Small-step operational semantics and SML

Lecture 3

MPhil ACS & Part III course, Functional Programming:
Implementation, Specification and Verification

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Non-termination

From last lecture:

Big-step operational semantics

- inductive relation describes terminating evaluations
- "big-step" because term-to-result evaluation is described by a single transition

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Small-step operational semantics

- models evaluation in "small steps"
- possibly infinite sequence of steps

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Also in this lecture:

Sketch of a formal semantics for a real FP language (SML)

Evaluation as small steps

The steps execute an abstract machine.

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Abstract machines:

- Turing machine for general computation

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Abstract machines:

- Turing machine for general computation
- Landin's SECD machine for idealised FP implementation (evaluates lambda calculus)

  SECD = Stack, Environment, Code and Dump

- We will use a modern version of SECD, a Felleisen and Friedman's CEK machine

  CEK = Control, Environment and (K)ontinuation

  sometimes called CESK, where S = store

A simple expression language

We start with a very simple language:

```
val ::= num

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      | Add exp exp
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A big-step semantics for this:

$$\frac{}{(\text{Const } n, env) \Downarrow_{\text{ev}} n} \qquad \frac{(e_1, env) \Downarrow_{\text{ev}} s_1 \qquad (e_2, env) \Downarrow_{\text{ev}} s_2}{(\text{Add } e_1\ e_2, env) \Downarrow_{\text{ev}} (s_1 + s_2)}$$

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a single step

Leaving holes

How to evaluate in steps?

Add e_1 e_2

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A small-step machine should:

- evaluate e_1 first, but
- remember to come back and evaluate e_2 and Add

Leaving holes

How to evaluate in steps?

Add e_1 e_2

A small-step machine should:

- evaluate e_1 first, but
- remember to come back and evaluate e_2 and Add

Solution: keep a 'TO-DO list' (continuation) with holes in exps

Sketch of the idea

```
exp: Add (Add 1 2) 3    continuation:
```

Sketch of the idea

```
exp: Add (Add 1 2) 3
```

```
exp: Add 1 2
```

```
continuation:
```

```
continuation: Add ?? 3
```

Sketch of the idea

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exp: Add (Add 1 2) 3
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continuation: Add ?? 3
```

hole in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
```

hole in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
val: 1                    continuation: Add ?? 2, Add ?? 3
```

hole in the exp

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exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
val: 1                    continuation: Add ?? 2, Add ?? 3
exp: 2                    continuation: Add 1 ??, Add ?? 3
```

hole in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3        continuation:
exp: Add 1 2                continuation: Add ?? 3
exp: 1                      continuation: Add ?? 2, Add ?? 3
val: 1                      continuation: Add ?? 2, Add ?? 3
exp: 2                      continuation: Add 1 ??, Add ?? 3
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hole in the exp

val in the exp

Sketch of the idea

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exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
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val: 1                    continuation: Add ?? 2, Add ?? 3
exp: 2                    continuation: Add 1 ??, Add ?? 3
val: 2                    continuation: Add 1 ??, Add ?? 3
```

hole in the exp

val in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
val: 1                    continuation: Add ?? 2, Add ?? 3
exp: 2                    continuation: Add 1 ??, Add ?? 3
val: 2                    continuation: Add 1 ??, Add ?? 3
val: 3                    continuation: Add ?? 3
```

hole in the exp

val in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
val: 1                    continuation: Add ?? 2, Add ?? 3
exp: 2                    continuation: Add 1 ??, Add ?? 3
val: 2                    continuation: Add 1 ??, Add ?? 3
val: 3                    continuation: Add ?? 3
exp: 3                    continuation: Add 3 ??
```

hole in the exp

val in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3      continuation:
exp: Add 1 2              continuation: Add ?? 3
exp: 1                    continuation: Add ?? 2, Add ?? 3
val: 1                    continuation: Add ?? 2, Add ?? 3
exp: 2                    continuation: Add 1 ??, Add ?? 3
val: 2                    continuation: Add 1 ??, Add ?? 3
val: 3                    continuation: Add ?? 3
exp: 3                    continuation: Add 3 ??
val: 3                    continuation: Add 3 ??
```

hole in the exp

val in the exp

Sketch of the idea

```
exp: Add (Add 1 2) 3        continuation:
exp: Add 1 2                continuation: Add ?? 3
exp: 1                      continuation: Add ?? 2, Add ?? 3
val: 1                      continuation: Add ?? 2, Add ?? 3
exp: 2                      continuation: Add 1 ??, Add ?? 3
val: 2                      continuation: Add 1 ??, Add ?? 3
val: 3                      continuation: Add ?? 3
exp: 3                      continuation: Add 3 ??
val: 3                      continuation: Add 3 ??
val: 6                      continuation:
```

hole in the exp

val in the exp

Defining a step

We define what expressions with holes and values means:

```
ctxt ::= CAdd1 hole exp
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```

The state consists of env, exp-or-val and a list of ctxt (continuation):

```
state = (env, exp_or_val, (ctxt * env) list)

exp_or_val ::= Exp exp | Val val
```

Defining a step

We define what expressions with holes and values means:

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The state consists of env, exp-or-val and a list of ctxt (continuation):

```
state = (env, exp_or_val, (ctxt * env) list)

exp_or_val ::= Exp exp | Val val
```

A step can either return or get stuck:

```
result = OK state | STUCK
```

The step function

```
step (env,e_or_v,c) =
  case e_or_v of
  | Val v => continue v c
  | Exp (Const n) => return env n c
  | Exp (Add e1 e2) => push env e1 (CAdd1 () e2) c

push env e c1 c = OK (env, Exp e, (c1, env) :: c)

return env v c = OK (env, Val v, c)

continue v c =
  case c of
  | [] => STUCK
  | ((CAdd1 () e2,env)::cs) => OK (env, Exp e2, (CAdd2 v ())::cs)
  | ((CAdd2 v1 (),env)::cs) => OK (env, Val (v1 + v), cs)
```

A few more functions

A function which recognises a final state:

```
is_final state =
  case state of
  | (env,Val v,[]) => true
  | _ => false
```

A function that can be used to run the step function:

```
steps state =
  if is_final state then [OK state] else
    case step state of
    | STUCK => [STUCK]
    | OK new_state => OK state :: steps new_state
```

NB: `steps` cannot be defined in HOL because it might not terminate.

Sample run

```
steps (env,Exp (Add (Const 1) (Const 2)),[]) =
 [ OK (env, Exp (Add (Const 1) (Const 2)), []),
   OK (env, Exp (Const 1),                 [CAdd1 () (Const 2)]),
   OK (env, Val 1,                         [CAdd1 () (Const 2)]),
   OK (env, Exp (Const 2),                 [CAdd2 1 ()]),
   OK (env, Val 2,                         [CAdd2 1 ()]),
   OK (env, Val 3,                         []) ]
```

Formal model

We define a relation describing a good step:

$$s_1 \to s_2 \;\equiv\; (\text{step } s_1 = \text{OK } s_2)$$

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Using this we can define:

$$\mathsf{terminating\_eval}\ env\ e\ v \;\equiv\; (env, \mathsf{Exp}\ e, []) \rightarrow^* (\_, v, [])$$

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reflexive-transitive closure

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We define a relation describing a good step:

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Using this we can define:

reflexive-transitive closure

$$\text{terminating\_eval } env\ e\ v \;\equiv\; (env, \text{Exp } e, []) \rightarrow^* (\_, v, [])$$

$$\text{non\_terminating\_eval } env\ e\ v \;\equiv$$
$$(\forall s.\ (env, \text{Exp } e, []) \rightarrow^* s \implies \exists t.\ s \rightarrow t)$$

Extending the language

```
val ::= num

exp ::= Const num
      | Var string
      | Add exp exp
      | If exp exp exp
      | Let string exp exp

ctxt ::= CAdd1 hole exp
       | CAdd2 val hole
       | CIf hole exp exp
       | CLet string hole exp
```

Extending the functions

```
step (env,e_or_v,c) =
  case e_or_v of
  ...
  | Exp (Var n) => return env (lookup n env) c
  | Exp (If e1 e2 e3) => push env e1 (CIf () e2 e3) c
  | Exp (Let name e1 e2) => push env e1 (CLet name () e2) c
```

```
continue v c =
  case c of
  ...
  | ((CIf () e2 e3,env)::cs) =>
      OK (env, Exp (if isTrue v then e2 else e3), cs)
  | ((CLet name () body,env)::cs) =>
      OK (env[name := v], Exp body, cs)
```

Adding support for functions

```
val ::= Num num
      | Closure string env exp

exp ::= Const num
      | Var string
      | Add exp exp
      | If exp exp exp
      | Let string exp exp
      | App exp exp
      | Fn string exp

ctxt ::= CAdd1 hole exp
       | CAdd2 val hole
       | CIf hole exp exp
       | CLet string hole exp
       | CApp1 hole exp
       | CApp2 val hole
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function value

Adding support for functions

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function value

function application

Adding support for functions

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val ::= Num num
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       | CLet string hole exp
       | CApp1 hole exp
       | CApp2 val hole
```

function value

function application

anonymous function

Additions to functions

```
step (env,e_or_v,c) =
  case e_or_v of
  ...
  | Exp (App e1 e2) => push env e1 (CApp1 () e2) c
  | Exp (Fn name e1) => return env (Closure name env e1) c

do_app v1 v2 =
  case v1 of
  | Closure name env exp => SOME (env[name := v2], exp)
  | _ => NONE

continue v c =
  case c of
  ...
  | ((CApp1 () e2,env)::cs) =>
      OK (env, Exp e2, CApp2 v () :: cs)
  | ((CApp2 v1 (),env)::cs) =>
      case do_app v1 v of
      | SOME (env,e) => OK (env, Exp e, cs)
      | NONE => STUCK
```

Additions to functions

```
step (env,e_or_v,c) =
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      | SOME (env,e) => OK (env, Exp e, cs)
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closure value created

Additions to functions

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  case e_or_v of
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do_app v1 v2 =
  case v1 of
  | Closure name env exp => SOME (env[name := v2], exp)
  | _ => NONE

continue v c =
  case c of
  ...
  | ((CApp1 () e2,env)::cs) =>
      OK (env, Exp e2, CApp2 v () :: cs)
  | ((CApp2 v1 (),env)::cs) =>
      case do_app v1 v of
      | SOME (env,e) => OK (env, Exp e, cs)
      | NONE => STUCK
```

closure value created

function application fails if value is not a function

Additions to functions

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do_app v1 v2 =
  case v1 of
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  | ((CApp1 () e2,env)::cs) =>
      OK (env, Exp e2, CApp2 v () :: cs)
  | ((CApp2 v1 (),env)::cs) =>
      case do_app v1 v of
      | SOME (env,e) => OK (env, Exp e, cs)
      | NONE => STUCK
```

closure value created

function application fails if value is not a function

failed application causes STUCK state

Sample run

In ML syntax: `(fn n => n + 1) 2`

```
steps
  ([],Exp (App (Fn "n" (Add (Var "n") (Const 1))) (Const 2)),[])
=
 [OK ([],Exp (App (Fn "n" (Add (Var "n") (Const 1))) (Const 2)),[]),
  OK ([],Exp (Fn "n" (Add (Var "n") (Const 1))),[CApp1 () (Const 2)]),
  OK ([],Val (Closure "n" [] (Add (Var "n") (Const 1))),[CApp1 () (Const 2)]),
  OK ([],Exp (Const 2),[CApp2 (Closure "n" [] (Add (Var "n") (Const 1))) ()]),
  OK ([],Val (Num 2),[CApp2 (Closure "n" [] (Add (Var "n") (Const 1))) ()]),
  OK ([("n",Num 2)],Exp (Add (Var "n") (Const 1)),[]),
  OK ([("n",Num 2)],Exp (Var "n"),[CAdd1 () (Const 1)]),
  OK ([("n",Num 2)],Val (Num 2),[CAdd1 () (Const 1)]),
  OK ([("n",Num 2)],Exp (Const 1),[CAdd2 (Num 2) ()]),
  OK ([("n",Num 2)],Val (Num 1),[CAdd2 (Num 2) ()]),
  OK ([("n",Num 2)],Val (Num 3),[])]
```

Another sample run

In ML syntax: `1(2)`

```
steps ([],Exp (App (Const 1) (Const 2)),[])
=
 [OK ([],Exp (App (Const 1) (Const 2)),[]),
  OK ([],Exp (Const 1),[CApp1 () (Const 2)]),
  OK ([],Val (Num 1),[CApp1 () (Const 2)]),
  OK ([],Exp (Const 2),[CApp2 (Num 1) ()]),
  STUCK]
```

Another sample run

In ML syntax: `1(2)`

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steps ([],Exp (App (Const 1) (Const 2)),[])
=
 [OK ([],Exp (App (Const 1) (Const 2)),[]),
  OK ([],Exp (Const 1),[CApp1 () (Const 2)]),
  OK ([],Val (Num 1),[CApp1 () (Const 2)]),
  OK ([],Exp (Const 2),[CApp2 (Num 1) ()]),
  STUCK]
```

Properly set up, we can prove that well-typed programs do not get stuck.
(later lecture)

Towards full SML

Full Standard ML (SML) includes:

- recursive functions
- datatypes (constructors and pattern-matching)
- user-declared exceptions (`raise`, `handle`)
- references (`ref`, `!`, `:=`)
- modules

Extending the val type

```
env = (string * val) list

val ::= Num num
      | Bool bool
      | Datatype string (val list)
      | Closure string env exp
      | Recclosure string string env exp
      | Loc of num
```

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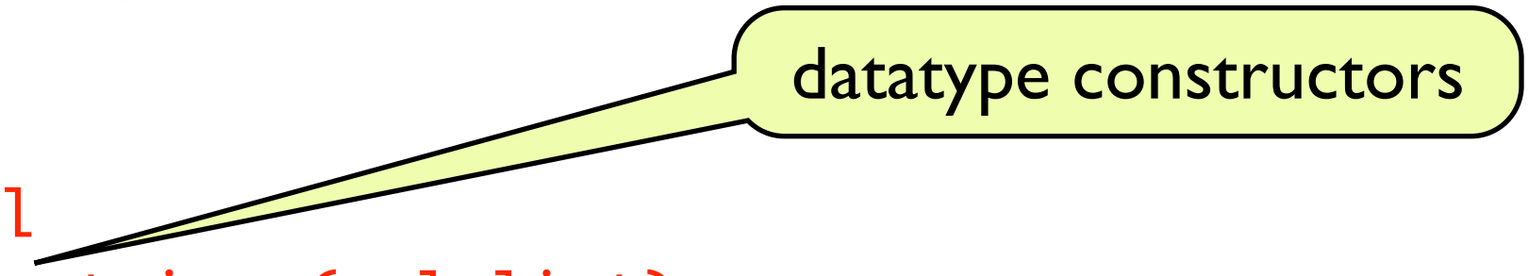

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```

datatype constructors

In ML syntax: `[1,2,3]`

In model:

```
Datatype “cons” [Num 1,
                 Datatype “cons” [Num 2,
                                  Datatype “cons” [Num 3,
                                                   Datatype “nil” []]]]
```

Extending the val type

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datatype constructors

value of a rec. closure

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      | Datatype string (val list)
      | Closure string env exp
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      | Loc of num
```

datatype constructors

value of a rec. closure

value of an ML ref, i.e. pointer into global store (imperative feature)

In ML syntax: `[1,2,3]`

In model:

```
Datatype “cons” [Num 1,
                 Datatype “cons” [Num 2,
                                  Datatype “cons” [Num 3,
                                                   Datatype “nil” []]]]
```

Recursive functions

```
do_app v1 v2 =
  case v1 of
  | Closure name env exp => SOME (env[name := v2], exp)
  | Recclosure f name env exp =>
      SOME (env[name := v2, f := Recclosure f name env exp], exp)
  | _ => NONE
```

Recursive functions

```
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  case v1 of
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body of rec. closure can refer to itself via this name

Recursive functions

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      SOME (env[name := v2, f := Recclosure f name env exp], exp)
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```

body of rec. closure can refer to itself via this name

Producing a rec. closure in SML:

```
let
  fun fac n = if n = 0 then 1 else n * fac (n-1)
in fac end
```

Exceptions

Example use of `raise` and `handle` in SML:

```
let
  val _ = print “Hello”
  val _ = raise (ErrorNumber 3)
  val _ = print “ there!”
in 1 end
handle ErrorNumber n => n
```

This SML expression returns `3` and prints only `Hello`.

Exceptions

Example use of `raise` and `handle` in SML:

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  val _ = print “Hello”
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execution jumps to enclosing `handle`

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execution jumps to enclosing `handle`

execution continues here

This SML expression returns `3` and prints only `Hello`.

Semantics for excpetions

```
step (env,e_or_v,c) =
  case e_or_v of
  ...
  | Exp (Raise e) => push env e (CRaise ()) c
  | Exp (Handle e1 name e2) => push env e1 (CHandle () name e2) c
```

```
continue env v c =
  case c of
  ...
  | ((Craise (),env)::cs) =>
    case c of
    | [] => UncaughtExc v
    | ((Chandle () name e2) :: c) =>
        OK (env[name := v], Exp e2, c)
    | _::c => OK (env,Val v,((Craise (),env) ::c))
  | ((Chandle () name e2,env)::cs) => return env v c
```

Semantics for excpetions

```
step (env,e_or_v,c) =
  case e_or_v of
  ...
  | Exp (Raise e) => push env e (CRaise ()) c
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  ...
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    | [] => UncaughtExc v
    | ((Chandle () name e2) :: c) =>
        OK (env[name := v], Exp e2, c)
    | _::c => OK (env,Val v,((Craise (),env) ::c))
  | ((Chandle () name e2,env)::cs) => return env v c
```

raise looks for enclosing handle

Semantics for excpetions

```
step (env,e_or_v,c) =
  case e_or_v of
  ...
  | Exp (Raise e) => push env e (CRaise ()) c
  | Exp (Handle e1 name e2) => push env e1 (CHandle () name e2) c
```

```
continue env v c =
  case c of
  ...
  | ((Craise (),env)::cs) =>
    case c of
    | [] => UncaughtExc v
    | ((Chandle () name e2) :: c) =>
        OK (env[name := v], Exp e2, c)
    | _::c => OK (env,Val v,((Craise (),env) ::c))
  | ((Chandle () name e2,env)::cs) => return env v c
```

raise looks for enclosing handle

handle just returns, i.e. transparent

Support for ML references

To support references (stateful), we need to pass around state:

```
step (env,e_or_v,c,state) =
  case e_or_v of
  | Val v => continue env v c state
  | Exp (Const n) => return env n c state
  | Exp (Add e1 e2) => push env e1 (CAdd1 () e2) c state

push env e c1 c state = OK (env, Exp e, (c1, env) :: c, state)

return env v c state = OK (env, Val v, c, state)

continue env v c state =
  ...
```

The state is a finite mapping from num to val.

Long definition ... help?

Use tool support!

Lem, a tool for lightweight executable mathematics
http://www.cs.kent.ac.uk/people/staff/sao/lem/

Ott, a tool for writing definitions of programming languages and calculi
http://www.cl.cam.ac.uk/~pes20/ott/

These tools are not examinable material.

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- possibly infinite sequence of steps
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Semantics for SML

- function values modelled as closures (values containing code)
- for 'real-world' semantics: best use a specification tool