Generational Random Graphs – a “natural” model for heterogeneous temporal networks?

Jon crowcroft 9/3/2017

Graphs aren't static or homoegenous

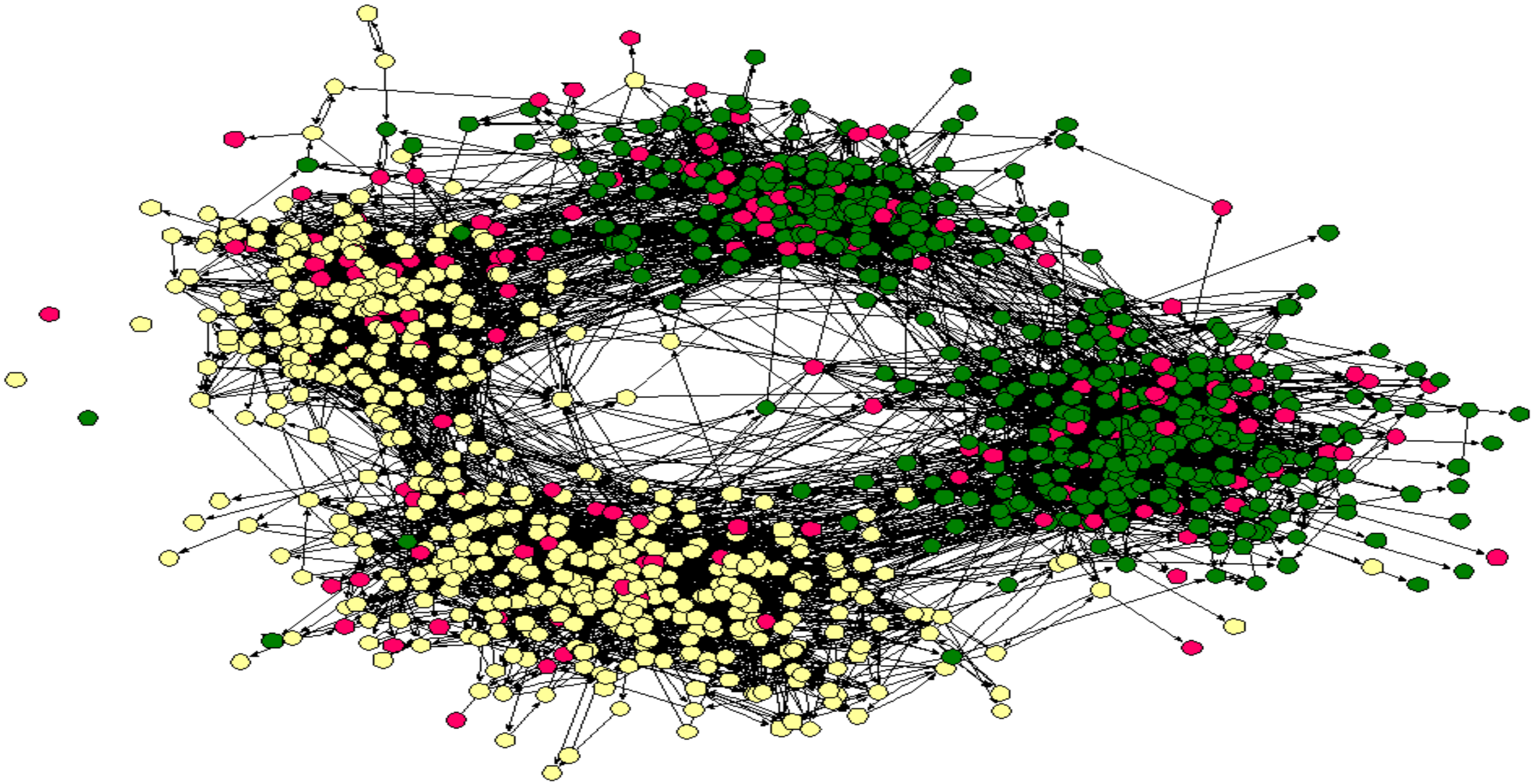
- Re-do two simple small world & clustered models
  - Preferential attachment & re-wiring (alpha & beta) models
- Add one simple idea, but in two guises:
  - Nodes are taken (in batches) from a sequence of generations
  - There's a birth (death) process of new (old) generations
  - To note: discrete generations, but continuous time…
- Two use cases
  - Social media/graphs – parents, siblings, children
  - Tech nets (internet, transport) - dialup, broadband, fiber, 3G/4,/ISP/IXP or horse, car, plane, drone…

    http://www.ee.ucl.ac.uk/~mrio/papers/hamedjrnl_camera.pdf

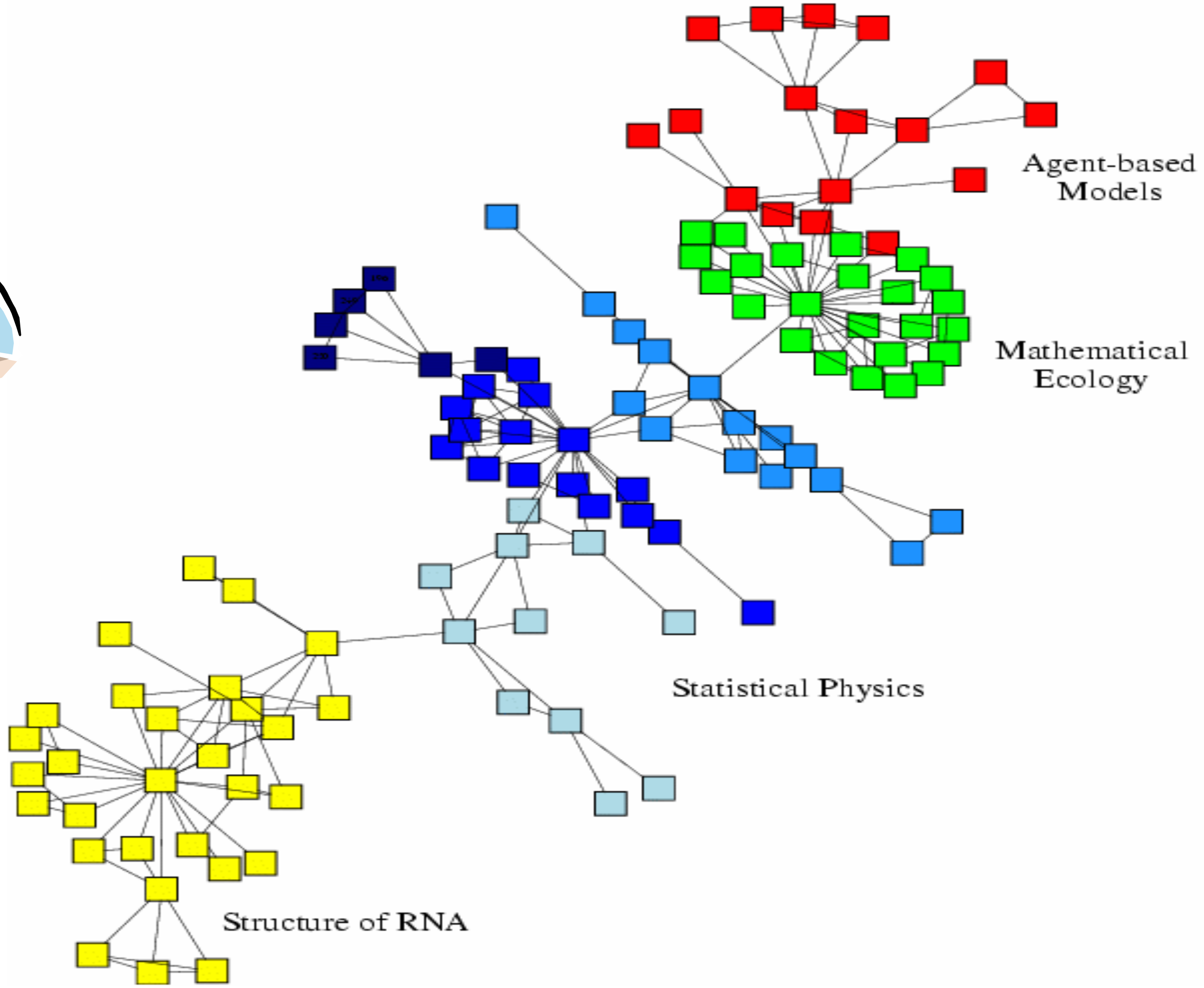
What makes a problem graph-like?

- There are two components to a graph
  - Nodes and edges
- In graph-like problems, these components have natural correspondences to problem elements
  - Entities are nodes and interactions between entities are edges
- Most complex systems are graph-like

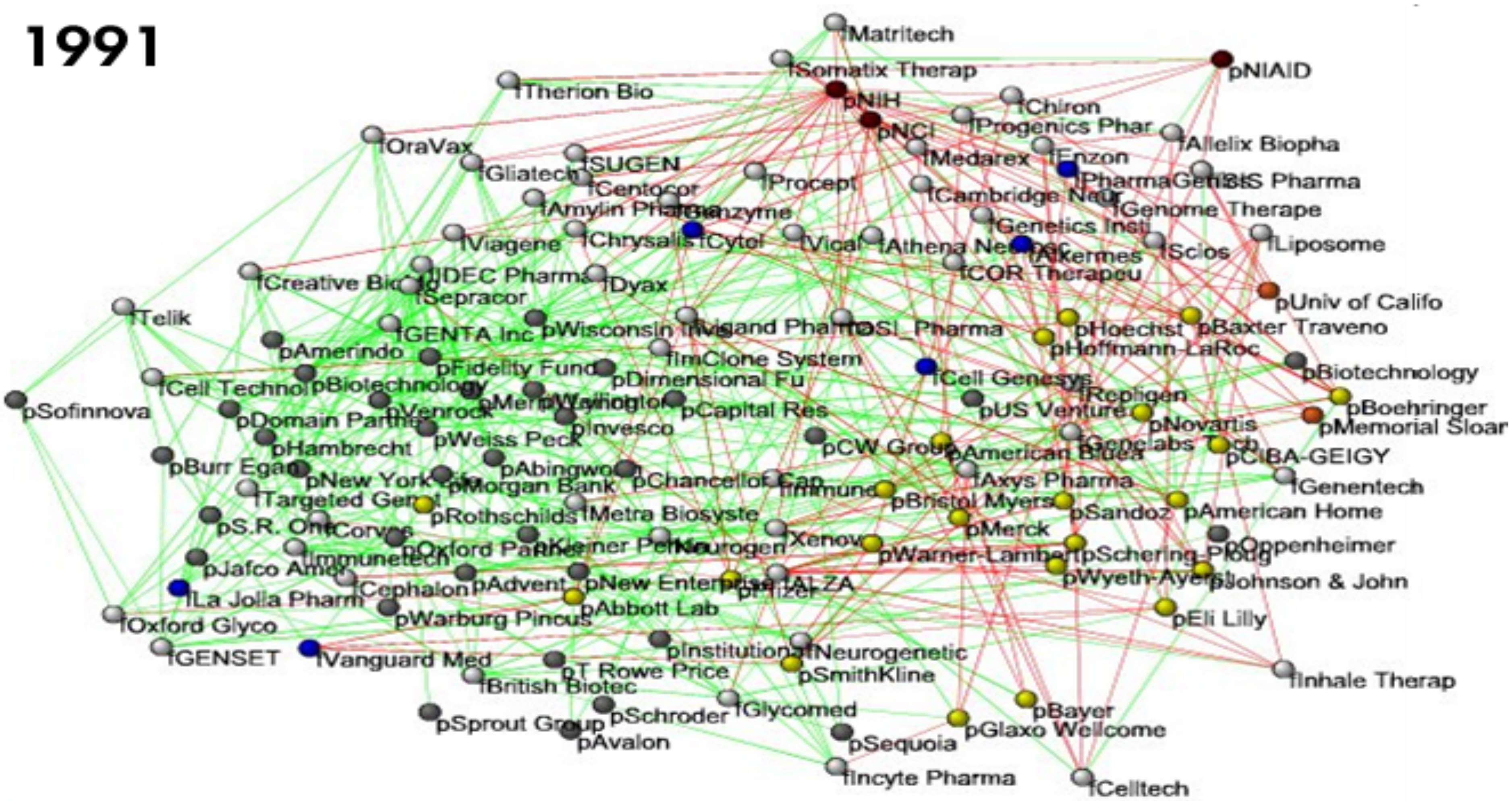
Friendship Network



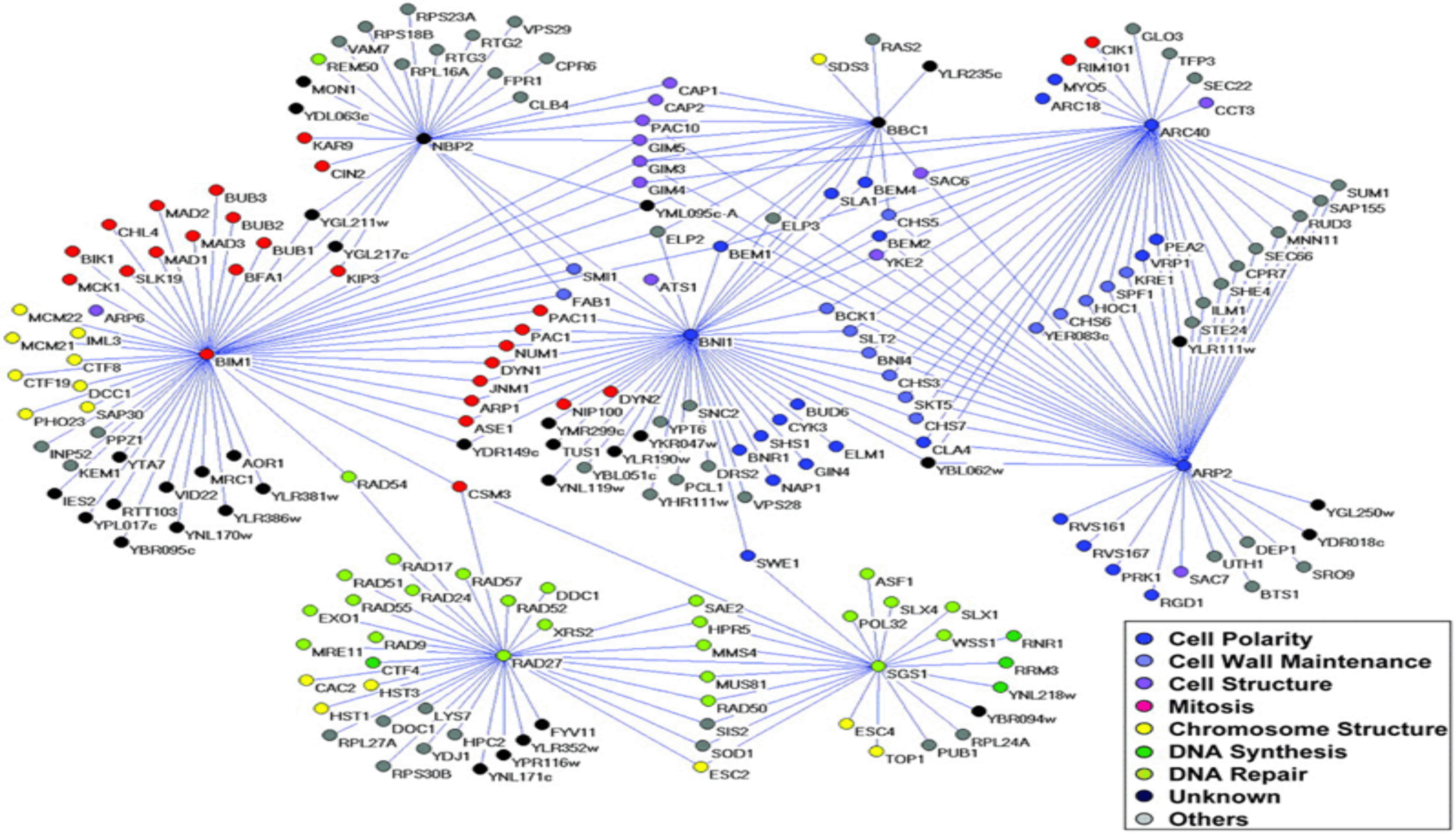
Scientific collaboration network



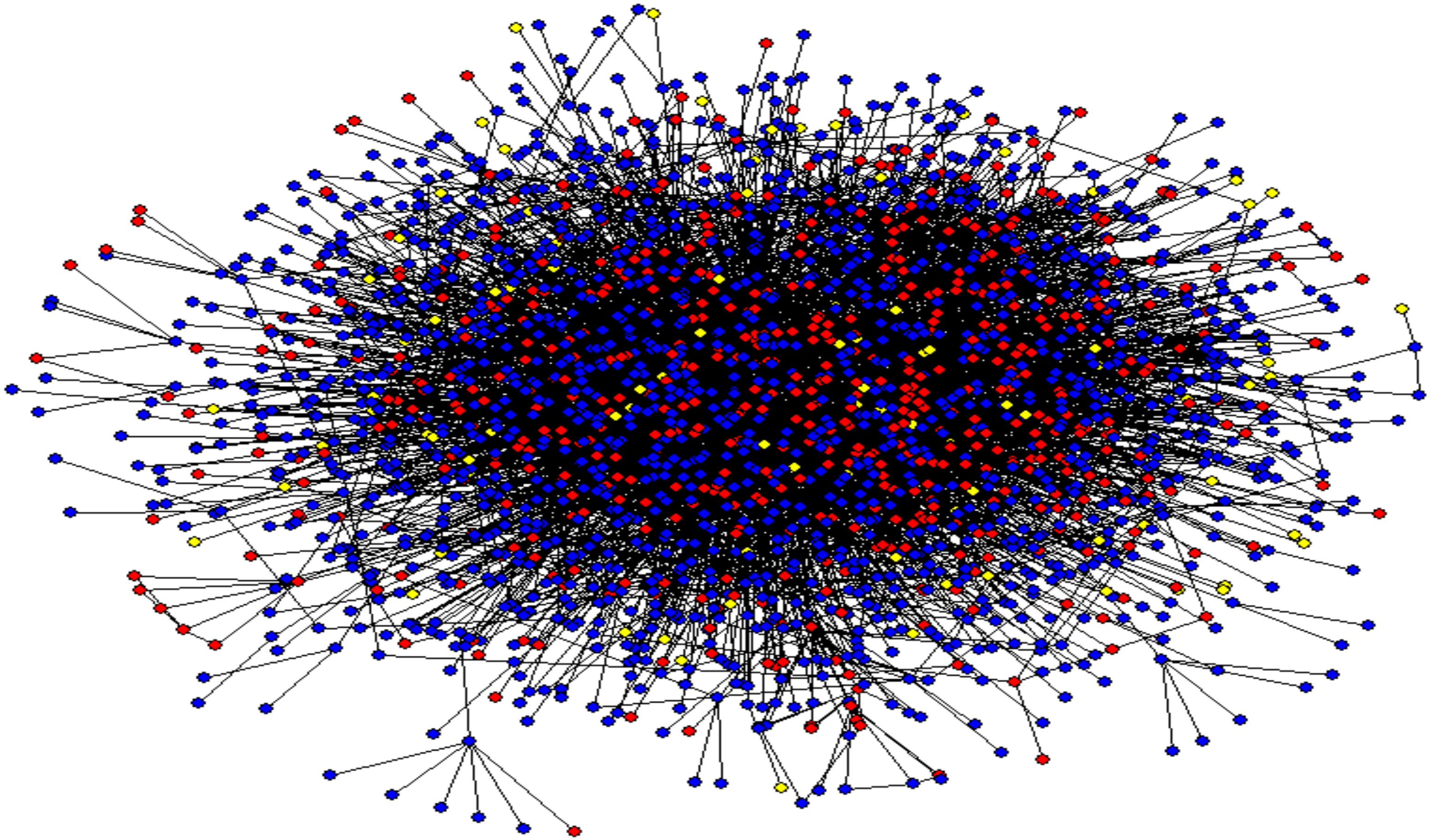
Business ties in US biotech-industry



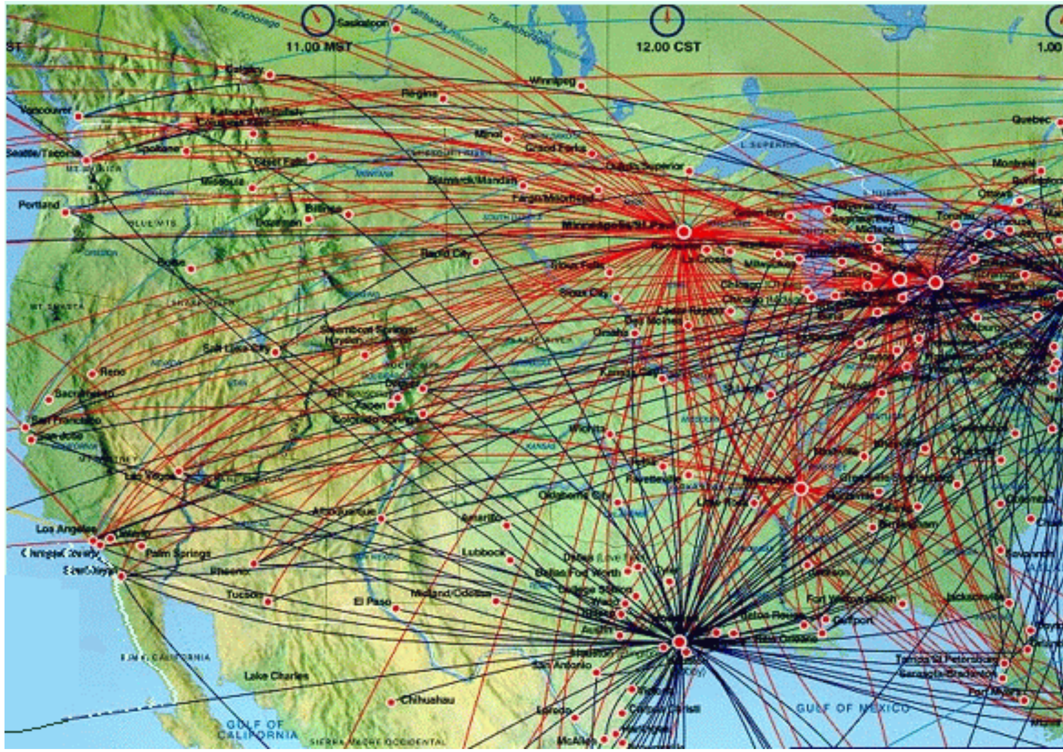
Genetic interaction network



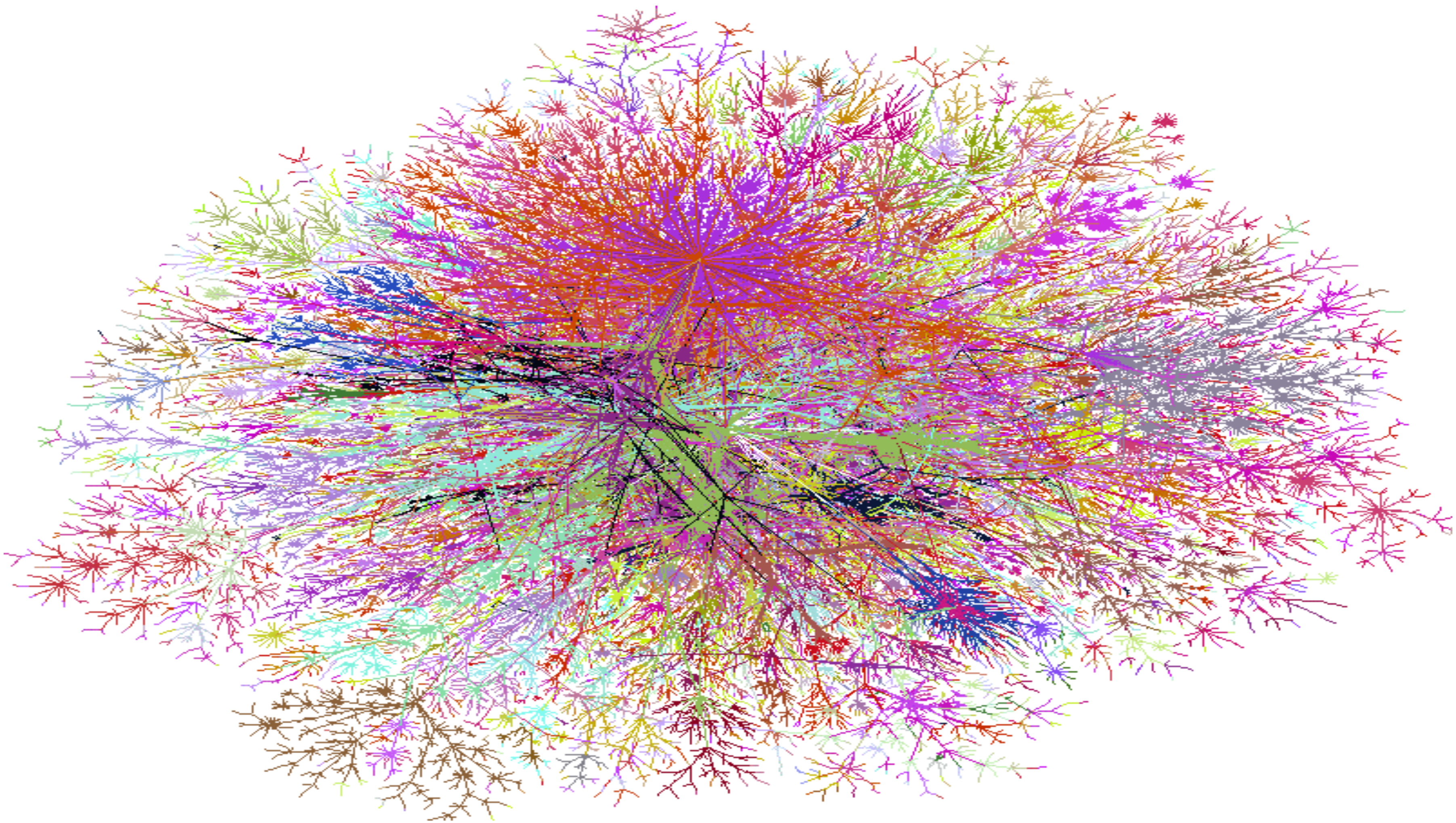
Protein-Protein Interaction Networks



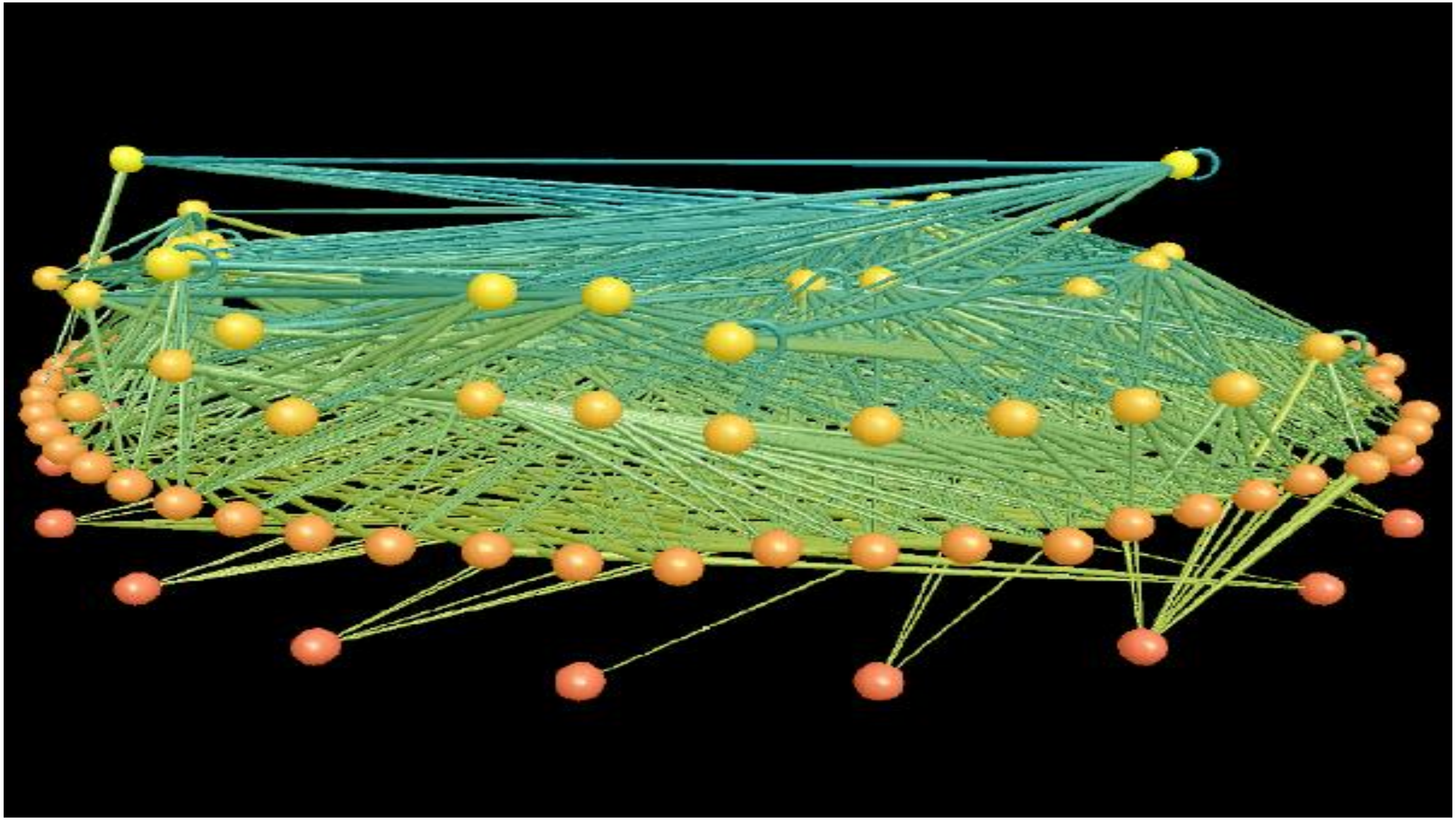
Transportation Networks



Internet



Ecological Networks

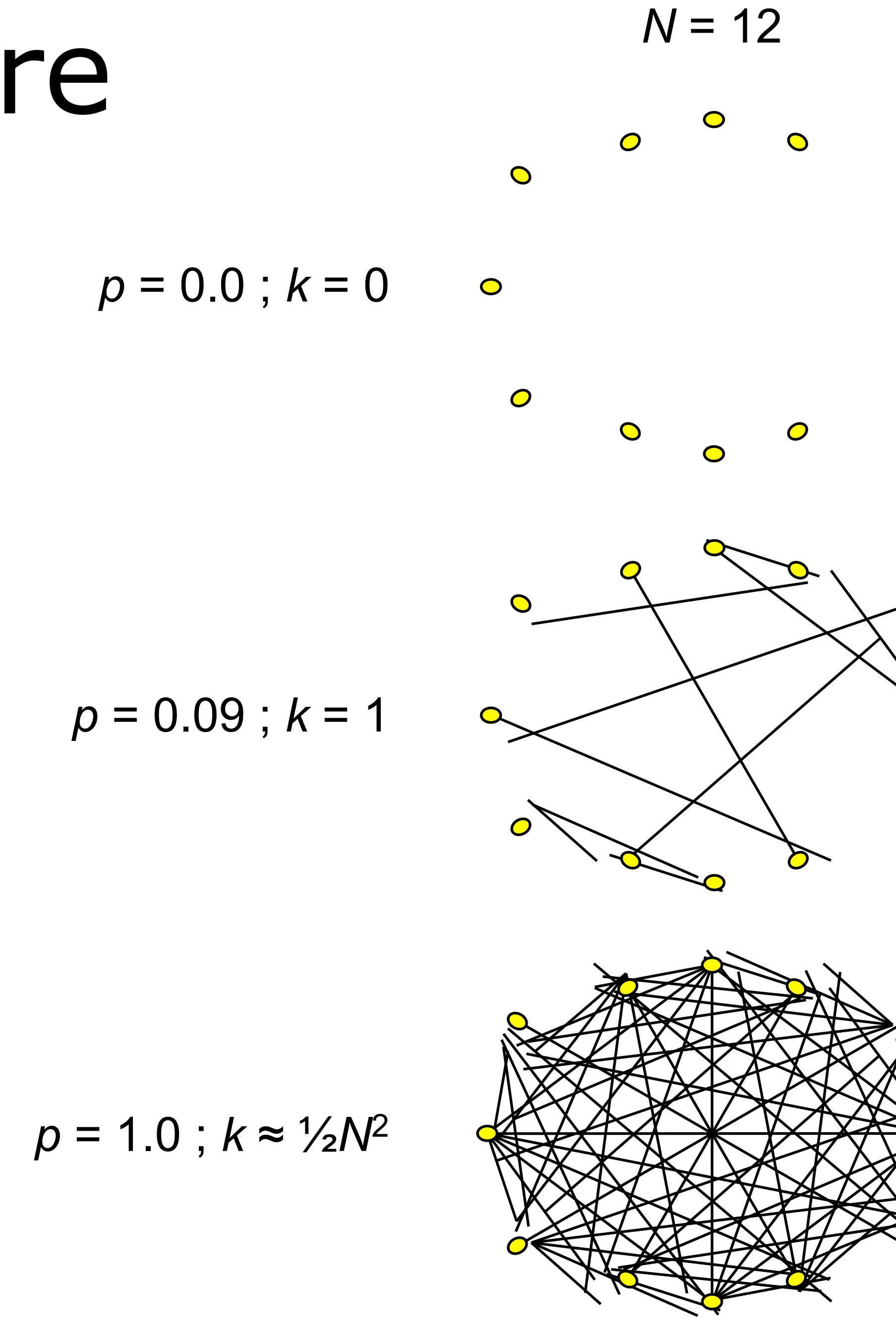


Random Graphs & Nature

Erdős and Renyi (1959)

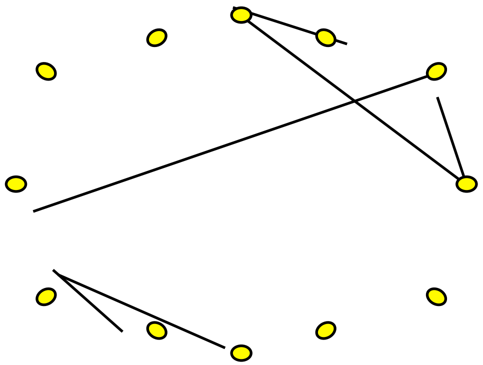
- N nodes
- A pair of nodes has probability *p* of being connected.
- Average degree, $k \approx pN$
- *What interesting things can be said for different values of p or k ?*

  (that are true as $N \rightarrow \infty$)



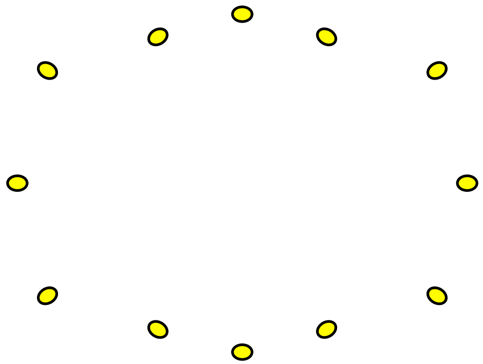
Random Graphs

Erdős and Renyi (1959)

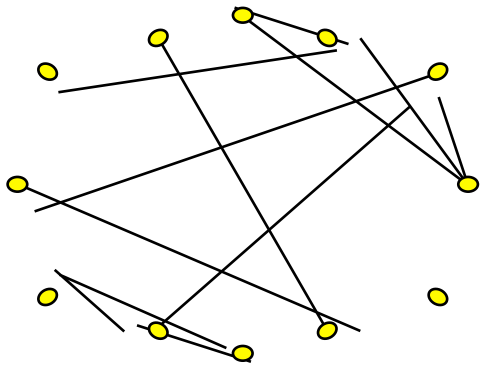


p = 0.045 ; k = 0.5

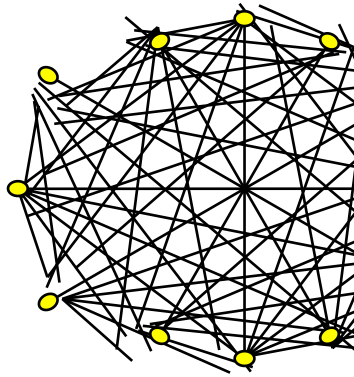
p = 0.0 ; k = 0



p = 0.09 ; k = 1



p = 1.0 ; $k \approx \frac{1}{2}N^2$



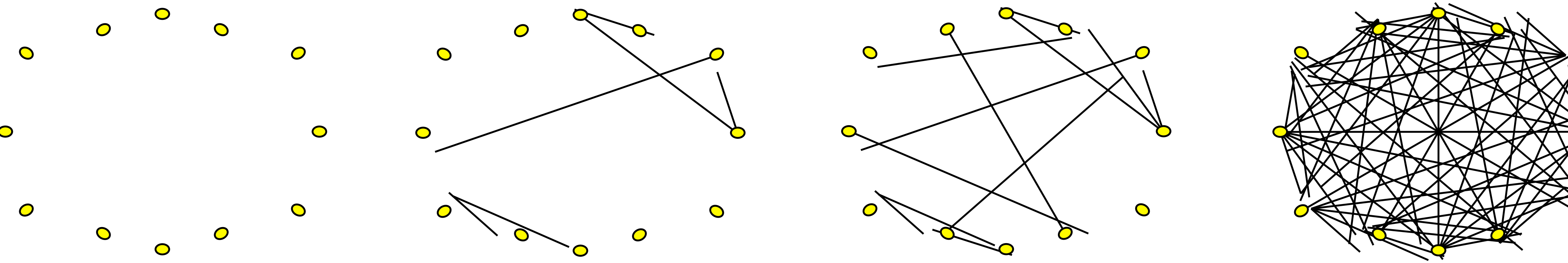
e of the largest connected cluster

ameter (maximum path length between nodes) of the largest cluster

erage path length between nodes (if a path exists)

Random Graphs

Erdős and Renyi (1959)



p = 0.0 ; k = 0	p = 0.045 ; k = 0.5	p = 0.09 ; k = 1	p = 1.0 ; k ≈ ½

of largest component

1	5	11	12

eter of largest component

0	4	7	1

age path length between nodes

0.0	2.0	4.2	1.0

Random Graphs

Erdős and Renyi (1959)

If k < 1:

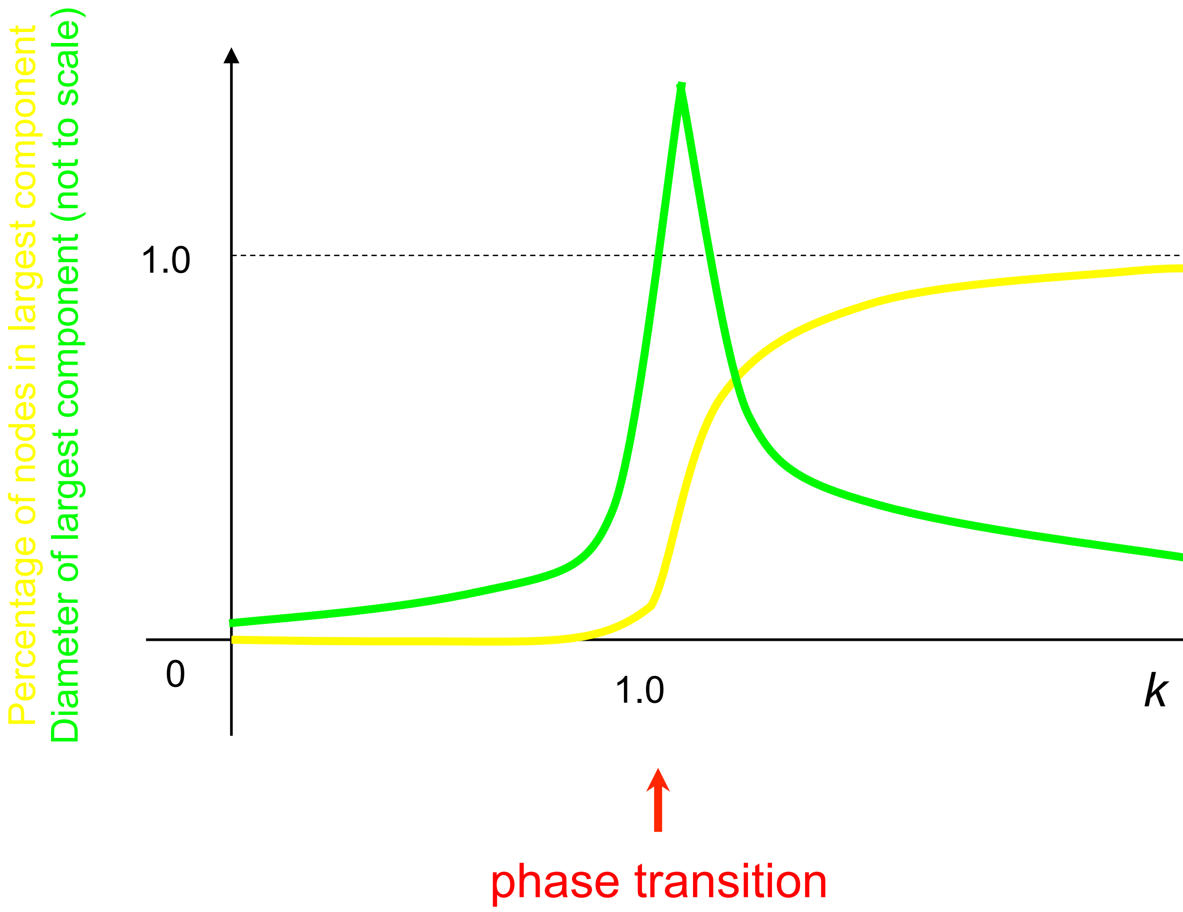
- small, isolated clusters
- small diameters
- short path lengths

At k = 1:

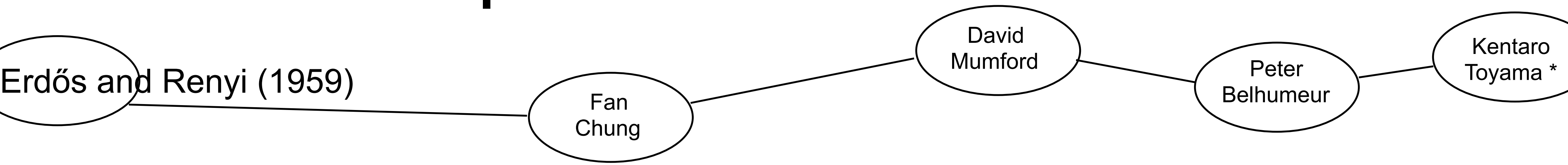
- a *giant component* appears
- diameter peaks
- path lengths are high

For k > 1:

- almost all nodes connected
- diameter shrinks
- path lengths shorten



Random Graphs



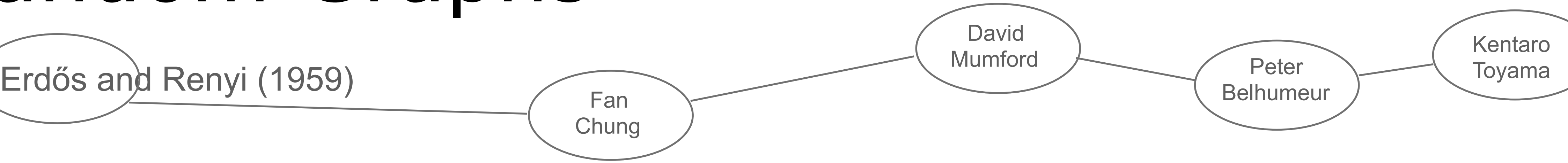
What does this mean?

- If connections between people can be modeled as a random graph, then…
  - Because the average person easily knows more than one person (k >> 1),
  - We live in a "small world" where within a few links, we are connected to anyone in the world.
  - Erdős and Renyi showed that average path length between connected nodes is

$$\frac{\ln N}{\ln k}$$

* An example researcher with Erdos #=4

andom Graphs



What does this mean?

BIG “IF”!!!

- If connections between people can be modeled as a random graph, then…
  - Because the average person easily knows more than one person ($k >> 1$),
  - We live in a “small world” where within a few links, we are connected to anyone in the world.
  - Erdős and Renyi computed average
    path length between connected nodes to be:

$$\frac{\ln N}{\ln k}$$

The Alpha Model

Watts (1999)

The people you know aren't randomly chosen.

People tend to get to know those who are two links away (Rapoport *, 1957).

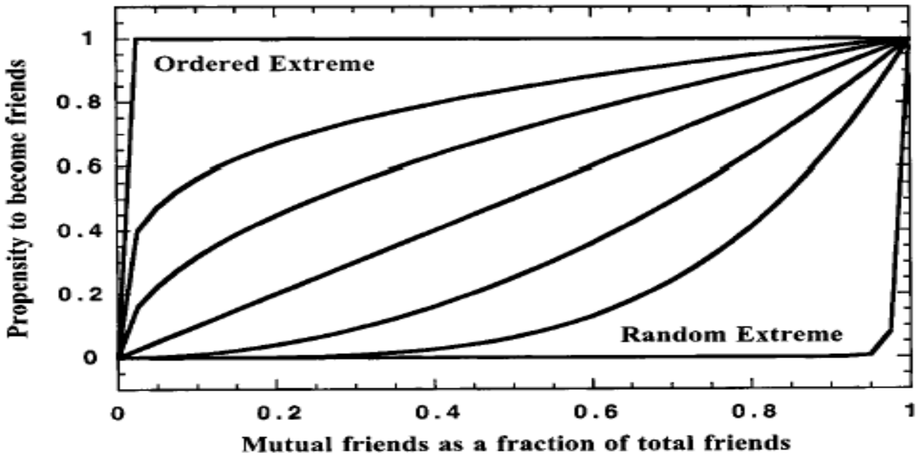
The real world exhibits a lot of *clustering.*

The Personal Map
by MSR Redmond's Social Computing Group

* *Same Anatol Rapoport, known for TIT FO*

The Alpha Model

Watts (1999)



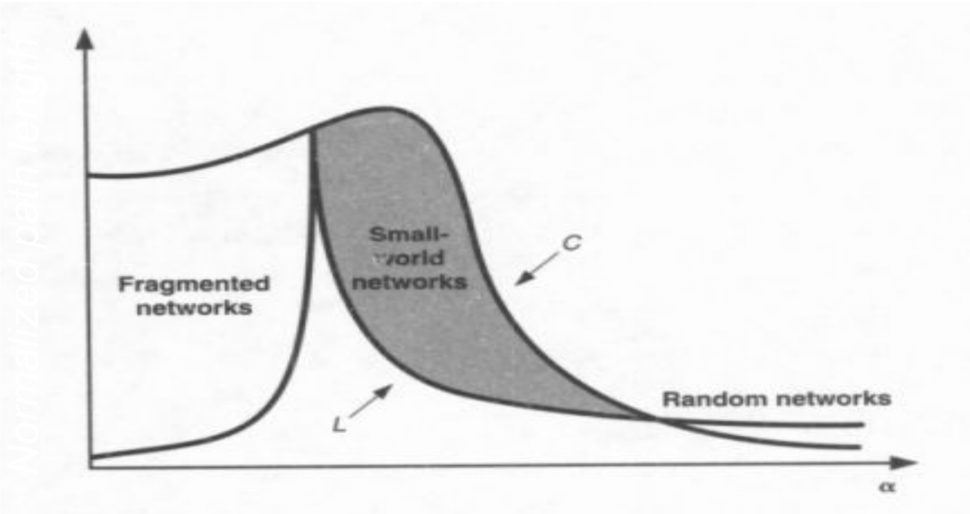
Probability of linkage as a function
of number of mutual friends
(α is 0 in upper left,
1 in diagonal,
and ∞ in bottom right curves.)

α model: Add edges to nodes, as in random graphs, but makes links more likely when two nodes have a common friend.

The Alpha Model

Watts (1999)

Clustering coefficient /



Clustering coefficient (*C*) and
average path length (*L*)
plotted against α

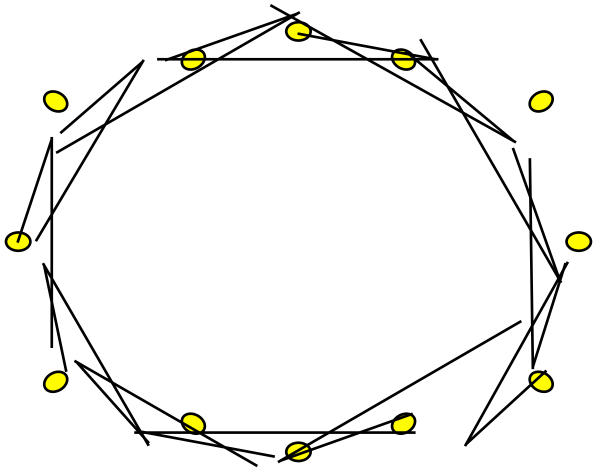
α model: Add edges to nodes, as in random graphs, but makes links more likely when two nodes have a common friend.

For a range of α values:

- The world is small (average path length is short), and
- Groups tend to form (high clustering coefficient).

The Beta Model

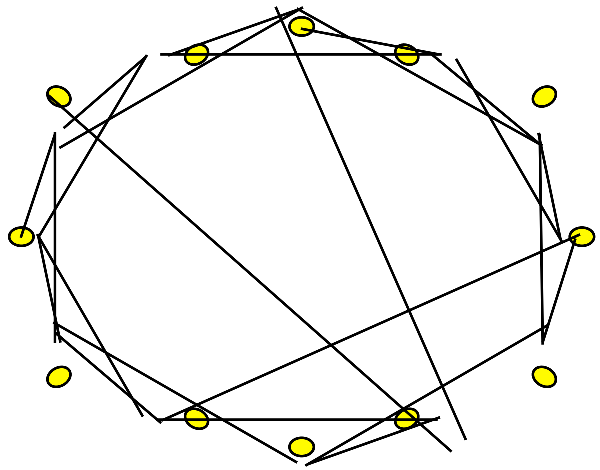
atts and Strogatz (1998), circular lattice,
ewiring to random other link w/ probability β



$\beta = 0$

People know
their neighbors.

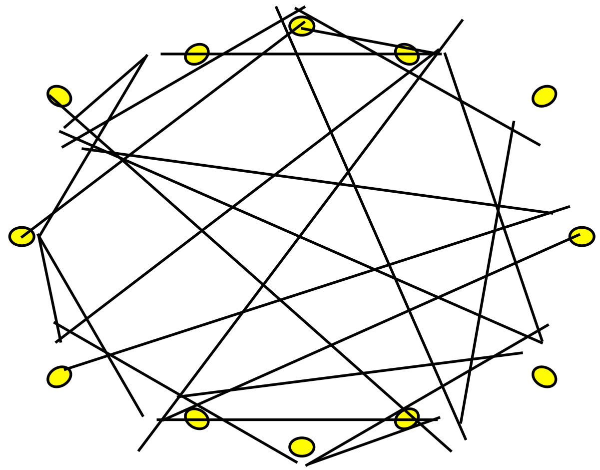
Clustered, but
not a “small world”



$\beta = 0.125$

People know
their neighbors,
and a few distant people.

Clustered and
“small world”



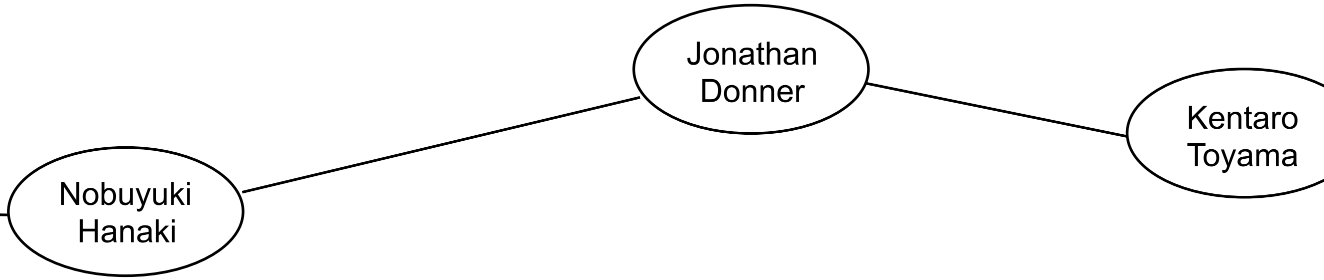
$\beta = 1$

People know
others at
random.

Not clustered,
but “small world”

ne Beta Model

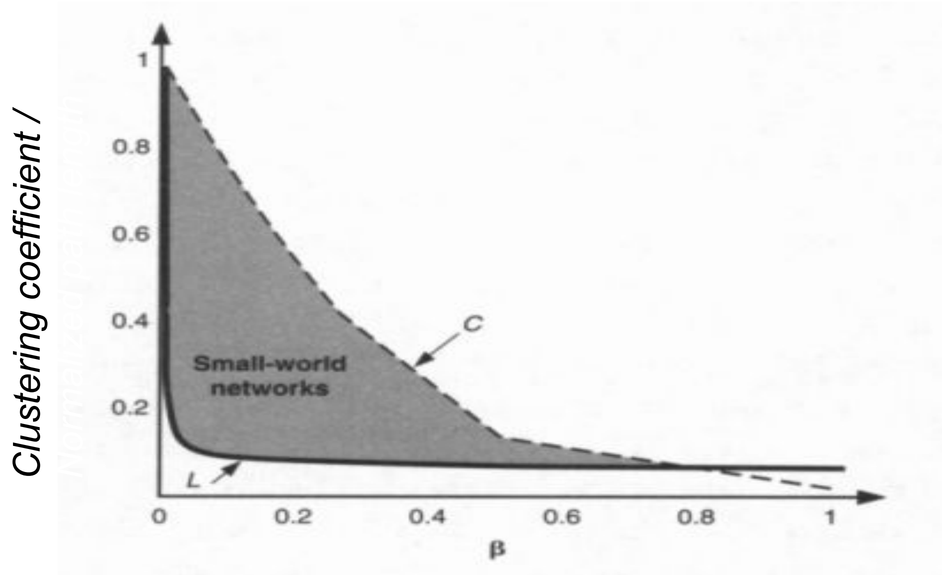
Watts and Strogatz (1998)



rst five random links reduce the average path length of the network by half, regardless of *N*!

oth α and β models reproduce short-path results of random graphs, but also allow for clustering.

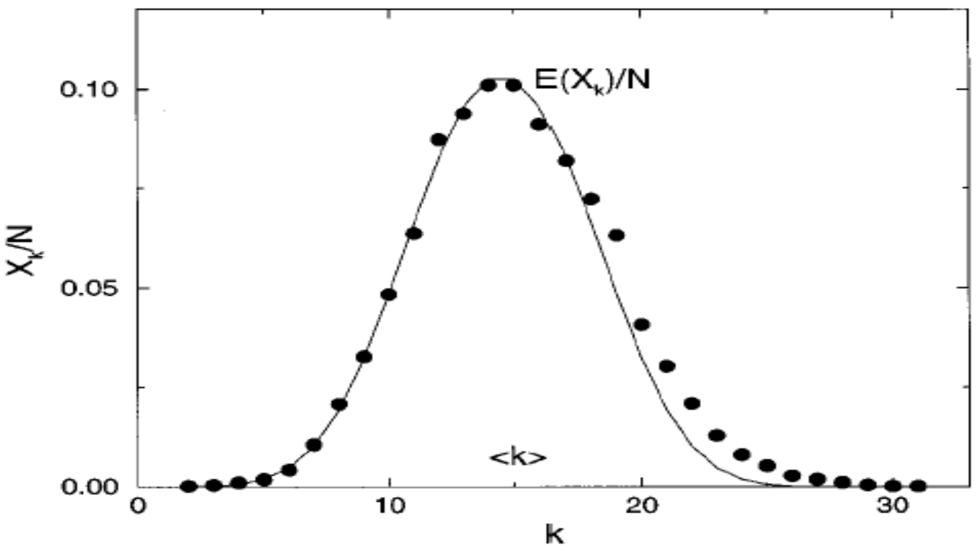
mall-world phenomena occur at threshold between order and chaos.



Clustering coefficient (*C*) and average path length (*L*) plotted against β

Power Laws

Albert and Barabasi (1999)



Degree distribution of a random graph,
N = 10,000 *p* = 0.0015 *k* = 15.
(Curve is a Poisson curve, for comparison.)

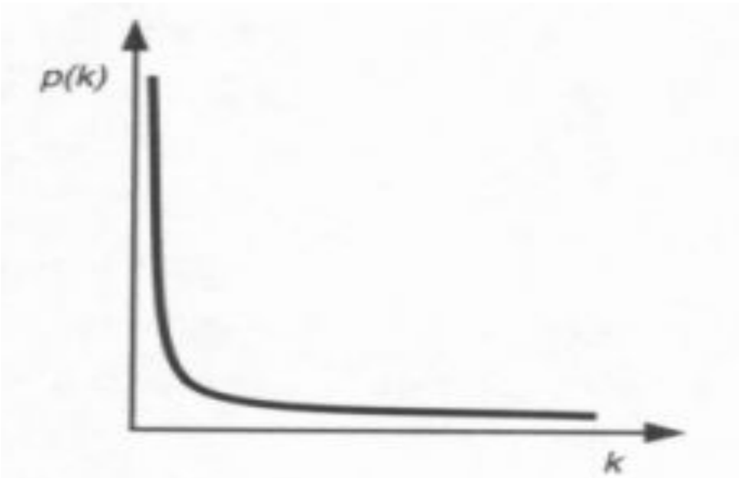
What's the degree (number of edges) distributi over a graph, for real-world graphs?

Random-graph model results in Poisson distribu

But, many real-world networks exhibit a *power-* distribution.

Power Laws

Albert and Barabasi (1999)



Typical shape of a power-law distribution.

What's the degree (number of edges) distributi
over a graph, for real-world graphs?

Random-graph model results in Poisson distribu

But, many real-world networks exhibit a *power-*
distribution.

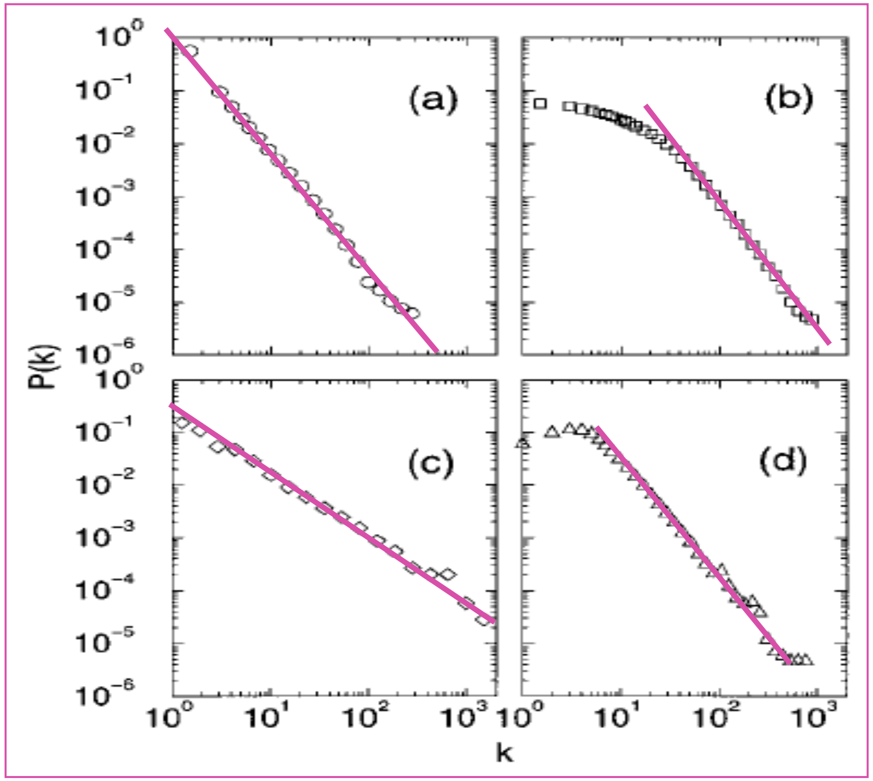
Power Laws

- Albert and Barabasi (1999)

- Power-law distributions are straight lines in log-log space.

- *How should random graphs be generated to create a power-law distribution of node degrees?*

- Hint:
  - Pareto's* Law: Wealth distribution follows a power law.



Power laws in real networks:
(a) WWW hyperlinks
(b) co-starring in movies
(c) co-authorship of physicists
(d) co-authorship of neuroscientists

** Same Velfredo Pareto, who defined Pareto optimality in game t*

Hippogriffically

- Spatial parameter(s) –
  - #generations – e.g. 1,3, infinity
  - Alpha' (Beta') now – ratio of preferential attachment (rewire) probabilty within and between generations –
  - e.g. between siblings, children, parents e.g. (.25, .5, .25) for 3 generations,
  - could be 1/n for n generations or could have a 1/d_i,j for distance between generations or whatever, or pick your distr…
- Temporal parameter(s) markovish…
  - #New Nodes/generation epoch
  - Removal process (perhaps)

For genes, this is a natural fit

- Generations accummulate more mutations
- There's a lot of modularity….

So lots of data out there (fb, internet topo over time)

- Fit model params
- Properties now indexed by generation (for example)
  - E.g. cliques for sibling v. family, centrality for grandparents, etc
- What other nets does this describe, intuitively?
- Is it still too complicated/complex?
- Does it make some things easier (or harder)?
- Do we need generational properties to keep global properties
  - Global mean diameter, cluster science, centrality=mean of mean each generation, etc
  - Or can they deviate in weird ways?