $r \in m.\text{red}.*$  and instance  $lr$  of the redex of  $r$  such that  $m = C_j[lr]$ .

**Proof**

We prove that if  $N$  has no module bindings then:

- 1) If  $C_j \neq \_$  then  $C_j[m]$  not in MVAL
- 2) For any  $r$ , L( $r$ ) and MVAL are disjoint.
- 3) For  $r \neq r'$ , L( $r$ ) and L( $r'$ ) are disjoint.
- 4) If exists  $S$  such that  $N \vdash m:S$  then exactly one of the following hold:
  - $m$  is in MVAL
  - there is a unique  $(j, C_j, r, lr)$  such that  $m = C_j[lr]$ ,  $r$  is a rule of  $j$ ,  $lr$  in L( $r$ )

1,2,3 are by inspection. 4 is by induction on typing judgements.

[m.var] Vacuous as  $N$  has no module bindings

[m.S<] By induction hyp

[m.struct] Consider  $m = [T1, e]$  as  $[X::K, T2]$ .

Case  $K = \text{Type}$  Take  $j=2$ ,  $C_j = \_$ ,  $r = m.\text{red}.abstype$ , and  $lr = m$ .

By (2)  $m$  not in MVAL. By the form of rules  $m \neq C_j'[lr']$  for any other

$(j',Cj',r',lr')$ , as no other rule has a struct outermost.  
Case T=EQ( $T1'$ )  $m$  in MVAL.  $m$  is not an instance of  $m.red.abstype$ .  
By the form of rules  $m \neq Cj'[lr']$  for any other  
 $(j',Cj',r',lr')$ , as no other rule has a struct outermost.  
**[m.self-type]** By induction hyp.  
**[m.fun]**  $m$  in MVAL. By the form of rules  $m \neq Cj[lr]$  for any  
 $(j,Cj,r,lr)$ , as no rule or context clause has a functor outermost.  
**[m.app]** Consider  $m m'$ . By the form of  $m.app E\vdash m:S$  and  $E\vdash m':SS1$  for  
some functor and structure signatures  $S, SS1$ . In all cases  $(m m')$  not in MVAL. (a)

Case  $m$  in MVAL. Here  $m$  must be a functor expression  
Case  $m'$  in MVAL. Here  $m'$  must be a struct expression, so take  
 $j=1, Cj=_ , r = m.red.beta, lr = m m'$ .  
By the form of rules  $(m m') \neq Cj'[lr']$  for any  
 $(j',Cj',r',lr')$ , as no other rule has an app outermost and  
neither  $m$  nor  $m'$  can be  $Cj''[lr'']$  for a redex instance  $lr''$ .  
Case there is a unique  $(j,Cj',r,lr)$  such that  $m=Cj'[lr]$ ,  $r$  is a rule of  $j,lr$  in  $L(r)$   
Take  $Cj = (m Cj')$ .  
Consider  $(j',Cj'',r',lr')$  such that  $(m m')=Cj''[lr']$ ,  $r'$  is a rule of  $j',lr'$  in  $L(r')$   
Case  $Cj''=_$ , then  $(m m')$  in  $L(r')$ , but it cannot be as  $m'$  not in MVAL. (b)  
Case  $Cj'' = (Cj''' m')$ , then  $m = Cj'''[lr']$ . This contradicts  
uniqueness for  $m$  as  $m$  in MVAL. (c)  
Case  $Cj'' = (mval Cj''')$ , then  $m = mval$  and  $m' = Cj'''[lr']$ .  
This contradicts uniqueness for  $m'$  unless  $j=j', Cj'=Cj'''$ ,  
 $r=r', lr=lr'$ , in which case  $Cj = (m Cj') = Cj''$ . (d)  
By (a)  $(m m')$  not in MVAL, and by (b,c,d) there is no competing 4-tuple.

Case there is a unique  $(j,Cj',r,lr)$  such that  $m=Cj'[lr]$ ,  $r$  is a rule of  $j,lr$  in  $L(r)$   
Take  $Cj = (Cj' m')$ .  
Consider  $(j',Cj'',r',lr')$  with  $r'$  a rule of  $j', lr'$  in  $L(r')$  and  $(m m')=Cj''[lr']$   
Case  $Cj''=_$ , then  $(m m')$  in  $L(r')$ , but it cannot be as  $m$  not in MVAL. (b)  
Case  $Cj'' = (Cj''' m')$ , then  $m = Cj'''[lr']$ .  
This contradicts uniqueness for  $m$  unless  $j=j', Cj'=Cj'''$ ,  
 $r=r', lr=lr'$ , in which case  $Cj = (Cj' m') = Cj''$ . (c)  
Case  $Cj'' = (mval Cj''')$ . Cannot be, as  $m$  not in MVAL (d)  
By (a)  $(m m')$  not in MVAL, and by (b,c,d) there is no competing 4-tuple.

**[m.seal]** Consider  $(m:SS)$ . By the rule  $N\vdash m:SS$ . Applying the induction hypothesis to  $m$ ,  
Case  $m$  in MVAL. As  $N\vdash m:SS$ ,  $m$  must be a struct (and as an MVAL, with an EQ kind).

Take  $j=1, Cj=_ , r = m.red.seal, and lr=(m:SS)$ .  
Consider  $(j',Cj',r',lr')$  with  $r'$  a rule of  $j', lr'$  in  $L(r')$  and  $(m:SS)=Cj'[lr']$   
Case  $Cj'=_$ , then  $lr'=(m:SS)=lr$ . By (3)  $j'=j, r'=r$ .  
Case  $Cj' = (Cj'' :SS)$ , then  $m = Cj''[lr']$ . But  $m$  in MVAL.  
Clearly  $(m:SS)$  is not in MVAL, and by the above there is no competing 4-tuple.  
Case there is a unique  $(j,Cj',r,lr)$  such that  $m=Cj'[lr]$ ,  $r$  is a rule of  $j,lr$  in  $L(r)$   
Take  $Cj = (Cj' :SS)$

**[m.with]** Consider  $m with! m'$ . By the  $m.with$  rule we have  $N\vdash m : SS$   
and  $N\vdash m' : SS'$  for two struct sigs. Clearly  $m with! m'$  is not in MVAL.  
Applying the induction hypothesis to  $m'$ :  
Case  $m'$  in MVAL

Applying the induction hypothesis to  $m$ :  
Case  $m$  in MVAL.  $(m with! m')$  is an instance of  $r = m.red.err.with!3$   
Take  $j=1, Cj=_ , r = m.red.err.with!3, lr = m with! m'$ .

Case there is a unique  $(j, Cj, r, lr)$  such that  $m = Cj[lr]$ ,  $r$  is a rule of  $j, lr$  in  $L(r)$   
 Case  $j=2$  (so  $r = m.red.abstype$ ) and  $Cj = \_$ . Take  $j'=1$ ,  $Cj' = \_$ ,  $lr' = m$  with!  $m'$ ,  
 and  $r$  the unique applicable rule from  $m.red.with!$ ,  $m.red.err.with!1$ ,  
 $m.red.err.with!2$ .  
 Case  $j=2$  (so  $r = m.red.abstype$ ) and  $Cj = C2h$ . Take  $Cj' = Cj$  with!  $m'$ .  
 Case  $j=1$ . Take  $Cj' = Cj$  with!  $m'$ .  
 Case there is a unique  $(j, Cj, r, lr)$  such that  $m' = Cj[lr]$ ,  $r$  is a rule of  $j, lr$  in  $L(r)$   
 Take  $Cj' = (m$  with!  $Cj)$ .  
**[m.new]** Consider  $m = \text{new } x:\text{chan } T$  in  $m'$ .  
 Take  $r = m.red.new$ ,  $Cj = \_$  and  $lr = m$ .  
 By (2,3)  $m$  not in  $L(r')$  for any  $r' \neq r$ , and  $m$  not in  $MVAL$ .  
 Consider  $(j', Cj', r', lr')$  with  $Cj' \neq \_$ ,  $r'$  a rule of  $j'$ ,  $lr'$  in  
 $L(r')$  and  $\text{new } x:\text{chan } T$  in  $m' = Cj'[lr']$ .  
 Must have  $Cj' = \text{new } x:\text{chan } T$  in  $Cj''$ , but it cannot be.

## B.8 Subject Reduction - build time

**Lemma 32** If

$N \vdash \text{ok}$ ,  
 $N$  has no module bindings,  
 forall  $U$  in  $\text{dom}(F)$ .  $N \vdash F(U) : \text{sig}(F(U))$ , and  
 $\text{typify}(N), \text{env}(F) \vdash m : S$

then  $N \vdash F(m) : F(S)$ .

**Proof**

Call the premises a,b,c,d. By d, 30 (for the  $x:T$  parts), narrowing  
 (for the  $X::K$  parts), and  $N$  has no module bindings (for the  $U:S$  parts),  
 we have  $N, \text{env}(F) \vdash m : S$  (e).

By b,c each  $F(U)$  contains no free module identifiers.

Proceed by induction on the length of  $F$ .

For  $F$  empty the result is immediate.

Consider  $N, \text{env}(F, (U, mval)) \vdash m : S$   
 By defn env  $N, \text{env}(F), U : \text{sig}(mval) \vdash m : S$   
 By c and 16  $N, \text{env}(F) \vdash mval : \text{sig}(mval)$   
 By 30  $N, \text{env}(F) \vdash \{mval/U\}m : \{mval/U\}S$   
 By the ind hyp  $N \vdash F(m) : F(S)$ .

**Lemma 33** For  $i=1,2$ , if  $N \vdash m : S$  and  $N \vdash m \rightarrow_i \text{New } N'$  in  $m'$  and  $N, N'$  disjoint  
 then  $N, N' \vdash m' : S$ . Moreover if  $N$  atomic and has no module bindings then  
 $N, N'$  is likewise.

**Proof**

**[m.red.beta]** By 28 have:

$N \vdash \lambda U : SS1.m : S2 : \Pi U : SS1'. S2'$	a
$N \vdash mval : SS1'$	b
$U$ not in $S2'$	c
$N \vdash (\lambda U : SS1.m : S2) mval : S2'$ <span style="float: right;">m.app</span>	

and  $N \vdash S2' < S$ . By Lemma 28  $N, U : SS1 \vdash m : S2$  (d),  $N \vdash SS1' < SS1$  sig (e)  
 and  $N, U : SS1' \vdash S2 < S2'$  sig (f)

By (b,e),m.S<             $\vdash \text{mval} : \text{SS1} \qquad \qquad \qquad \text{g}$   
 By (d,g,30)             $\vdash \{\text{mval}/\text{U}\}_m : \{\text{mval}/\text{U}\}_S \text{S2} \qquad \text{h}$   
 By (f,b,30)             $\vdash \{\text{mval}/\text{U}\}_S \text{S2} < \{\text{mval}/\text{U}\}_S \text{S2}' \text{ sig}$   
 By m.S<             $\vdash \{\text{mval}/\text{U}\}_m : \{\text{mval}/\text{U}\}_S \text{S2}'$   
 By U not in S2'       $\vdash \{\text{mval}/\text{U}\}_m : \text{S2}'$   
 By m.S<             $\vdash \{\text{mval}/\text{U}\}_m : \text{S}$

**[m.red.seal]**

Consider  $\vdash ([\text{T1},\text{e}] \text{ as } \text{SS} : \text{SS}') : \text{S}$  where  $\text{SS}=[\text{X}::\text{EQ}(\text{T1}'),\text{T2}]$ ,  $\text{SS}'=[\text{X}::\text{K}',\text{T2}']$   
 and  $\vdash ([\text{T1},\text{e}] \text{ as } \text{SS} : \text{SS}') \longrightarrow_1 [\text{T1},\text{e}] \text{ as } \text{SS}'$  m.red.seal  
 By 28  $\vdash [\text{T1},\text{e}] \text{ as } \text{SS} : \text{SS}'$  and  $\vdash \text{SS}' < \text{S}$  sig  
 By 28  $\vdash \text{T1}::\text{EQ}(\text{T1}')$  a  
           $\text{N},\text{X}::\text{EQ}(\text{T1}') \vdash \text{T2} :: \text{Type}$  b  
           $\text{N},\text{X}::\text{EQ}(\text{T1}) \vdash \text{e} : \text{T2}$  c  
          X not free in e d  
           $\vdash \text{EQ}(\text{T1}') <:: \text{K}'$  e  
           $\text{N},\text{X}::\text{EQ}(\text{T1}') \vdash \text{T2} == \text{T2}' :: \text{Type}$  f  
 By T.K<             $\vdash \text{T1}::\text{K}'$   
 By 15             $\vdash [\text{X}::\text{K}',\text{T2}'] \text{ sig}$  so (cf S.Struct)  $\text{N},\text{X}::\text{K}' \vdash \text{T2}' :: \text{Type}$   
 By (a)  $\vdash \text{EQ}(\text{T1}) <:: \text{EQ}(\text{T1}')$ . By (c) and narrowing and  $\text{e.T} = \text{N},\text{X}::\text{EQ}(\text{T1}) \vdash \text{e} : \text{T2}'$   
 By m.structure       $\vdash [\text{T1},\text{e}] \text{ as } [\text{X}::\text{K}',\text{T2}'] : [\text{X}::\text{K}',\text{T2}']$   
 By m.S<             $\vdash [\text{T1},\text{e}] \text{ as } [\text{X}::\text{K}',\text{T2}'] : \text{S}$

**[m.red.new]** Immediate from (28).

**[m.red.with!]** Consider a redex  $(\text{m with! } \text{m}') \text{ and } \vdash \text{m with! } \text{m}' : \text{S}$ . Have  
 $\text{m} = [\text{T1},\text{e}] \text{ as } [\text{X}::\text{Type},\text{T2}]$  a  
 $\text{m}' = [\text{Z},\text{e}'] \text{ as } [\text{Y}::\text{EQ}(\text{Z}),\text{T2}']$  c  
 $\text{N}(\text{Z}) = \text{EQ}(\text{TZ})$  f  
 $\text{typify}(\text{N}) \vdash \text{T1} = \text{TZ} :: \text{Type}$  g  


---

 $\vdash \text{m with! } \text{m}' \longrightarrow_1 [\text{Z},\text{e}] \text{ as } [\text{X}::\text{EQ}(\text{Z}),\text{T2}] \text{ m.red.with!}$   
 ie  
 $\vdash [\text{T1},\text{e}] \text{ as } [\text{X}::\text{Type},\text{T2}] \text{ with! } [\text{Z},\text{e}'] \text{ as } [\text{Y}::\text{EQ}(\text{Z}),\text{T2}'] \longrightarrow_1 [\text{Z},\text{e}] \text{ as } [\text{X}::\text{EQ}(\text{Z}),\text{T2}]$

and by (28)

$\vdash [\text{T1},\text{e}] \text{ as } [\text{X}::\text{Type},\text{T2}] : [\text{X}::\text{K}'',\text{T2}'']$  h  
 $\vdash [\text{Z},\text{e}'] \text{ as } [\text{Y}::\text{EQ}(\text{Z}),\text{T2}'] : [\text{Y}::\text{EQ}(\text{T1}'''),\text{T2}''']$  i  
 $\vdash [\text{X}::\text{EQ}(\text{T1}'''),\text{T2}'''] < \text{S}$  sig m  
 (wlg choose X,Y distinct from dom N and fv e). By well-formedness  $\vdash \text{ok}$ . j  
 By (j),(f) and sundry rules  $\vdash \text{Z} :: \text{EQ}(\text{Z})$  l

By (h),(28), (l) and narrowing     $\text{N},\text{X}::\text{EQ}(\text{Z}) \vdash \text{T2} :: \text{Type}$

By (h) and 28             $\text{N},\text{X}::\text{EQ}(\text{T1}) \vdash \text{e} : \text{T2}$  k  
 By (f),(j)  $\text{N} \vdash \text{Z}::\text{EQ}(\text{TZ})$ . By 1       $\vdash \text{EQ}(\text{Z}) <:: \text{EQ}(\text{TZ})$   
 By (g) and N-narrowing 21       $\vdash \text{T1} = \text{TZ} :: \text{Type}$   
 By sundry rules             $\vdash \text{EQ}(\text{TZ}) <:: \text{EQ}(\text{T1})$   
 By K<.tran             $\vdash \text{EQ}(\text{Z}) <:: \text{EQ}(\text{T1})$   
 By this, (k) and narrowing       $\text{N},\text{X}::\text{EQ}(\text{Z}) \vdash \text{e} : \text{T2}$

By assumption X not in e

By these four and `m.struct`  $N \vdash [Z, e] \text{ as } [X::EQ(Z), T2] : [X::EQ(Z), T2]$

By (i) and 28  $N \vdash EQ(Z) <:: EQ(T1''')$

By (h), 28 and narrowing  $N, X::EQ(Z) \vdash T2 == T2'' :: \text{Type}$

By these two and `S<.Struct`  $N \vdash [X::EQ(Z), T2] < [X::EQ(T1'''), T2'''] \text{ sig}$

then by `m.S<`  $N \vdash [Z, e] \text{ as } [X::EQ(Z), T2] : [X::EQ(T1'''), T2''']$

and by (m), `m.S<`  $N \vdash [Z, e] \text{ as } [X::EQ(Z), T2] : S$

**[m.red.abstype]**

Have  $N \vdash [T1, e] \text{ as } [X::\text{Type}, T2] : [X::K', T2']$

with no top-level news or module ids free in that struct and  $Y$  not in  $\text{dom } N$   
wlg take  $X$  not in  $\text{dom } N$  and  $X \neq Y$ .

By 28

$N \vdash T1 :: \text{Type}$	c
$N, X::\text{Type} \vdash T2 :: \text{Type}$	j
$N, X::EQ(T1) \vdash e : T2$	k
$X$ not free in $e$	l

$N \vdash \text{Type} <:: K'$	m
$N, X::\text{Type} \vdash T2 == T2' :: \text{Type}$	n

By (c) and `K.EQ, E.type`  $N, Y::EQ(T1) \vdash \text{ok.}$  f

By `T.var, T.K<, K<.EQ, T=.refl, T.T=`  $N, Y::EQ(T1) \vdash Y::EQ(Y)$

By (j), weakening and permutation 19  $N, Y::EQ(T1), X::\text{Type} \vdash T2 :: \text{Type}$

By f, `T.var` and 1  $N, Y::EQ(T1) \vdash EQ(Y) <:: EQ(T1)$  g

By narrowing 20  $N, Y::EQ(T1), X::EQ(Y) \vdash T2 :: \text{Type}$

By (k), weakening, permutation  $N, Y::EQ(T1), X::EQ(T1) \vdash e : T2$

By narrowing 20  $N, Y::EQ(T1), X::EQ(Y) \vdash e : T2$

From these three, (l) and `m.struct`  $N, Y::EQ(T1) \vdash [Y, e] \text{ as } [X::EQ(Y), T2 ] : [X::EQ(Y), T2 ]$

By (c), 1  $N \vdash EQ(T1) <:: \text{Type}$ . By (m) and `K<.tran`  $N \vdash EQ(T1) <:: K'$ .

By weakening  $N, Y::EQ(T1) \vdash EQ(T1) <:: K'$

By (f), `T.var, T=.EQ, K=.T=, K<.K=`  $N, Y::EQ(T1) \vdash EQ(Y) <:: EQ(T1)$

By `K<.tran`  $N, Y::EQ(T1) \vdash EQ(Y) <:: K'$

(could be more direct, as by (d) we really know  $K' = \text{Type}$ )

By (n), weakening, permutation and narrowing  $N, Y::EQ(T1), X::EQ(Y) \vdash T2 == T2' :: \text{Type}$

By these two and `S<.Struct`  $N, Y::EQ(T1) \vdash [X::EQ(Y), T2] < [X::K', T2'] \text{ sig}$

By `m.S<`  $N, Y::EQ(T1) \vdash [Y, e] \text{ as } [X::EQ(Y), T2] : [X::K', T2']$

**Lemma 34** If  $E \vdash C1[m] : S$  then there exists  $S'$  such that  $E \vdash m : S'$  and for all  $E', m'$ , if  $E, E' \vdash m' : S'$  then  $E, E' \vdash C1[m'] : S$ .

**Proof** Induction on the pair of  $C1$  and the type derivation.

If  $C1 = \_$  the result is immediate, so consider otherwise and take

cases of the last type rule in the derivation.

**[m.S<]**  
 $E \vdash C1[m] : S1$   
 $E \vdash S1 < S2 \text{ sig}$   
 $\hline E \vdash C1[m] : S2$  **m.S<**

By the ind hyp there exists  $S1'$  such that  $E \vdash m : S1'$  and for all  $E', m'$ , if  $E, E' \vdash m' : S1'$  then  $E, E' \vdash C1[m'] : S1$   
 By weak  $E, E' \vdash S1 < S2 \text{ sig}$ . By **m.S<**  $E, E' \vdash C1[m'] : S2$ .

**[C1 m2 /m.app]**  
 $E \vdash C1[m] : \text{IIU} : SS1.S$   
 $E \vdash m2 : SS1$   
 $U \text{ not free in } S2$   
 $\hline E \vdash C1[m] m2 : S$  **m.app**

By the ind hyp there exists  $S'$  such that  $E \vdash m : S'$  and for all  $E', m'$ , if  $E, E' \vdash m' : S'$  then  $E, E' \vdash C1[m'] : \text{IIU} : SS1.S$   
 By weak  $E, E' \vdash m2 : SS1$ . By **m.app**  $E, E' \vdash C1[m'] m2 : S$ .  
**[mval C1 /m.app] [C1 with! mval /m.with] [m1 with! C1 /m.with] [C1:SS /m.seal]**  
 All similar to the previous case.  
**[m.var] [m.struct] [m.self-type] [m.fun] [m.new]** All  
 vacuous, as there is no  $A1$  that can match them.

**Lemma 35 (Module subject reduction)** If  $N \vdash m : S$  and  
 $N \vdash m \implies \text{New } N' \text{ in } m'$  and  $N, N'$  disjoint then  $N, N' \vdash m' : S$ .  
 Moreover if  $N$  atomic and  $N$  has no module bindings then so does  $N, N'$ .

**Proof**

**[m.wred.refl]** Immediate  
**[m.wred.tran]** By the induction hypothesis twice.  
**[m.wred.red]**

Suppose  $N \vdash Ci[m] : S$ . By (34) (using the fact that any  $C2$  is also a  $C1$ )  
 there exists  $S'$  such that  $N \vdash m : S'$  and for all  $N', m'$ , if  $N, N' \vdash m' : S'$   
 then  $N, N' \vdash Ci[m'] : S$ . By Lemma 33  $N, N' \vdash m' : S$ , so using (a)  $N, N' \vdash Ci[m'] : S$ .

## B.9 Subject Reduction - run time

If  $s$  is a finite partial function from term variables to expressions and  
 $E'$  is a type environment with term variable bindings only  
 then say  $E \vdash s : E'$  iff  $\text{dom}(s) = \text{dom}(E')$  and forall  $x$  in  $\text{dom}(s)$ .  $E \vdash s(x) : E'(x)$

**Lemma 36 (Well-formed term substitutions)** If  $E$  atomic and has no module bindings,  
 $E \vdash e : T$  and  $E \vdash \text{pat} : T \triangleright E'$  then  $\{e/\text{pat}\}'$  is defined and  $E \vdash \{e/\text{pat}\}' : E'$ .

**Proof** Induction on  $E \vdash \text{pat} : T \triangleright E'$ , using 29.

**Lemma 37 (Substitution - term)**

If  $E, E', E'' \vdash e : T$  and  $E \vdash s : E'$  then  $E, E'' \vdash se : T$ .



**Proof** Induction on  $E, E', E'' \vdash e : T$ , using 16 and 5.

**Lemma 38 (Type soundness of structural congruence)**

If  $E \vdash e : \text{proc}$  and  $e == e'$  then  $E \vdash e' : \text{proc}$ .

**Proof**

We show that if  $E \vdash e : \text{proc}$  and  $(e == e' \text{ or } e' == e)$  then  $E \vdash e' : \text{proc}$ , by induction on the derivation of structural congruence.

[sc.id] (a) If  $E \vdash 0 | e1 : \text{proc}$ . By (29)  $E \vdash e1 : \text{proc}$ .

(b) If  $E \vdash e1 : \text{proc}$  then by well-formedness  $E \vdash \text{ok}$ , so by  $e.\text{nil}$   $E \vdash 0 : \text{proc}$ , and by  $e.\text{par}$   $E \vdash 0 | e1 : \text{proc}$ .

[sc.com] [sc.ass] Similar.

[sc.ext] Wlg take  $x$  not in  $\text{dom } E$ . (a) If  $E \vdash e1 | \text{new } x : T \text{ in } e2 : \text{proc}$  then by 29  $E \vdash e1 : \text{proc}$  and  $E \vdash \text{new } x : T \text{ in } e2 : \text{proc}$ .

By Lemma 29  $E, x : T \vdash e2 : \text{proc}$  and there exists  $T_0$  such that  $T = \text{chan } T_0$ .

By weakening  $E, x : T \vdash e1 | e2 : \text{proc}$ . By  $e.\text{new}$   $E \vdash \text{new } x : T \text{ in } e1 | e2 : \text{proc}$ .

(b) If  $E \vdash \text{new } x : T \text{ in } e1 | e2 : \text{proc}$  by 29  $E, x : T \vdash e1 | e2 : \text{proc}$

and  $T = \text{chan } T_0$  for some  $T_0$ . By 29  $E, x : T \vdash e1 : \text{proc}$

and  $E, x : T \vdash e2 : \text{proc}$ . By strengthening 16  $E \vdash e1 : \text{proc}$ .

By  $e.\text{new}$  and  $e.\text{par}$   $E \vdash e1 | \text{new } x : T \text{ in } e2 : \text{proc}$ .

[sc.cong.par] By 29 and the ind hyp.

[sc.cong.new] By 29 and the ind hyp.

[sc.refl] Immediate. [sc.sym] by the ind hyp. [sc.tran] by the ind hyp twice.

**Lemma 39 (Process subject reduction)** If  $E$  atomic and has no module bindings,

$E \vdash e : \text{proc}$  and  $e \longrightarrow e'$  then  $E \vdash e' : \text{proc}$ .

**Proof**

By induction on derivation of  $e \longrightarrow e'$ .

[e.red.comm] Wlg take  $\text{vars}(\text{pat})$  and  $\text{dom } E$  disjoint.

By 29  $E \vdash x ! e1 : \text{proc}$  and  $E \vdash x ? \text{pat}. e2 : \text{proc}$ .

Clearly for some  $T, T', E'$  we have  $E \vdash x : \text{chan } T'$ ,  $E \vdash x : \text{chan } T$ ,  $E \vdash e1 : T'$ ,  $E \vdash \text{pat} : T \triangleright E'$ , and  $E, E' \vdash e2 : \text{proc}$ . Clearly also  $E \vdash \text{chan } T == \text{chan } T' :: \text{Type}$ .

By 25  $E \vdash T == T' :: \text{Type}$  so  $E \vdash e1 : T$ . By 36  $\{e1/\text{pat}\}'$  is defined and  $E \vdash \{e1/\text{pat}\}' : E'$ .

By 37  $E \vdash \{e1/\text{pat}\}' e2 : \text{proc}$ .

[e.red.par] By 29, the ind hyp and  $e.\text{par}$ .

[e.red.res] Wlg take  $x$  not in  $\text{dom } E$ .

By 29, the ind typ (noting that  $E, x : \text{chan } T_0$  is atomic) and  $e.\text{new}$

[e.red.sc] By 38 twice and the ind hyp.

**Lemma 40 (Process soundness)** If  $E$  atomic and has no module bindings and  $E \vdash e : \text{proc}$

then not  $(e \xrightarrow{\text{err}})$ .

**Proof** We show  $(e \xrightarrow{\text{err}}$  and  $E$  atomic and has no module bindings and  $E \vdash e : \text{proc})$

implies false, by induction on  $e \xrightarrow{\text{err}}$ .

[e.err.comm] Wlg take  $\text{vars}(\text{pat})$  and  $\text{dom } E$  disjoint.

By 29  $E \vdash x ! e1 : \text{proc}$  and  $E \vdash x ? \text{pat}. e2 : \text{proc}$ .

Clearly for some  $T, T', E'$  we have  $E \vdash x : \text{chan } T'$ ,  $E \vdash x : \text{chan } T$ ,  $E \vdash e1 : T'$ ,  $E \vdash \text{pat} : T \triangleright E'$ , and  $E, E' \vdash e2 : \text{proc}$ . Clearly also  $E \vdash \text{chan } T == \text{chan } T' :: \text{Type}$

By 25  $E \vdash T == T' :: \text{Type}$  so  $E \vdash e1 : T$

By 36  $\{e1/\text{pat}\}'$  is defined, so not  $(e \xrightarrow{\text{err}})$ .

[e.err.sc] By 38 and the ind hyp.

[e.err.par] By 29 twice and ind.

[e.err.res] Wlg take  $x$  not in  $\text{dom } E$ .

By 29, the ind typ (noting that  $E, x:\text{chan } T_0$  is atomic) and  $e.\text{new}$

## B.10 Subject Reduction and Soundness - whole systems

**Theorem 1** If  $\vdash N, F, e$  ok and  $N, F, e \xrightarrow{\text{Com}} N', F', e'$  then  $\vdash N', F', e'$  ok.

**Proof**

**[build]** By 32  $N \vdash F(m) : F(S)$ . By this and 35  $N, N' \vdash \text{mval} : F(S)$   
and moreover  $N, N'$  atomic and contains no module bindings.

By well-formedness  $N, N' \vdash \text{ok}$ . Now consider typing clause of  $\vdash (N, N'), F+(U, \text{mval}), e$  ok

– For  $U$  consider cases of  $\text{mval}$  a struct or a functor, then apply  
28 and  $\text{m.struct}$  or 28 and  $\text{m.fun}$  respectively.

– For  $U'$  in  $\text{dom}(F+(U, \text{mval})) - \{U\}$ , the clause follows by weakening.

The proc clause of  $\vdash (N, N'), F+(U, \text{mval}), e$  ok follows by weakening.

**[load]** wlg suppose  $X$  not in  $\text{dom } N$ . The typing and  $N$  clauses of  $N, F, e|e'$  are immediate.  
For the proc clause, by hypothesis  $N \vdash e:\text{proc}$ .

By hypothesis  $N \vdash [T, e']$  as  $[X::K, \text{proc}] : [X::K, \text{proc}]$

By 28  $N, X::\text{EQ}(T) \vdash e' : \text{proc}$  and  $X$  not free in  $e'$

By strengthening 18  $N \vdash e':\text{proc}$

By  $e.\text{par}$   $N \vdash e|e':\text{proc}$

**[compute]** The typing and  $N$  clauses of  $N, F, e'$  are immediate.

The proc clause follows from 39.

**Theorem 2** If  $\vdash N, F, e$  ok then there is no transition  $N, F, e \xrightarrow{\text{tau}} \text{err}(\text{runtime error})$ .

**Proof** Immediate from the system rule and 40.

## B.11 Proof of Theorem 3

**Lemma 41** If  $E, Y::\text{Type}, U: [X::\text{EQ}(Y), T], E' \vdash J$  and  $Y$  not in  $T$   
then  $E, U: [X::\text{Type}, T], \{U.\text{Type}/Y\}E' \vdash \{U.\text{Type}/Y\}J$ .

**Proof**

**[T.var]** **[E.mod]** Straightforward.

**[m.var]** Case before, after: routine.

Case same: by ind  $E, U: [X::\text{Type}, T], \{U.\text{Type}/Y\}E' \vdash \text{ok}$ .

By  $\text{m.var}$   $E, U: [X::\text{Type}, T], \{U.\text{Type}/Y\}E' \vdash U: [X::\text{Type}, T]$

By  $T.\text{Proj}, T=\text{refl}, T.T=$   $E, U: [X::\text{Type}, T], \{U.\text{Type}/Y\}E' \vdash U.\text{Type} :: \text{EQ}(U.\text{Type})$

By  $\text{m.self-type}$   $E, U: [X::\text{Type}, T], \{U.\text{Type}/Y\}E' \vdash U: [X::\text{EQ}(U.\text{Type}), T]$

**Theorem 3** If  $m_1$  and  $m_2$  are structure expressions

$$\begin{aligned} m_1 &= [T_1, e_1] \text{ as } [X::K_1, T_1'] && \text{with } SS_1 = [X::K_1, T_1'] \\ m_2 &= [T_2, e_2] \text{ as } [X::K_2, T_2'] && SS_2 = [X::K_2, T_2'] \end{aligned}$$

and

$$m_2' = (\lambda U_1:SS_1.m_2:SS_2) [T_1, e_1] \text{ as } [X::EQ(T_1), T_1']$$

then

$$(\exists N, F, e. \text{ empty}, \emptyset, 0 \xrightarrow{U_1:=m_1} \xrightarrow{U_2:=m_2} N, F, e) \text{ iff } (\exists N, F, e. \text{ empty}, \emptyset, 0 \xrightarrow{U_2:=m_2'} N, F, e).$$

**Proof** For the left-to-right direction, first consider  $K_1=Type$ .

By the build rule there exist  $SS_1', Y, mval_1, SS_2', N_2, mval_2$  such that

- a  $\text{empty} \vdash m_1 : SS_1'$
- b  $mval_1 = [Y, e_1] \text{ as } [X::EQ(Y), T_1']$
- c  $\text{empty} \vdash m_1 \implies \text{New } Y::EQ(T_1) \text{ in } mval_1$
- d  $Y::Type, U_1:[X::EQ(Y), T_1'] \vdash m_2 : SS_2'$
- e  $Y::EQ(T_1) \vdash \{mval_1/U_1\}m_2 \implies \text{New } N_2 \text{ in } mval_2$
- f  $\{Y\}$  disjoint from  $fv(m_2)$  and  $dom(N_2)$
- g  $N = Y::EQ(T_1), N_2$

By (a,28)  $\text{empty} \vdash m_1 : SS_1$

By (d,28)  $Y::Type, U_1:[X::EQ(Y), T_1'] \vdash m_2 : SS_2$

By (f,41)  $U_1:[X::Type, T_1'] \vdash m_2 : SS_2$

By m.fun  $\text{empty} \vdash \lambda U_1:[X::Type, T_1'].m_2:SS_2 : \Pi U_1:[X::Type, T_1'].SS_2$

By sundry rules  $\text{empty} \vdash \lambda U_1:[X::Type, T_1'].m_2:SS_2 : \Pi U_1:[X::EQ(T_1), T_1'].\{T_1/U_1.Type\}SS_2$

By m.app  $\text{empty} \vdash (\lambda U_1:[X::Type, T_1'].m_2:SS_2) [T_1, e_1] \text{ as } [X::EQ(T_1), T_1']$   
 $: \{T_1/U_1.Type\}SS_2$

ie  $\text{empty} \vdash m_2' : \{T_1/U_1.Type\}SS_2$

There is then some  $N'$  and  $mval_2'$  such that  $\text{empty} \vdash m_2' \implies \text{New } N' \text{ in } mval_2'$

so  $\text{empty}, \emptyset, 0 \xrightarrow{U_2 := m_2'} N', \{U_2 \mapsto mval_2'\}, 0$

Now consider  $K_1=EQ(T)$ .  $m_1$  is a value, so by the build rule there exist  $SS_1', SS_2', mval_2$  such that

- a  $\text{empty} \vdash m_1 : SS_1'$
- b  $U_1:SS_1' \vdash m_2 : SS_2'$
- c  $\text{empty} \vdash \{m_1/U_1\}m_2 \implies \text{New } N \text{ in } mval_2$
- d  $F = \{U_1 \mapsto m_1, U_2 \mapsto mval_2\}$  and  $e=0$

By (a,28)  $\text{empty} \vdash m_1 : SS_1$

By (b,28)  $U_1:SS_1' \vdash m_2 : SS_2$

By m.fun  $\text{empty} \vdash \lambda U_1:SS_1'.m_2:SS_2 : \Pi U_1:SS_1'.SS_2$

By sundry rules  $\text{empty} \vdash \lambda U_1:SS_1'.m_2:SS_2 : \Pi U_1:SS_1'.\{T/U_1.Type\}SS_2$

By m.app  $\text{empty} \vdash (\lambda SS_1'.m_2:SS_2) m_1 : \{T/U_1.Type\}SS_2$

ie  $\text{empty} \vdash m_2' : \{T/U_1.Type\}SS_2$

There is then some  $N'$  and  $mval_2'$  such that  $\text{empty} \vdash m_2' \implies \text{New } N' \text{ in } mval_2'$

so  $\text{empty}, \emptyset, 0 \xrightarrow{U_2 := m_2'} N', \{U_2 \mapsto mval_2'\}, 0$

For the right-to-left direction, by the build rule there exists  $SS2', mval2'$  such that

$empty \vdash m2' : SS2'$   
 $empty \vdash m2' \implies \text{New } N \text{ in } mval2'$

By 28 there exist  $SS1''$  and  $SS2''$  with  $U1$  not in  $SS2''$  such that

$empty \vdash [T1, e1] \text{ as } [X::EQ(T1), T1'] : SS1''$   
 $empty \vdash (\lambda U1:SS1.m2) : \Pi U1:SS1''.SS2''$

By 28  $U1:SS1 \vdash m2 : SS2$ ,  $empty \vdash SS1'' < SS1 \text{ sig}$  and  $U1:SS1'' \vdash SS2 < SS2'' \text{ sig}$   
 By  $m.S <$   $empty \vdash [T1, e1] \text{ as } [X::EQ(T1), T1'] : SS1$  (recall  $SS1 = [X::K1, T1']$ )  
 so  $empty \vdash [T1, e1] \text{ as } SS1 : SS1$

There is then some  $N1$  and  $mval1$  such that  $empty \vdash m1 \implies \text{New } N1 \text{ in } mval1$

so  $empty, \emptyset, 0 \xrightarrow{U1:=m1} N1, \{U1 \mapsto mval1\}, 0$ .

By weakening  $typify(N1), U1:SS1 \vdash m2 : SS2$   
 By refl or  $m.red.abstype$   $typify(N1) \vdash sig(mval1) < SS1 \text{ sig}$   
 By narrowing for modules  $typify(N1), U1:sig(mval1) \vdash m2 : SS2$

There is then some  $N2$  and  $mval2$  such that  $N1 \vdash \{mval1/U1\}m2 \implies \text{New } N2 \text{ in } mval2$

and  $empty, \emptyset, 0 \xrightarrow{U1:=m1} \xrightarrow{U2:=m2} (N1, N2), \{U1 \mapsto mval1, U2 \mapsto mval2\}, 0$ .

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