Logic and Proof - Supervision 3

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Exercise 37. Apply Fourier-Motzkin variable elimination to the set of constraints

$$x \le 2y \quad x \le y+3 \quad z \le x \quad 0 \le z \quad y \le 4x.$$

Exercise 39. Compute the BDD for each of the following formulas, taking the variables as alphabetically ordered:

(a) $\neg(p \vee q) \vee p$

(b) $\neg(p \wedge q) \leftrightarrow (p \vee q)$

Exercise 41. Verify the following equivalence using BDDs:

$$(p \vee q) \to r \simeq (p \to r) \wedge (q \to r).$$

Exercise M. Prove the following sequents:

(a) $\Diamond(p \to q), \Box p \Rightarrow \Diamond q$

(b) $\Box p \wedge \Box q \Rightarrow \Box(p \wedge q)$

(c) $\Box\Diamond\Box p, \Box\Diamond\Box q \Rightarrow \Box\Diamond\Box(p \wedge q)$

Inspired by Exam 2012 - Paper 6 Question 6.

(a) Demonstrate the sequent calculus, the free-variable tableau calculus and resolution by using each of them to prove the following formula:

$$(P(a,b) \vee \exists z P(z,z)) \to \exists x \exists y P(x,y).$$

Comment briefly on the similarities and differences among these three methods.

(b) Prove $\Box\Diamond P \to \Diamond\Box P$ using the sequent calculus for S4 modal logic, or exhibit a falsifying interpretation.

(c) Briefly outline the procedure for converting a formula to a BDD, illustrating your answer by constructing the BDD that represents the conjunction of those found in Exercise 39. (a) and (b).

If you feel keen, you can also do questions (a) and (c) on Exam 2014 Paper 6 Question 5.