typevar, X	type variable			
termvar, x	term variable			
label, l, k	field label			
index, i, j, n, m	indices			
$T,\ S,\ U$	::=			type
	|	X		type variable
	|	**Top**		maximum type
	|	$T \rightarrow T'$		type of functions
	|	$\forall X{<:}T.\,T'$	bind X in T'	universal type
	|	$\{\, l_1 : T_1 , .. , l_n : T_n \,\}$		record
	|	$(\, T \,)$	S	
	|	$[\, X \mapsto T \,]\, T'$	M	
t	::=			term
	|	x		variable
	|	$\lambda x{:}T.\, t$	bind x in t	abstraction
	|	$t\, t'$		application
	|	$\Lambda X{<:}T.\, t$	bind X in t	type abstraction
	|	$t\, [\, T \,]$		type application
	|	$\{\, l_1 = t_1 , .. , l_n = t_n \,\}$		record
	|	$t\, .\, l$		projection
	|	**let** $p = t$ **in** t'	bind $\mathrm{b}(p)$ in t'	pattern binding
	|	$(\, t \,)$	S	
	|	$[\, x \mapsto t \,]\, t'$	M	
	|	$[\, X \mapsto T \,]\, t$	M	
	|	$\sigma\, t$	M	
p	::=			pattern
	|	$x : T$	$\mathrm{b} = x$	variable pattern
	|	$\{\, l_1 = p_1 , .. , l_n = p_n \,\}$	$\mathrm{b} = \mathrm{b}(p_1 .. p_n)$	record pattern
v	::=			values
	|	$\lambda x{:}T.\, t$	bind x in t	abstraction
	|	$\Lambda X{<:}T.\, t$	bind X in t	type abstraction
	|	$\{\, l_1 = v_1 , .. , l_n = v_n \,\}$		record
$\Gamma,\ \Delta$	::=			type environment
	|	**empty**		
	|	$\Gamma , X <: T$		
	|	$\Gamma , x : T$		
	|	$\Gamma_1 , .. , \Gamma_n$	M	
σ	::=			multiple term substitution
	|	$[\, x \mapsto t \,]$		
	|	$\sigma_1 , .. , \sigma_n$		
terminals	::=			
	|	λ		
	|	$\rightarrow$		
	|	$\Rightarrow$		
	|	$\vdash$		

	|	$\longrightarrow$	
	|	$\forall$	
	|	$<:$	
	|	$\mapsto$	
	|	$\wedge$	
	|	$\vee$	
	|	$=$	
formula	::=		
	|	*judgement*	
	|	$x = x'$	
	|	$X = X'$	
	|	$(\, formula\,)$	
	|	$\neg formula$	
	|	$\forall i \in 1..m.formula$	
	|	$\exists i \in 1..m.formula$	
	|	$formula \wedge formula'$	
	|	$l = l'$	
	|	$formula_1 \quad \ldots \quad formula_n$	
Judgement_in	::=		
	|	$x \in \mathbf{dom}(\Gamma)$	
	|	$X \in \mathbf{dom}(\Gamma)$	
	|	$x : T \in \Gamma$	
	|	$X <: U \in \Gamma$	
Jtype	::=		
	|	$\Gamma \vdash \mathbf{ok}$	type environment Γ is well-formed
	|	$\Gamma \vdash T$	type T is well-formed in type environment Γ
	|	$\Gamma \vdash S <: T$	S is a subtype of T
	|	$\Gamma \vdash t : T$	term t has type T
	|	$\vdash p : T \Rightarrow \Delta$	pattern p matches type T giving bindings Δ
Jop	::=		
	|	$t_1 \longrightarrow t_2$	t_1 reduces to t_2
	|	$\mathbf{match}\,(\, p\, ,\, v\,) = \sigma$	
judgement	::=		
	|	*Judgement_in*	
	|	*Jtype*	
	|	*Jop*	
user_syntax	::=		
	|	*typevar*	
	|	*termvar*	
	|	*label*	
	|	*index*	

$|\quad T$
$|\quad t$
$|\quad p$
$|\quad v$
$|\quad \Gamma$
$|\quad \sigma$
$|\quad terminals$
$|\quad formula$

$\boxed{x \in \mathbf{dom}(\Gamma)}$

$$\frac{}{x \in \mathbf{dom}(\Gamma\,,\; x\; :\; T)} \quad \textsc{xinG\_1}$$

$$\frac{x \in \mathbf{dom}(\Gamma)}{x \in \mathbf{dom}(\Gamma\,,\; X'\; <:\; U')} \quad \textsc{xinG\_2}$$

$$\frac{x \in \mathbf{dom}(\Gamma)}{x \in \mathbf{dom}(\Gamma\,,\; x'\; :\; T')} \quad \textsc{xinG\_3}$$

$\boxed{X \in \mathbf{dom}(\Gamma)}$

$$\frac{}{X \in \mathbf{dom}(\Gamma\,,\; X\; <:\; U)} \quad \textsc{XinG\_1}$$

$$\frac{X \in \mathbf{dom}(\Gamma)}{X \in \mathbf{dom}(\Gamma\,,\; X'\; <:\; U')} \quad \textsc{XinG\_2}$$

$$\frac{X \in \mathbf{dom}(\Gamma)}{X \in \mathbf{dom}(\Gamma\,,\; x'\; :\; T')} \quad \textsc{XinG\_3}$$

$\boxed{x : T \in \Gamma}$

$$\frac{}{x : T \in \Gamma\,,\; x\; :\; T} \quad \textsc{tin\_1}$$

$$\frac{x : T \in \Gamma}{x : T \in \Gamma\,,\; X'\; <:\; U'} \quad \textsc{tin\_2}$$

$$\frac{x : T \in \Gamma}{x : T \in \Gamma\,,\; x'\; :\; T'} \quad \textsc{tin\_3}$$

$\boxed{X <: U \in \Gamma}$

$$\frac{}{X <: U \in \Gamma\,,\; X\; <:\; U} \quad \textsc{Tin\_1}$$

$$\frac{X <: U \in \Gamma}{X <: U \in \Gamma\,,\; X'\; <:\; U'} \quad \textsc{Tin\_2}$$

$$\frac{X <: U \in \Gamma}{X <: U \in \Gamma\,,\; x'\; :\; T'} \quad \textsc{Tin\_3}$$

$\boxed{\Gamma \vdash \mathbf{ok}}$ type environment Γ is well-formed

$$\frac{}{\mathbf{empty} \vdash \mathbf{ok}} \quad \textsc{Gok\_1}$$

$$\frac{\Gamma \vdash T \quad \neg(\, x \in \mathbf{dom}(\Gamma)\,)}{\Gamma\, ,\, x \,:\, T \vdash \mathbf{ok}} \quad \textsc{Gok\_2}$$

$$\frac{\Gamma \vdash T \quad \neg(\, X \in \mathbf{dom}(\Gamma)\,)}{\Gamma\, ,\, X \,<:\, T \vdash \mathbf{ok}} \quad \textsc{Gok\_3}$$

$\boxed{\Gamma \vdash T}$ type T is well-formed in type environment Γ

$$\frac{\Gamma \vdash \mathbf{ok} \quad X <: U \in \Gamma}{\Gamma \vdash X} \quad \textsc{GT\_Var}$$

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash \mathbf{Top}} \quad \textsc{GT\_Top}$$

$$\frac{\Gamma \vdash T \quad \Gamma \vdash T'}{\Gamma \vdash T \to T'} \quad \textsc{GT\_Fun}$$

$$\frac{\Gamma\, ,\, X \,<:\, T \vdash T'}{\Gamma \vdash \forall X {<:} T.\, T'} \quad \textsc{GT\_Forall}$$

$$\frac{\Gamma \vdash T_1 \quad .. \quad \Gamma \vdash T_n}{\Gamma \vdash \{\, l_1 \,:\, T_1\, ,\, ..\, ,\, l_n \,:\, T_n \,\}} \quad \textsc{GT\_Rcd}$$

$\boxed{\Gamma \vdash S <: T}$ S is a subtype of T

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash S <: \mathbf{Top}} \quad \textsc{SA\_Top}$$

$$\frac{\Gamma \vdash \mathbf{ok}}{\Gamma \vdash X <: X} \quad \textsc{SA\_Refl\_TVar}$$

$$\frac{X <: U \in \Gamma \quad \Gamma \vdash U <: T}{\Gamma \vdash X <: T} \quad \textsc{SA\_Trans\_TVar}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma \vdash S_2 <: T_2}{\Gamma \vdash S_1 \to S_2 <: T_1 \to T_2} \quad \textsc{SA\_Arrow}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma\, ,\, X <: T_1 \vdash S_2 <: T_2}{\Gamma \vdash \forall X {<:} S_1.\, S_2 <: \forall X {<:} T_1.\, T_2} \quad \textsc{SA\_All}$$

$$\frac{\forall i \in 1..m. \exists j \in 1..n. (\, k_i = l_j \wedge \Gamma \vdash S_i <: T_j \,)}{\Gamma \vdash \{\, k_1 \,:\, S_1\, ,\, ..\, ,\, k_m \,:\, S_m \,\} <: \{\, l_1 \,:\, T_1\, ,\, ..\, ,\, l_n \,:\, T_n \,\}} \quad \textsc{SA\_Rcd}$$

$\boxed{\Gamma \vdash t : T}$ term t has type T

$$\frac{\Gamma \vdash \mathbf{ok} \quad x : T \in \Gamma}{\Gamma \vdash x : T} \quad \textsc{Ty\_Var}$$

$$\frac{\Gamma\, ,\, x \,:\, T_1 \vdash t_2 \,:\, T_2}{\Gamma \vdash \lambda x{:}T_1.\, t_2 \,:\, T_1 \to T_2} \quad \textsc{Ty\_Abs}$$

$$\frac{\Gamma \vdash t_1 : T_{11} \rightarrow T_{12} \quad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash t_1\, t_2 : T_{12}} \quad \textsc{Ty\_App}$$

$$\frac{\Gamma, X <: T_1 \vdash t_2 : T_2}{\Gamma \vdash \Lambda X{<:}T_1.\, t_2 : \forall X{<:}T_1.\, T_2} \quad \textsc{Ty\_TAbs}$$

$$\frac{\Gamma \vdash t_1 : \forall X{<:}T_{11}.\, T_{12} \quad \Gamma \vdash T_2 <: T_{11}}{\Gamma \vdash t_1\,[\,T_2\,] : [\,X \mapsto T_2\,]\, T_{12}} \quad \textsc{Ty\_TApp}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \vdash p : T_1 \Rightarrow \Delta \quad \Gamma, \Delta \vdash t_2 : T_2}{\Gamma \vdash \mathbf{let}\, p = t_1\, \mathbf{in}\, t_2 : T_2} \quad \textsc{Ty\_Let}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad .. \quad \Gamma \vdash t_n : T_n}{\Gamma \vdash \{\, l_1 = t_1\,, .., \, l_n = t_n \,\} : \{\, l_1 : T_1\,, .., \, l_n : T_n \,\}} \quad \textsc{Ty\_Rcd}$$

$$\frac{\Gamma \vdash t : \{\, l_1 : T_1\,, .., \, l_n : T_n \,\}}{\Gamma \vdash t\,.\, l_j : T_j} \quad \textsc{Ty\_Proj}$$

$$\frac{\Gamma \vdash t : S \quad \Gamma \vdash S <: T}{\Gamma \vdash t : T} \quad \textsc{Ty\_Sub}$$

$\boxed{\vdash p : T \Rightarrow \Delta}$ pattern p matches type T giving bindings Δ

$$\frac{}{\vdash x : T : T \Rightarrow \mathbf{empty}\,, x : T} \quad \textsc{Pat\_Var}$$

$$\frac{\vdash p_1 : T_1 \Rightarrow \Delta_1 \quad .. \quad \vdash p_n : T_n \Rightarrow \Delta_n}{\vdash \{\, l_1 = p_1\,, .., \, l_n = p_n \,\} : \{\, l_1 : T_1\,, .., \, l_n : T_n \,\} \Rightarrow \Delta_1\,, .., \, \Delta_n} \quad \textsc{Pat\_Rcd}$$

$\boxed{t_1 \longrightarrow t_2}$ t_1 reduces to t_2

$$\frac{}{(\,\lambda x{:}T_{11}.\, t_{12}\,)\, v_2 \longrightarrow [\,x \mapsto v_2\,]\, t_{12}} \quad \textsc{reduce\_AppAbs}$$

$$\frac{}{(\,\Lambda X{<:}T_{11}.\, t_{12}\,)\,[\,T_2\,] \longrightarrow [\,X \mapsto T_2\,]\, t_{12}} \quad \textsc{reduce\_TappTabs}$$

$$\frac{\mathbf{match}\,(\,p\,,\, v_1\,) = \sigma}{\mathbf{let}\, p = v_1\, \mathbf{in}\, t_2 \longrightarrow \sigma\, t_2} \quad \textsc{reduce\_LetV}$$

$$\frac{}{\{\, l'_1 = v_1\,, .., \, l'_n = v_n \,\}\,.\, l'_j \longrightarrow v_j} \quad \textsc{reduce\_ProjRcd}$$

$$\frac{t_1 \longrightarrow t'_1}{t_1\, t \longrightarrow t'_1\, t} \quad \textsc{reduce\_Ctx\_app\_fun}$$

$$\frac{t_1 \longrightarrow t'_1}{v\, t_1 \longrightarrow v\, t'_1} \quad \textsc{reduce\_Ctx\_app\_arg}$$

$$\frac{t_1 \longrightarrow t'_1}{t_1\,[\,T\,] \longrightarrow t'_1\,[\,T\,]} \quad \textsc{reduce\_Ctx\_type\_fun}$$

$$\frac{t \longrightarrow t'}{\{\, l_1 = v_1\,, .., \, l_m = v_m\,, \, l = t\,, \, l'_1 = t'_1\,, .., \, l'_n = t'_n \,\} \longrightarrow \{\, l_1 = v_1\,, .., \, l_m = v_m\,, \, l = t'\,, \, l'_1 = t'_1\,, .., \, l'_n = t'_n \,\}} \quad \textsc{reduce\_C}$$

$$\frac{t_1 \longrightarrow t_1'}{\mathbf{let}\, p = t_1 \,\mathbf{in}\, t_2 \longrightarrow \mathbf{let}\, p = t_1' \,\mathbf{in}\, t_2} \quad \textsc{reduce\_Ctx\_let\_binding}$$

$\boxed{\mathbf{match}\,(\, p \,,\, v \,) = \sigma}$

$$\frac{}{\mathbf{match}\,(\, x \,:\, T \,,\, v \,) = [\, x \mapsto v \,]} \quad \textsc{M\_Var}$$

$$\frac{\forall i \in 1..m . \exists j \in 1..n . (\, l_i = k_j \wedge \mathbf{match}\,(\, p_i \,,\, v_j \,) = \sigma_i \,)}{\mathbf{match}\,(\, \{\, l_1 = p_1 \,,\, .. \,,\, l_m = p_m \,\} \,,\, \{\, k_1 = v_1 \,,\, .. \,,\, k_n = v_n \,\} \,) = \sigma_1 \,,\, .. \,,\, \sigma_m} \quad \textsc{M\_Rcd}$$

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Definition rules:        48 good    0 bad
Definition rule clauses: 99 good    0 bad
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