Optimal ec-PIN Guessing

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Known: 12 offset digits from magnetic stripe:

$$\begin{aligned}
\text{Offset 1:} \quad & O_1 = (O_{1,1}, O_{1,2}, O_{1,3}, O_{1,4}) \\
\text{Offset 2:} \quad & O_2 = (O_{2,1}, O_{2,2}, O_{2,3}, O_{2,4}) \\
\text{Offset 3:} \quad & O_3 = (O_{3,1}, O_{3,2}, O_{3,3}, O_{3,4})
\end{aligned}$$

Wanted: four most likely PIN digits

$$\hat{P} = (\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4)$$

Define:

$$\begin{aligned}
\tilde{P}_j &= \text{random variable for } j\text{-th digit in PIN} \\
\tilde{O}_{i,j} &= \text{random variable for } j\text{-th digit in offset } i
\end{aligned}$$

for all $1 \le i \le 3$ and $1 \le j \le 4$.

Distributions:

$$p(\tilde{P}_j = k) = \begin{cases} 0/16, & \text{if } j = 1 \text{ and } k = 0 \\ 4/16, & \text{if } j = 1 \text{ and } k = 1 \\ 2/16, & \text{if } j > 1 \text{ and } k \in \{0, 1\} \\ 2/16, & \text{if } k \in \{2, \ldots, 5\} \\ 1/16, & \text{if } k \in \{6, \ldots, 9\} \end{cases}$$

$$p(\tilde{O}_{i,j} = k | \tilde{P}_j = l) = \begin{cases} 2/16, & \text{if } (l - k) \bmod 10 \in \{0, \ldots, 5\} \\ 1/16, & \text{if } (l - k) \bmod 10 \in \{6, \ldots, 9\} \end{cases}$$

A most likely PIN $\hat{P}$ is a P for which

$$p(\tilde{P} = P | \forall i : \tilde{O}_i = O_i)$$

is maximal. PIN digits are independent, therefore we look at per-digit probability

$$p(\tilde{P}_j = P_j | \forall i : \tilde{O}_{i,j} = O_{i,j})$$

and get best PIN as the combination of most likely digits.

We turn around this conditional probability (BAYES' theorem)

$$\begin{aligned}
& p(\tilde{P}_j = P_j | \forall i : \tilde{O}_{i,j} = O_{i,j}) \\
= \quad & \frac{p(\tilde{P}_j = P_j \wedge \forall i : \tilde{O}_{i,j} = O_{i,j})}{p(\forall i : \tilde{O}_{i,j} = O_{i,j})} \\
= \quad & \frac{p(\forall i : \tilde{O}_{i,j} = O_{i,j} | \tilde{P}_j = P_j) \cdot p(\tilde{P}_j = P_j)}{p(\forall i : \tilde{O}_{i,j} = O_{i,j})} \\
= \quad & \frac{p(\forall i : \tilde{O}_{i,j} = O_{i,j} | \tilde{P}_j = P_j) \cdot p(\tilde{P}_j = P_j)}{\displaystyle\sum_{k=0}^{9} p(\forall i : \tilde{O}_{i,j} = O_{i,j} | \tilde{P}_j = k) \cdot p(\tilde{P}_j = k)}
\end{aligned}$$

and since all three offsets are independent

$$= \quad \frac{\displaystyle\prod_{i=1}^{3} p(\tilde{O}_{i,j} = O_{i,j} | \tilde{P}_j = P_j) \cdot p(\tilde{P}_j = P_j)}{\displaystyle\sum_{k=0}^{9} \prod_{i=1}^{3} p(\tilde{O}_{i,j} = O_{i,j} | \tilde{P}_j = k) \cdot p(\tilde{P}_j = k)}$$

Now calculate this for all $P_j \in \{0, \ldots, 9\}$ and determine the $\hat{P}_j$ with maximum probability.

What success rate do we expect with a randomly picked card?

For PIN digit j: Try all 16^4 combinations of hexadecimal digits (W, X, Y, Z). Like the bank, determine the PIN and offsets:

$$
\begin{aligned}
P_j &:= \begin{cases} W \bmod 10, & \text{if } W \bmod 10 > 0 \text{ or } j > 1 \\ 1, & \text{if } W \bmod 10 = 0 \text{ and } j = 1 \end{cases} \\
O_{1,j} &:= (P_j - X) \bmod 10 \\
O_{2,j} &:= (P_j - Y) \bmod 10 \\
O_{3,j} &:= (P_j - Z) \bmod 10
\end{aligned}
$$

We have now 16^4 simulated cards with realistic PIN and offset digit distribution.

Now, determine most likely PIN digit $\hat{P}_j$ for all of those 16^4 cards and compare $\hat{P}_j$ with P_j. The measured success rates are:

$$
\begin{aligned}
&\text{digit 1:} \quad 0.27856 \approx 28\% \approx 1/3.6 \\
&\text{digit 2:} \quad 0.20312 \approx 20\% \approx 1/4.9 \\
&\text{digit 3:} \quad 0.20312 \approx 20\% \approx 1/4.9 \\
&\text{digit 4:} \quad 0.20312 \approx 20\% \approx 1/4.9
\end{aligned}
$$

Note: With a good PIN-generation algorithm, we would have expected 1/9 for first digit and 1/10 for remaining three.

Single attempt success rate for all four digits:

$$0.27856 \cdot 0.20312^3 \approx 0.0023346 \approx 0.233\% \approx 1/428$$

A card thief has at least three attempts to enter a PIN and most second or third-best PINs have a similar success probability, therefore

$$3 \cdot 0.0023346 \approx 0.7\% \approx 1/150$$

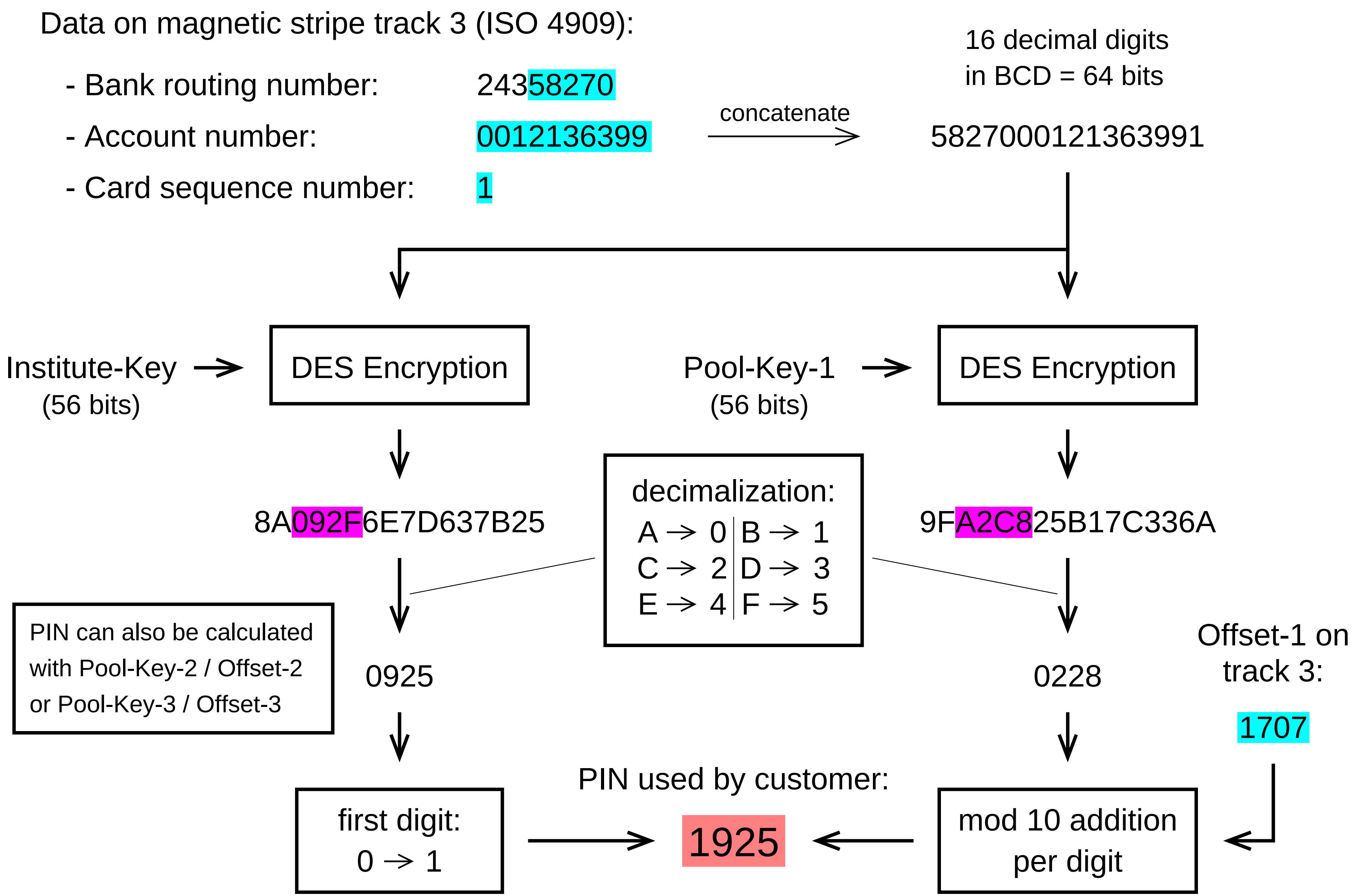
This is an expected value for a randomly selected card. Some individual cards with offsets like 0000/6666/6555 allow success rates as high as $1.896\% \approx 1/52.7$ in three attempts.

Comparison: With a good PIN algorithm, we would have expected

$$3 \cdot 1/9 \cdot 1/10 \cdot 1/10 \cdot 1/10 = 1/3000 \approx 0.033\%.$$

In other words, the security of the 4-digit ec-PIN system is worse than that of a good 3-digit system (with $1/300 \approx 0.33\%$ success rate).

PIN Calculation for EuroCheque ATM Debit Cards



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