Machine code, formal verification and functional programming

Magnus Myreen University of Cambridge, UK

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Machine Code

Machine code is what the CPU executes.

- 0: E3A00000
- 4: E3510000
- 8: 12800001
- 12: 15911000
- 16: 1AFFFFB

Ultimately all program verification ought to reach real machine code.

Machine code

Machine code,

E1510002 B0422001 C0411002 01AFFFFB

is impossible to read, write or maintain manually.

However, for theorem-prover-based formal verification:

machine code is clean and tractable!

Reason:

- all types are concrete: word32, word8, bool.
- state consists of a few simple components: a few registers, a memory and some status bits.
- each instruction performs only small well-defined updates.

Some C problems avoided

Machine code verification avoids some challenges in C verification:

- C has annoyingly weak type system, e.g. union and cast to/from void type
- multiple ambiguities in both syntax and semantics, e.g. C syntax preprocessing cpp, evaluation orders
- richer set of features compared to plain machine instructions, mdContext->in[mdi++] = *inBuf++ in-line assembly in C: __asm__(...), semantics?

Also, verified C code must be compiled, while verified machine code can be executed 'as is'.

Verification of Machine Code

Challenges:

machine code



ARM/x86/PowerPC model (1000...10,000 lines each) correctness
{P} code {Q}

Contribution: tools/methods which

- expose as little as possible of the big models to the user
- makes non-automatic proofs independent of the models

HOL: fully-expansive LCF-style prover

The aim is to prove deep functional properties of machine code.

Proofs are performed in HOL4 — a fully expansive theorem prover

HOL4 theorem prover



All proofs expand at runtime into primitive inferences in the HOL4 kernel.

The kernel implements the axioms and inference rules of higher-order logic.

Short demo

Talk outline

Part I: Tools and infrastructure

proof-producing decompiler:

translates machine code into equivalent functions in logic

proof-producing compiler:

translates functions in logic into correct-by-cons. machine code

Part 2: Case studies

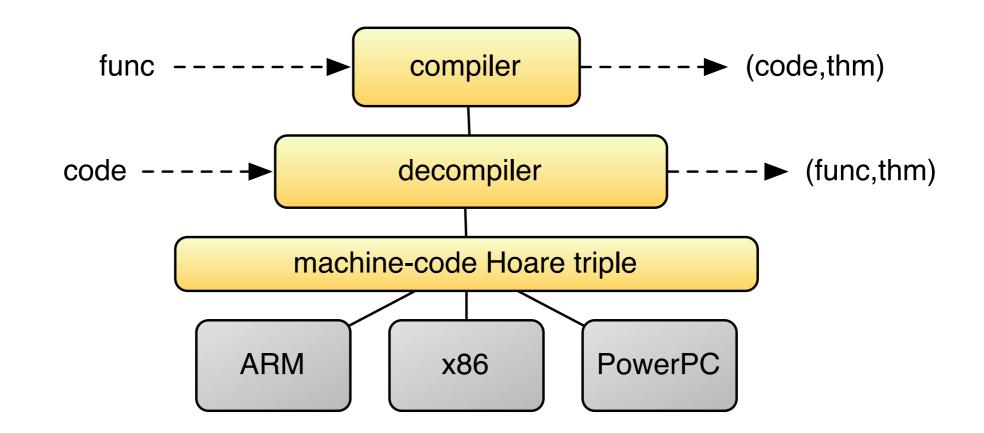
simple verified Lisp interpreter

verified just-in-time compiler for Lisp

verified read-eval-print-loop for a subset of Standard ML

Infrastructure

During my PhD, I developed the following infrastructure:



...each part will be explained in the next slides.

Models of machine code

Machine models borrowed from work by others:

ARM model, by Fox [TPHOLs'03]

- covers practically all ARM instructions, for old and new ARMs
- still actively being developed

x86 model, by Sarkar et al. [POPL'09]

- covers all addressing modes in 32-bit mode x86
- includes approximately 30 instructions

PowerPC model, originally from Leroy [POPL'06]

- ▶ manual translation (Coq \rightarrow HOL4) of Leroy's PowerPC model
- instruction decoder added

Hoare triples

Each model can be evaluated, e.g. ARM instruction add r0,r0,r0 is described by theorem:

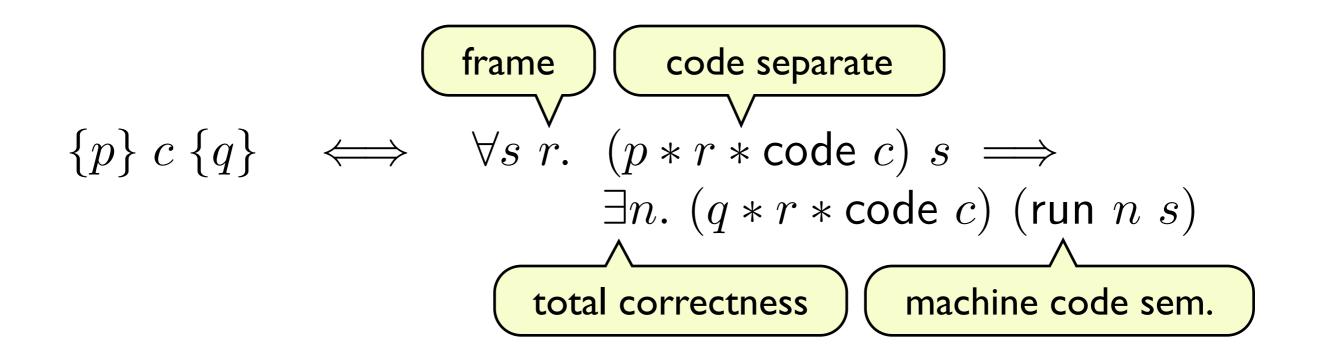
|- (ARM_READ_MEM ((31 >< 2) (ARM_READ_REG 15w state)) state =
 OxE0800000w) ∧ ¬state.undefined ⇒
 (NEXT_ARM_MMU cp state =
 ARM_WRITE_REG 15w (ARM_READ_REG 15w state + 4w)
 (ARM_WRITE_REG 0w
 (ARM_READ_REG 0w state + ARM_READ_REG 0w state) state))</pre>

As a total-correctness machine-code Hoare triple:

- SPEC ARM_MODEL	Informal syntax for this talk:
(aR Ow x * aPC p)	{ R0 x * PC p }
$\{(p, 0xE080000w)\}$	<i>p</i> : E0800000
(aR Ow (x+x) * aPC (p+4w))	$\{ R0 (x+x) * PC (p+4) \}$

Short demo

Definition of Hoare triple



Decompiler

Decompiler automates Hoare triple reasoning.

Example: Given some ARM machine code,

0:	E3A00000	mov r0, #0
4:	E3510000	L: cmp r1, #0
8:	12800001	addne r0, r0, #1
12:	15911000	ldrne r1, [r1]
16:	1AFFFFFB	bne L

the decompiler automatically extracts a readable function:

$$f(r_0, r_1, m) = \text{let } r_0 = 0 \text{ in } g(r_0, r_1, m)$$

$$g(r_0, r_1, m) = \text{if } r_1 = 0 \text{ then } (r_0, r_1, m) \text{ else}$$

$$\text{let } r_0 = r_0 + 1 \text{ in}$$

$$\text{let } r_1 = m(r_1) \text{ in}$$

$$g(r_0, r_1, m)$$

Decompilation, correct?

Decompiler automatically proves a certificate theorem:

 $f_{pre}(r_0, r_1, m) \Rightarrow \\ \{ (R0, R1, M) \text{ is } (r_0, r_1, m) * PC \ p * S \} \\ p : E3A00000 \ E3510000 \ 12800001 \ 15911000 \ 1AFFFFB \\ \{ (R0, R1, M) \text{ is } f(r_0, r_1, m) * PC \ (p + 20) * S \} \end{cases}$

which informally reads:

for any initially value (r_0, r_1, m) in reg 0, reg 1 and memory, the code terminates with $f(r_0, r_1, m)$ in reg 0, reg 1 and memory.

Decompilation verification example

To verify code: prove properties of function f,

 $\forall x \ l \ a \ m. \ list(l, a, m) \Rightarrow f(x, a, m) = (length(l), 0, m)$ $\forall x \ l \ a \ m. \ list(l, a, m) \Rightarrow f_{pre}(x, a, m)$

since properties of f carry over to machine code via the certificate.

Proof reuse: Given similar x86 and PowerPC code:

31C085F67405408B36EBF7

38A000002C140000408200107E80A02E38A500014BFFFFF0

which decompiles into f' and f'', respectively. Manual proofs above can be reused if f = f' = f''.

Algorithm

Decompilation algorithm:

- Step I: evaluate underlying ISA model (prove Hoare triples for each instruction)
- Step 2: construct CFG and find 'decompilation rounds' (usually one round per loop)
- **Step 3:** for each round, compose a Hoare triple theorem:

 $\begin{aligned} &\{ pre[v_0 \dots v_n] \} \\ & code \\ &\{ \mathsf{let} \ (v'_0 \dots v'_n) = f(v_0 \dots v_n) \text{ in } post[v'_0 \dots v'_n] \} \end{aligned}$

if the code contains a loop, apply a loop rule

Decompiler implementation

Implementation:

- ML program which fully-automatically performs forward proof,
- no heuristics and no dangling proof obligations,
- Ioops are handled by a special loop rule which introduces tail-recursive functions:

tailrec(x) = if G(x) then tailrec(F(x)) else D(x)

with termination and side-conditions H collected as:

 $pre(x) = (if G(x) then pre(F(x)) else true) \land H(x)$

Details in Myreen et al. [FMCAD'08].

Compilation

Synthesis often more practical. Given function f,

 $f(r_1) = \text{if } r_1 < 10 \text{ then } r_1 \text{ else let } r_1 = r_1 - 10 \text{ in } f(r_1)$

our *compiler* generates ARM machine code:

E351000A	L:	cmp r1,#10
2241100A		subcs r1,r1,#10
2AFFFFFC		bcs L

and automatically proves a certificate HOL theorem:

 $\vdash \{ R1 r_1 * PC p * s \}$ p: E351000A 2241100A 2AFFFFC $\{ R1 f(r_1) * PC (p+12) * s \}$

Compilation, example cont.

One can prove properties of f since it lives inside HOL:

 $\vdash \forall x. \ f(x) = x \bmod 10$

Properties proved of *f* translate to properties of the machine code:

 $\vdash \{ \text{R1} \ r_1 * \text{PC} \ p * \text{s} \}$ p : E351000A 2241100A 2AFFFFC $\{ \text{R1} \ (r_1 \mod 10) * \text{PC} \ (p+12) * \text{s} \}$

Additional feature: the compiler can use the above theorem to extend its input language with: let $r_1 = r_1 \mod 10$ in _

User-defined extensions

Using our theorem about mod, the compiler accepts:

$$g(r_1, r_2, r_3) = \text{let } r_1 = r_1 + r_2 \text{ in}$$

 $\text{let } r_1 = r_1 + r_3 \text{ in}$
 $\text{let } r_1 = r_1 \mod 10 \text{ in}$
 (r_1, r_2, r_3)

Previously proved theorems can be used as building blocks for subsequent compilations.

Implementation

To compile function f:

- 1. generate, without proof, code from input f;
- 2. decompile, with proof, a function f' from generated code;
- 3. prove f = f'.

Features:

- code generation completely separate from proof
- supports many light-weight optimisations without any additional proof burden: instruction reordering, conditional execution, dead-code elimination, duplicate-tail elimination, ...
- allows for significant user-defined extensions

Details in Myreen et al. [CC'09]

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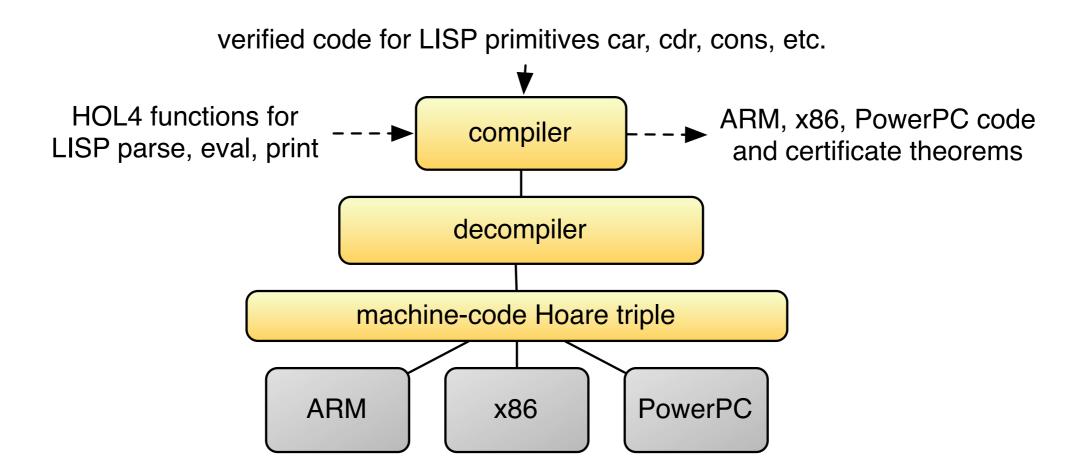
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A verified Lisp interpreter

Idea: create LISP implementations via compilation.



Lisp formalised

LISP s-expressions defined as data-type SExp:

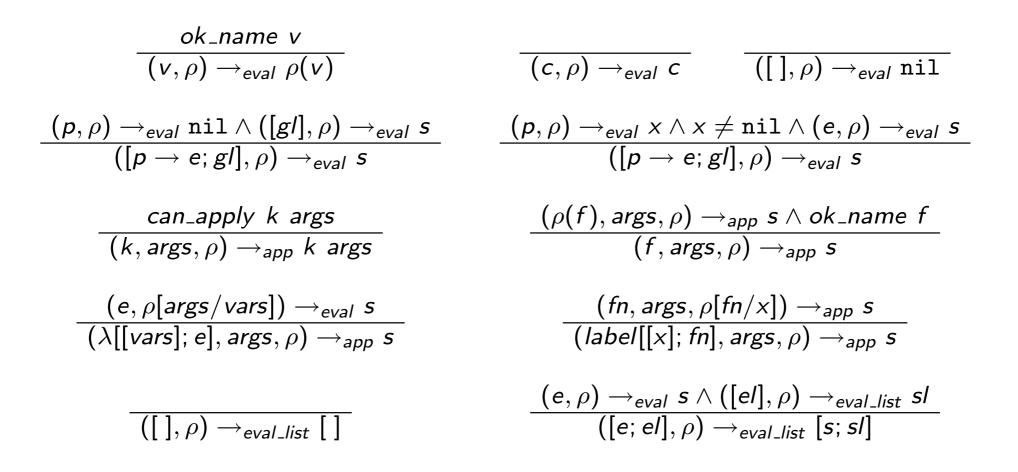
LISP primitives were defined, e.g.

$$cons x y = Dot x y$$
$$car (Dot x y) = x$$
plus (Num m) (Num n) = Num (m + n)

The semantics of LISP evaluation was taken to be Gordon's formalisation of 'LISP 1.5'-like evaluation, next slide...

Gordon's Lisp semantics from ACL2 workshop 2007

Defined using three mutually recursive relations \rightarrow_{eval} , \rightarrow_{app} and \rightarrow_{eval_list} .



Here c, v, k and f range over value constants, value variables, function constants and function variables, respectively.

Extending the compiler

We define heap assertion 'lisp $(v_1, v_2, v_3, v_4, v_5, v_6, I)$ ' and prove implementations for primitive operations, e.g.

is_pair $v_1 \Rightarrow$ { lisp $(v_1, v_2, v_3, v_4, v_5, v_6, l) * pc p$ } p : E5934000{ lisp $(v_1, car v_1, v_3, v_4, v_5, v_6, l) * pc (p + 4)$ } size $v_1 + size v_2 + size v_3 + size v_4 + size v_5 + size v_6 < l \Rightarrow$ { lisp $(v_1, v_2, v_3, v_4, v_5, v_6, l) * pc p$ } p : E50A3018 E50A4014 E50A5010 E50A600C ...{ lisp $(cons v_1 v_2, v_2, v_3, v_4, v_5, v_6, l) * pc (p + 332)$ }

with these the compiler understands:

```
let v_2 = \operatorname{car} v_1 in ...
let v_1 = \operatorname{cons} v_1 v_2 in ...
```

Short demo

Verified Lisp interpreters

Evaluation.

- 1. the compiler was extended with code for car, cons, plus, etc.
- 2. lisp_eval defined as tail-rec function, for which we proved:

3. the compiler automatically produced correct implementations.

Parsing/printing.

1. high-level definitions parsing/printing functions were defined,

 $\forall s. \text{ sexp_ok } s \Rightarrow \text{ string2sexp } (\text{sexp2string } s) = s$

- 2. low-level definitions were compiled to machine code,
- 3. manual proof related high- and low-level definitions.

Correctness theorem

The result is an interpreter which parses, evaluates and prints LISP:

 $\forall s \ r \ l \ p.$ $s \rightarrow_{eval} r \land \text{sexp_ok} \ s \land \text{lisp_eval_pre}(s, l) \Rightarrow$ $\{ \exists a. \text{R3} \ a \ast \text{string} \ a \ (\text{sexp2string} \ s) \ast \text{space} \ s \ l \ast \text{pc} \ p \ \}$ $p : \dots \text{ machine code not shown} \dots$ $\{ \exists a. \text{R3} \ a \ast \text{string} \ a \ (\text{sexp2string} \ r) \ast \text{space}' \ s \ l \ast \text{pc} \ (p+8968) \ \}$

where:

$s ightarrow_{eval} r$	is	"s evaluates to r in Gordon's semantics"
sexp_ok <i>s</i>	is	" <i>s</i> contains no bad symbols"
$lisp_eval_pre(s, I)$	is	"s can be evaluated with heap limit I"
string a str	is	"string <i>str</i> is stored in memory at address <i>a</i> "
space s /	is	"there is enough memory to setup heap of size <i>I</i> "

Running the Lisp interpreter



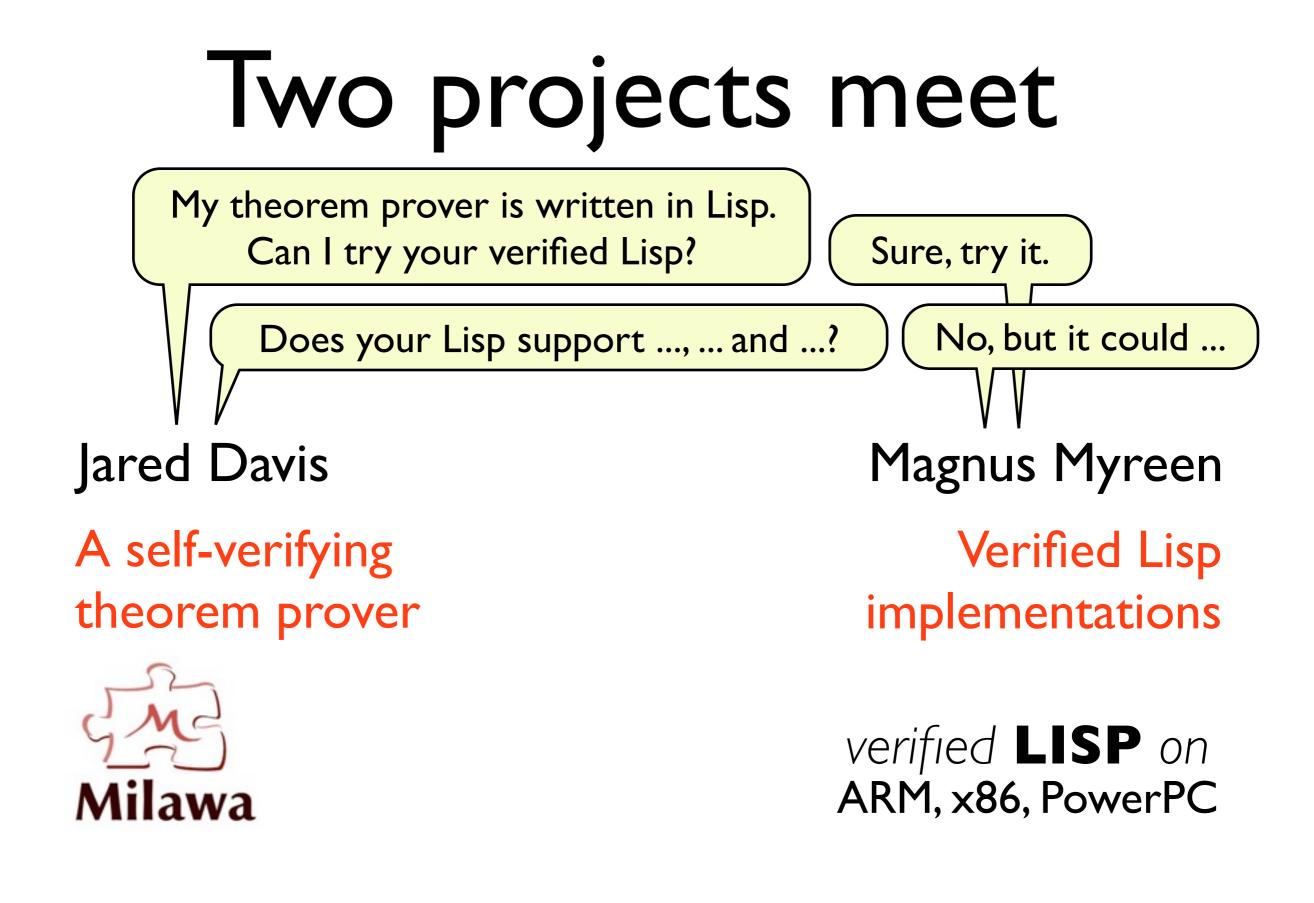
```
Nintendo DS lite (ARM) MacBook (x86) old MacMini (PowerPC)
```

```
(pascal-triangle '((1)) '6)
```

returns:

```
((1 6 15 20 15 6 1)
(1 5 10 10 5 1)
(1 4 6 4 1)
(1 3 3 1)
(1 2 1)
(1 1)
(1))
```

Next: can we do better than a simple Lisp interpreter?



Running Milawa



Milawa's bootstrap proof:

- 4 gigabyte proof file:
 >500 million unique conses
- takes 16 hours to run on a state-of-the-art runtime (CCL)

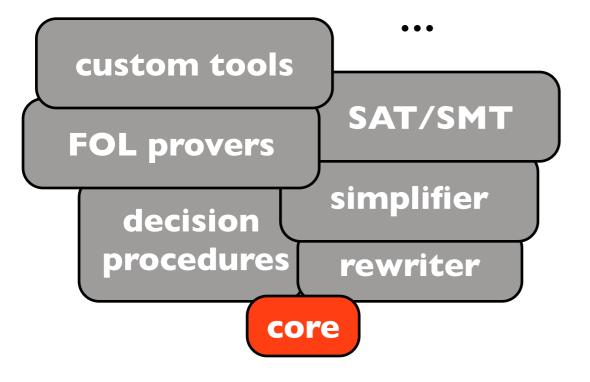
Jitotvių edrikus PISP ARM, x86, PovemPoiler (TPHOLs 2009)

- Contribution: "toy"
 - a new verified Lisp which is able to host the Milawa thm prover

A short introdution to

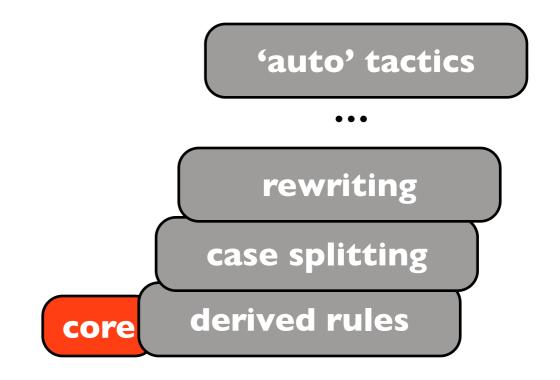
- Milawa is styled after theorem provers such as NQTHM and ACL2,
- has a small trusted logical kernel similar to LCF-style provers,
- ... but does not suffer the performance hit of LCF's fully expansive approach.

Comparison with LCF approach



LCF-style approach

- all proofs pass through the core's primitive inferences
- extensions steer the core



the Milawa approach

- all proofs must pass the core
- the core can be reflectively extended at runtime

Requirements on runtime

Milawa uses a subset of Common Lisp which

is for most part first-order pure functions over natural numbers, symbols and conses,

uses primitives: cons car cdr consp natp symbolp
 equal + - < symbol-< if</pre>

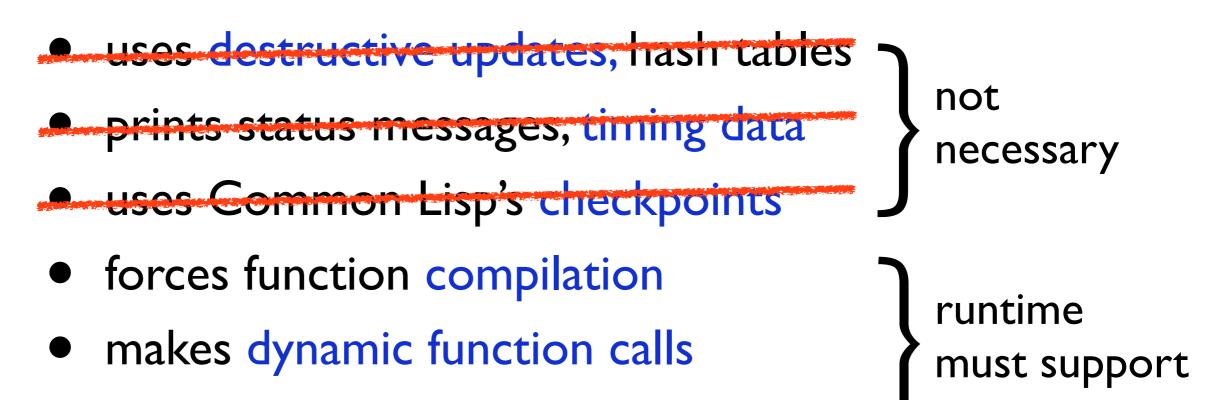
macros: or and list let let* cond first second third fourth fifth

and a simple form of lambda-applications.

(Lisp subset defined on later slide.)

Requirements on runtime

...but Milawa also



• can produce runtime errors

(Lisp subset defined on later slide.)

Runtime must scale

Designed to scale:

- just-in-time compilation for speed
 - functions compile to native code
- target 64-bit x86 for heap capacity
 - space for 2³¹ (2 billion) cons cells (16 GB)
- efficient scannerless parsing + abbreviations
 - must cope with 4 gigabyte input
- graceful exits in all circumstances
 - allowed to run out of space, but must report it

~30,000 lines of HOL4 scripts •specified input language: syntax & semantics 2.verified necessary algorithms, e.g. compilation from source to bytecode • parsing and printing of s-expressions copying garbage collection 3.proved refinements from algorithms to x86 code 4.plugged together to form read-eval-print loop

Workflow

AST of input language

Example of semantics for macros:

 $\begin{array}{c} (\mathsf{App}\ (\mathsf{PrimitiveFun}\ \mathsf{Car})\ [x], env, k, io) \xrightarrow{\mathsf{ev}} (ans, env', k', io') \\ (\mathsf{First}\ x, env, k, io) \xrightarrow{\mathsf{ev}} (ans, env', k', io') \end{array}$

		List (<i>term</i> list)	(macro)
		Let $((string \times term) \text{ list}) term$	(macro)
		LetStar $((string \times term) \text{ list}) term$	(macro)
		Cond $((term \times term) \text{ list})$	(macro)
		First <i>term</i> Second <i>term</i> Third <i>term</i>	(macro)
		Fourth term Fifth term	(macro)
func	::= 	Define Print Error Funcall PrimitiveFun <i>primitive</i> Fun <i>string</i>	
primitive	::= 	Equal Symbolp SymbolLess Consp Cons Car Cdr Natp Add Sub Less	

compile: $AST \rightarrow bytecode list$

bytecode

Pop PopN num PushVal num PushSym *string* LookupConst *num* Load *num* Store *num* DataOp *primitive* Jump num JumpIfNil *num* DynamicJump Call *num* DynamicCall Return Fail Print Compile

pop one stack element pop n stack elements push a constant number push a constant symbol push the nth constant from system state push the nth stack element overwrite the nth stack element add, subtract, car, cons, ... jump to program point nconditionally jump to njump to location given by stack top static function call (faster) dynamic function call (slower) return to calling function signal a runtime error print an object to stdout compile a function definition

How do we get just-in-time compilation?

Treating code as data:

 $\forall p \ c \ q. \quad \{p\} \ c \ \{q\} = \{p * \mathsf{code} \ c\} \ \emptyset \ \{q * \mathsf{code} \ c\}$ (POPL'10)

Definition of Hoare triple:

 $\begin{array}{ll} \{p\} \ c \ \{q\} & = & \forall s \ r. & (p \ast r \ast \mathsf{code} \ c) \ s \implies \\ & \exists n. \ (q \ast r \ast \mathsf{code} \ c) \ (\mathsf{run} \ n \ s) \end{array}$

I/O and efficient parsing

Jitawa implements a read-eval-print loop:

Use of external C routines adds assumptions to proof:

- reading next string from stdin
- printing null-terminated string to stdout

An efficient s-expression parser (and printer) is proved, which deals with abbreviations:

(append (cons (cons a b) c) (cons (cons a b) c))

(append #1=(cons (cons a b) c) #1#)

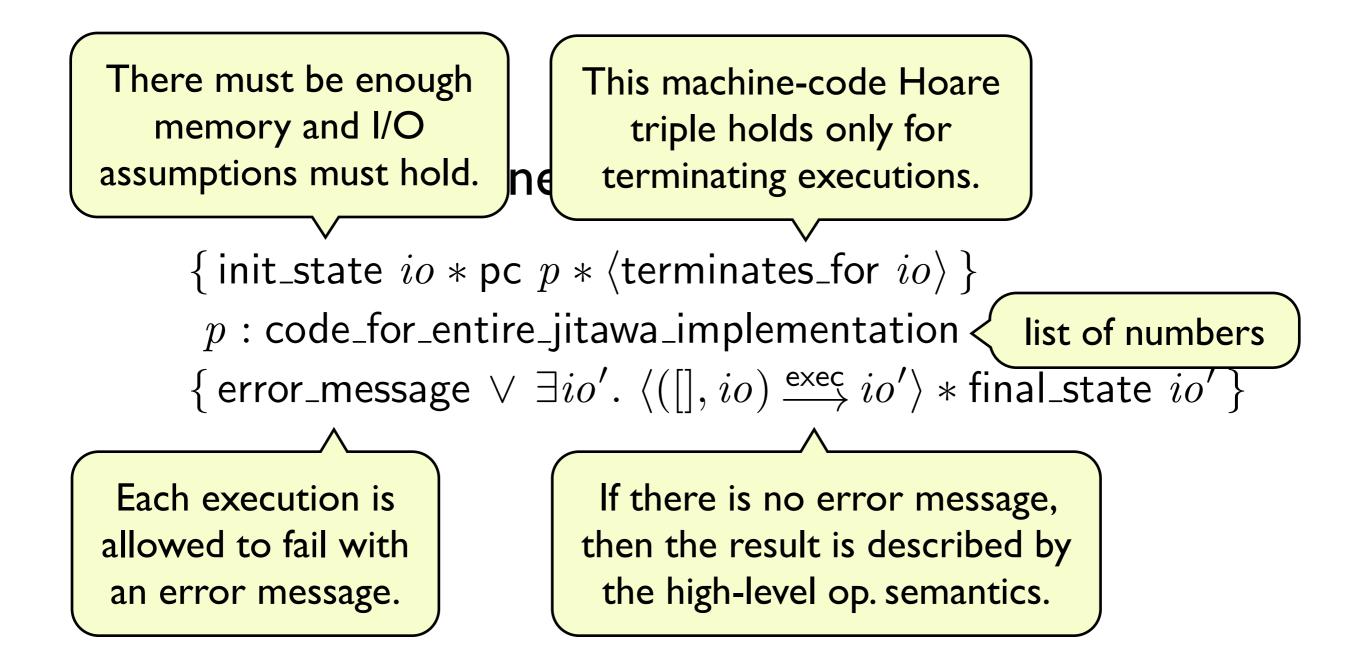
Read-eval-print loop

- Result of reading lazily, writing eagerly
- Eval = compile then jump-to-compiled-code
- Specification: read-eval-print until end of input

$$\neg \text{is_empty (get_input } io) \land \\ \text{next_sexp (get_input } io)) = (s, rest) \land \\ (\text{sexp2term } s, [], k, \text{set_input } rest \; io) \xrightarrow{\text{ev}} (ans, k', io') \land \\ (k', \text{append_to_output (sexp2string } ans) \; io') \xrightarrow{\text{exec}} io'' \\ (k, io) \xrightarrow{\text{exec}} io''$$

 $\begin{array}{c} \text{is_empty (get_input } io) \\ \hline (k, io) \xrightarrow{\text{exec}} io \end{array}$

Correctness theorem



Verified code

\$ cat verified_code.s

/* Machine code automatically extracted from a HOL4 theorem. */

*/

/* The code consists of 7423 instructions (31840 bytes).

.byte	0x48,	0x8B,	0x5F,	0x18		
.byte	0x4C,	0x8B,	0x7F,	0x10		
.byte	0x48,	0x8B,	0x47,	0x20		
.byte	0x48,	0x8B,	0x4F,	0x28		
.byte	0x48,	0x8B,	0x57,	0x08		
.byte	0x48,	0x8B,	0x37			
.byte	0x4C,	0x8B,	0x47,	0x60		
.byte	0x4C,	0x8B,	0x4F,	0x68		
.byte	0x4C,	0x8B,	0x57,	0x58		
.byte	0x48,	0x01,	0xC1			
.byte	0xC7,	0x00,	0x04,	0x4E,	0x49,	0x4C
.byte	0x48,	0x83,	0xC0,	0x04		
.byte	0xC7,	0x00,	0x02,	0x54,	0x06,	0x51
.byte	0x48,	0x83,	0xC0,	0x04		
• • •						

A short demo:

Jitawa – a verified runtime for Milawa

Running Milawa on Jitawa

Running Milawa's 4-gigabyte booststrap process:

CCLI6 hoursJitawa's compiler performsSBCL22 hoursalmost no optimisations.Jitawa128 hours(8x slower than CCL)

Parsing the 4 gigabyte input:

CCL 716 seconds (9x slower than Jitawa) Jitawa 79 seconds

Next: can we do better than Lisp? (on going work)

The CakeML project



Aim: to do the same for a subset of Standard ML:

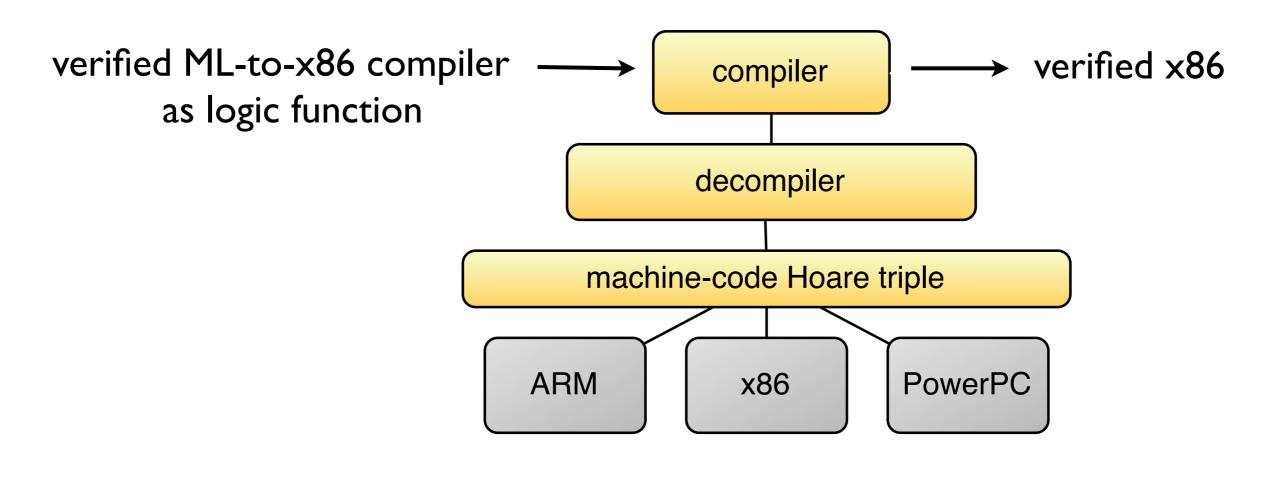
produce verified read-eval-print-loop for ML construct a proved-to-be-sound version of HOL light synthesise hardware that runs ML programs on 'bare metal'

Collaborators:

Scott Owens – semantics, type/module systems Ramana Kumar – compiler verification Michael Norrish – parsing, general HOL expertise David Greaves – hardware, FPGAs

Implementation of ML compiler

How to produce compile component?



Very cumbersome....

Bootstrapping the compiler

Instead: we bootstrap the verified compile function, we evaluate the compiler on a deep embedding of itself within the logic:

EVAL ``compile COMPILE``

derives a theorem:

compile COMPILE = compiler-as-machine-code

We believe this is the first bootstrapping of a formally verified compiler.

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