Logic and Proof - Supervision 2

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Exercise 26. For each of the following pairs of terms, give a most general unifier or explain why none exists. Do not rename variables prior to performing the unification.

(a) $f(g(x), z)$ and $f(y, h(y))$

(b) $j(x, y, z)$ and $j(f(y, y), f(z, z), f(a, a))$

(c) $j(x, z, x)$ and $j(y, f(y), z)$

(d) $j(f(x), y, a)$ and $j(y, z, z)$

(e) $j(g(x), a, y)$ and $j(z, x, f(z, z))$

2005 Paper 5 Question 9

(a) In order to prove the following formula by resolution, what set of clauses should be submitted to the prover? Justify your answer briefly.

$$\forall x\, [P(x) \lor Q \to \neg R(x)] \land \forall x\, [(Q \to \neg S(x)) \to (P(x) \land R(x))] \to \forall x S(x)$$

(b) Derive the empty clause using resolution with the following set of clauses, or give convincing reasons why it cannot be derived.

$$\{\neg P(x, x)\} \quad \{P(x, f(x))\} \quad \{\neg P(x, y), \neg P(y, z), P(x, z)\}$$

(c) Derive the empty clause using resolution with the following set of clauses, or give convincing reasons why it cannot be derived. (Note that a and b are constants.)

$$\{\neg P(a)\} \quad \{Q(a)\} \quad \{R(b)\} \quad \{s(b)\}$$

$$\{\neg Q(x), P(x), \neg R(y), \neg Q(y)\} \quad \{\neg S(x), \neg R(x), Q(x)\}$$

1999 Paper 6 Question 10

(a) Describe the role of Herbrand models in mechanical theorem proving. What may we infer when a set of clauses has no Herbrand model?

The remainder of this question concerns using clause methods to determine whether or not the formula

$$\exists x\,[P(x) \wedge Q(x)] \to \exists x\,[P(f(x,x)) \vee \forall y Q(y)]$$

is a theorem.

(b) Convert the problem into clause form. Justify each step you take and explain in what respect the set of clauses is equivalent to the original problem.

(c) Describe the Herbrand universe for your clauses.

(d) Produce a resolution proof from your clauses, or give reasons why none exists.

(e) Exhibit a Herbrand model for your clauses or give reasons why none exists.