Second Set of Exercises

Exercise 1

Show from the definition of the operator $A[\cdots U \cdots]$ that $A[\mathsf{T}\ U\ Q]$ holds if Q holds somewhere along every path.

What is the formula of CTL that is true if Q holds along every path starting from a state satisfying P?

If $AG\ Q$ is defined to be $\neg E[\mathsf{T}\ U\ \neg Q]$ then show that $AG\ Q$ holds if Q holds everywhere along every path.

Express "*if P is ever true, then it continues to be true until Q is false*" in CTL.

Describe in English the meaning of "$AG(P \wedge Q \ \Rightarrow\ AX(A[P\ U\ \neg Q]))$". □

Exercise 2

> **The Missionaries and Cannibals Problem**
>
> Three missionaries and three cannibals seek to cross a river. They have a boat which can carry up to two people at a time. All missionaries and cannibals are able to navigate the boat. If at any time the cannibals out number the missionaries on either side of the bank the cannibals will eat the missionaries.
>
> [See: `http://www-formal.stanford.edu/jmc/elaboration/node2.html`]

Formalise the Missionaries and Cannibals Problem as a state transition system. Describe carefully the type of states in your formalisation. Write down in logic:

1. a predicate characterising the starting state;
2. a predicate characterising the goal state (i.e. everyone has crossed the river);
3. the transition relation.

Write down a formula in CTL that is true if there is a path from the initial state to the goal state and is such that the cannibals won't eat the missionaries at any state in the path. □

Exercise 3
Consider the following transition relation:

$$\begin{array}{l}
\mathcal{R}(x,y,z),(x',y',z')) = \\
\quad (x' = E_x(x,y,z) \quad \wedge \quad y' = y \quad \wedge \quad z' = z) \ \vee \\
\quad (x' = x \quad \wedge \quad y' = E_y(x,y,z) \quad \wedge \quad z' = z) \ \vee \\
\quad (x' = x \quad \wedge \quad y' = y \quad \wedge \quad z' = E_z(x,y,z))
\end{array}$$

If Pre is defined by:

$$\mathsf{Pre}\ \mathcal{R}\ \mathcal{Q}\ s \ = \ \exists s'.\ \mathcal{R}(s,s') \ \wedge \ \mathcal{Q}\ s'$$

then show how the BDD of $\mathsf{Pre}\ \mathcal{R}\ \mathcal{Q}\ (x,y,z)$ can be computed without having to compute the BDD of $\mathcal{R}$.

What is the significance of this? $\square$