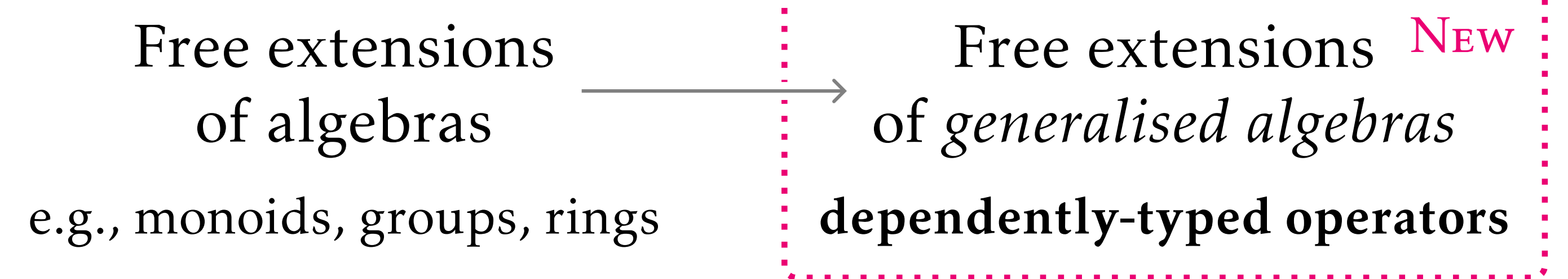
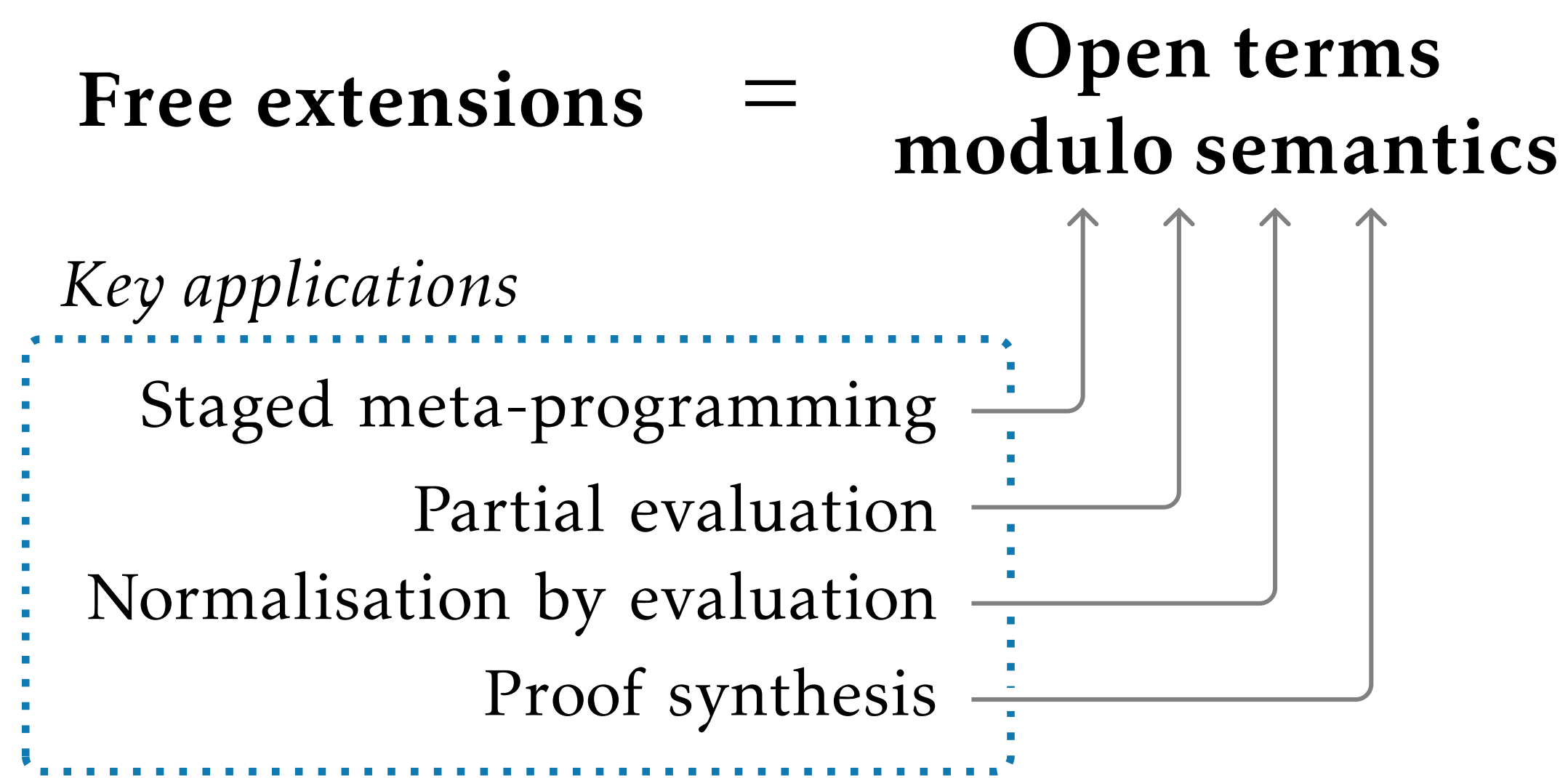


1. Background

2. Contributions



Generalised algebra	Applications	Free extension
Categories	Semantics	✓ (Agda)
Monoidal categories	Linear type systems	TODO
Cartesian categories	First order PLs	TODO
CCCs	Purely functional PLs	TODO

3. Examples

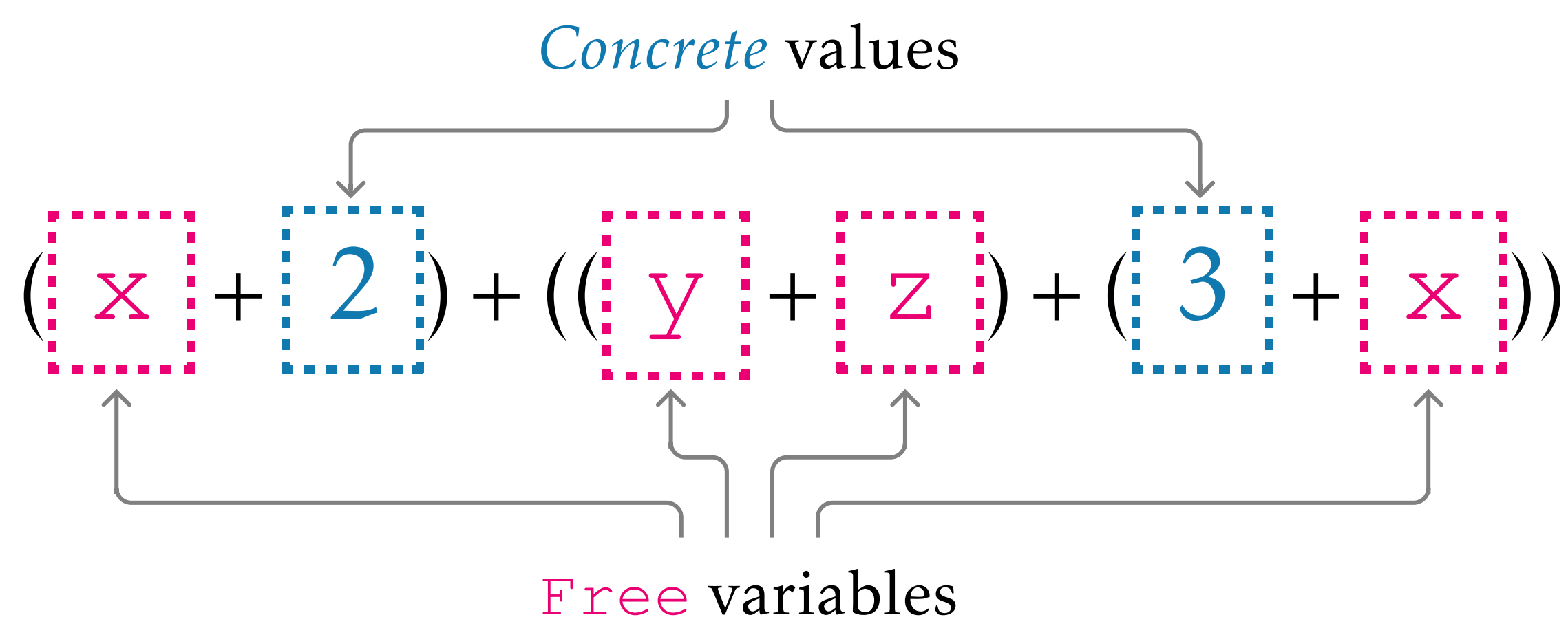
(i) Normalising in a commutative monoid

(ii) Normalising composites in a category

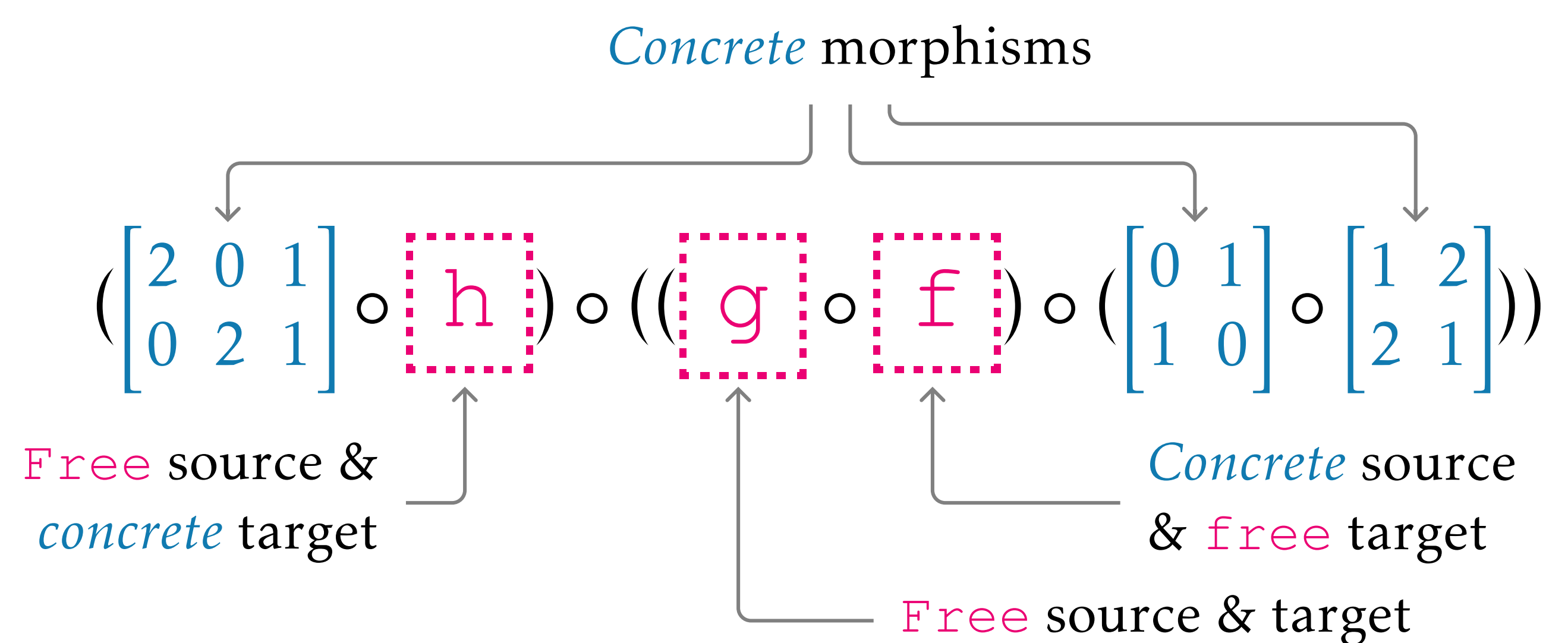


- **Syntax:** sums built from a binary operator with unit.

- **Syntax:** well-typed composites.

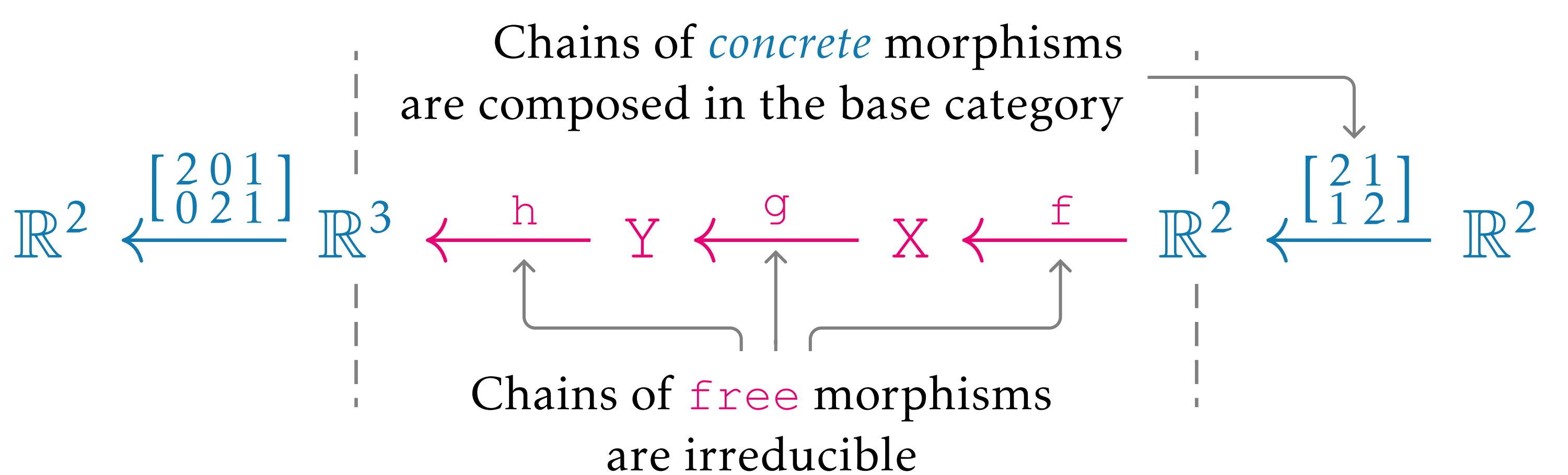
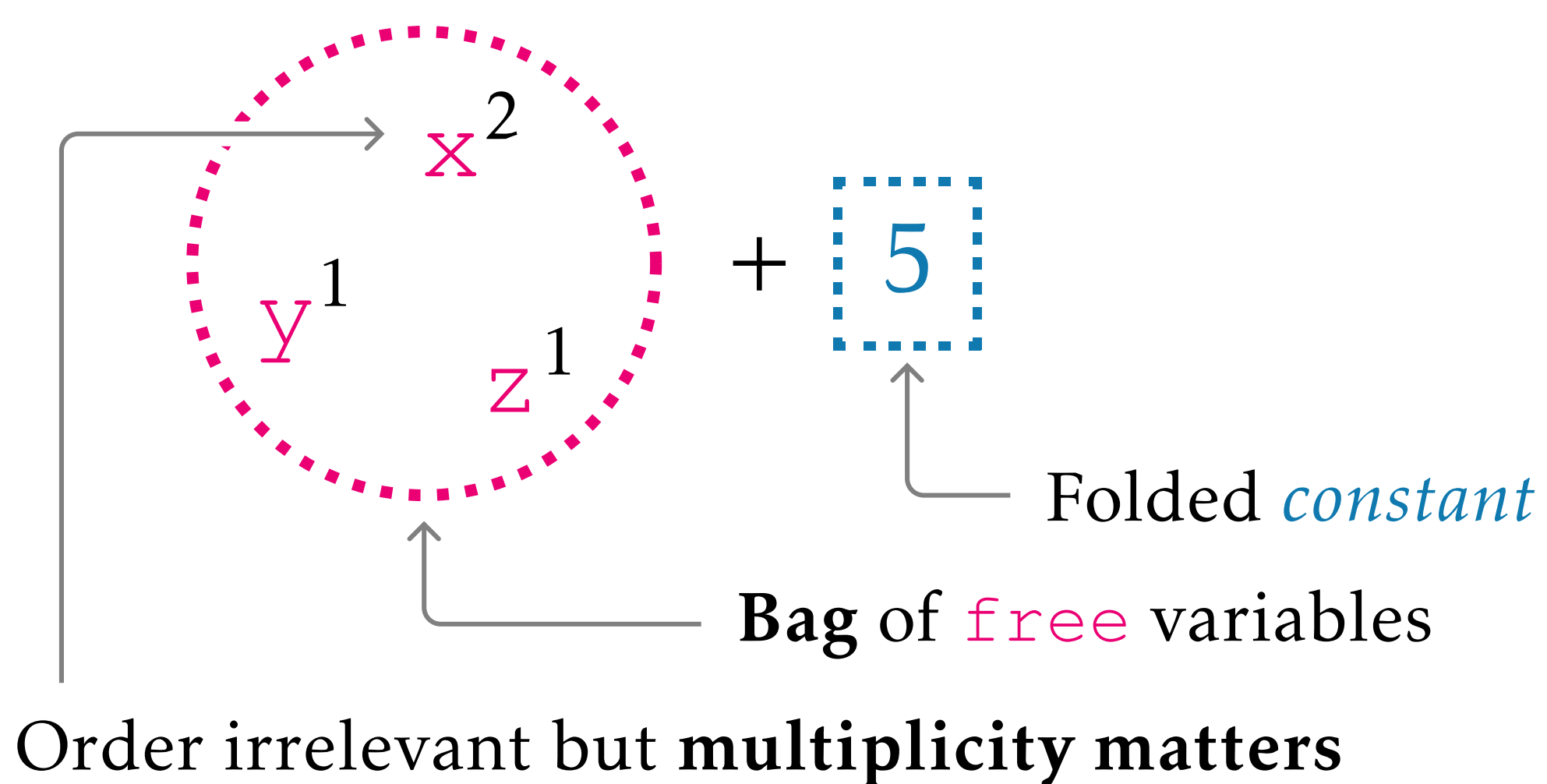


- **Syntax:** well-typed composites.



- **Normal forms:** bags of free variables and a constant.

- **Normal forms:** typed lists of composable morphisms.



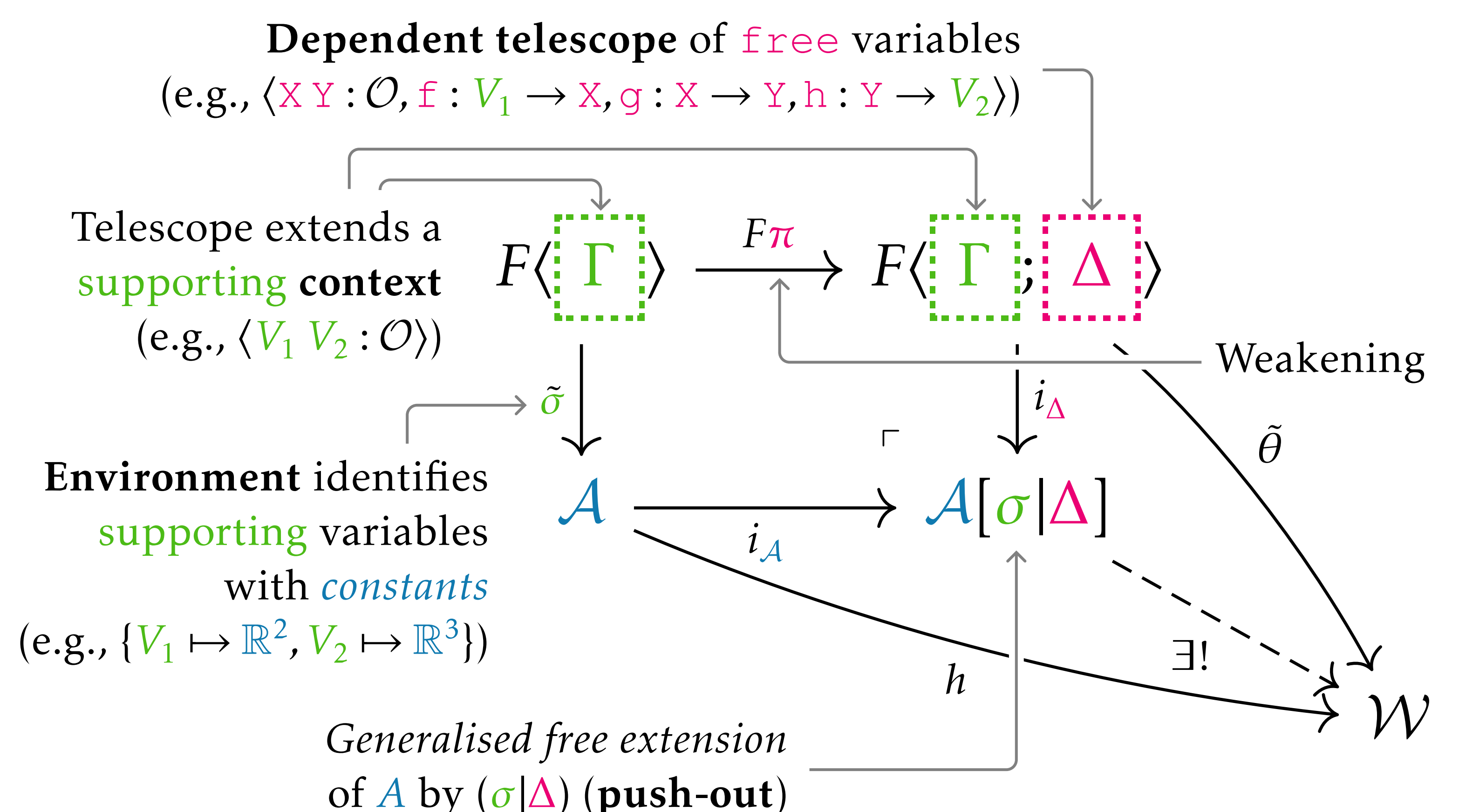
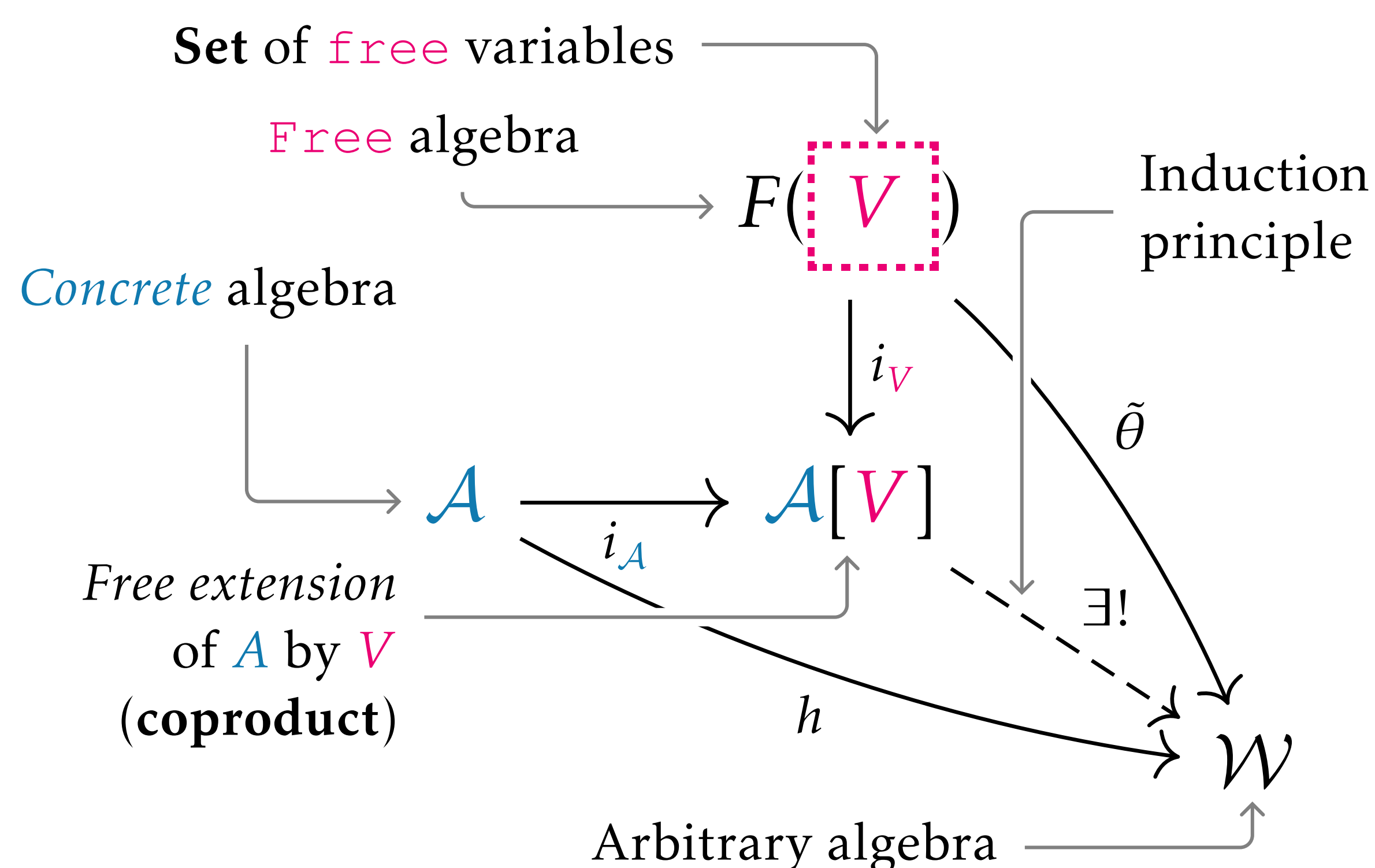
4. Abstraction

(i) Free extensions of algebras

(ii) Generalised free extensions

- Simple categorical description in terms of a universal property.

- Include types by generalising the universal property.



References

- [1] Guillaume Allais, Edwin Brady, Nathan Corbyn, Ohad Kammar, and Jeremy Yallop. Frex: dependently-typed algebraic simplification. Draft, 2022.
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