THE MARGARET STANIER MEMORIAL SUNDIAL
AN UNEQUAL-HOURS DIAL FOR NEWNHAM COLLEGE
Part 1. Some Design Considerations

FRANK H. KING

Unequal-hours sundials are rarely commissioned so there are very few modern dials whose sole gnomonic function is to indicate such hours. In this first part, of a two-part article, it is shown that although one cannot indicate unequal hours precisely using a gnomon (as distinct from a nodus) one can indicate unequal hours to an acceptable approximation. In Part 2, the development of the design into a memorial to Margaret Stanier will be described.

Genesis

Margaret Stanier, who is shown in Fig. 1 outside her home in Swaffham Prior, was the immediate past Editor of the *BSS Bulletin.* For many years she was a Fellow of Newnham College, Cambridge, and, following her death in 2007, the College invited me to design a sundial as a memorial.

Fig. 1 — Margaret Stanier

Margaret had a particular interest in mass dials and I proposed a design that was loosely based on the only historic mass dial in Cambridge. This is on a south buttress of Little S. Mary's Church and is shown in Fig. 2. The dial is featured in *Cambridge Sundials*[1], the book which Margaret co-authored with Alexis Brooks.

As usual, the gnomon is missing but it is assumed that this would have been perpendicular to the plane of the dial.

Fig. 2 — Mass Dial at Little S. Mary's Church, Cambridge

Analysing the angles of the hour-lines on any particular mass dial tends to be unrewarding but the overall appearance of the Little S. Mary's dial resembles that of the traditional design of unequal-hours dial illustrated schematically in Fig. 6. This thinking led to the following outline specification for the proposed sundial:

- There would be a straight-rod gnomon
- The hour-lines would radiate from close to the centre of the dial
- The outer ends of the hour-lines would be bounded by a circle
- The hour-lines would indicate unequal hours

The intention is that the angle of the shadow of the gnomon should indicate unequal hours and this is asking for the impossible. The specification can be satisfied only at the equator.

There is no such problem if a nodus is used and one could, for example, use the tip of the gnomon as a surrogate nodus. This idea was rejected, in part because this is not how mass dials are generally understood to have functioned.

There is a more pragmatic reason for using a gnomon in preference to a nodus. When using a nodus there will always be times when the sun is shining on the dial but the shadow of the nodus is off the dial. In an early e-mail[2], following the installation of the sundial at Selwyn College, the Master of the College noted that he had seen the shadow “about a foot to the left” of the dial itself. See Fig. 3.

Fig. 3 — The Shadow of a Nodus out of the Field of Play

This is not a photograph that would fare well in a sundial photographic competition! The sun is shining brightly on the dial and is clearly casting shadows but the crucial shadow is out of the field of play. With a gnomon, provided only that the root of the gnomon is on the dial, this cannot happen.

Designing a gnomon-driven unequal-hours sundial is indeed impossible but the finished design is a close approximation to this unattainable goal. The reader may judge just how acceptable an approximation the result is.

The Celestial Sphere

The most direct way of visualising the daily passage of the sun across the sky is to draw sketches such as those in Figs 4 and 5. In both cases, the view may be imagined to be that captured by a camera with an extra-wide-angle lens. There is blue sky above verdant pasture with a clean horizon between.

The views extend from due east via due south to due west with a small black triangle on the horizon marking due south. The three circular arcs represent the path traced by the sun at the summer solstice (top arc), at the equinoxes (middle arc) and at the winter solstice (bottom arc). These arcs are appropriate for the latitude of Cambridge.

The rising sun is shown at the eastern ends of the equinoctial arc and the winter solstice arc. The setting sun is shown at the western ends. The sky represents part of the inside surface of the celestial sphere. No great claims are made for the projection used but it is quite helpful for sketching in different kinds of hour-lines.

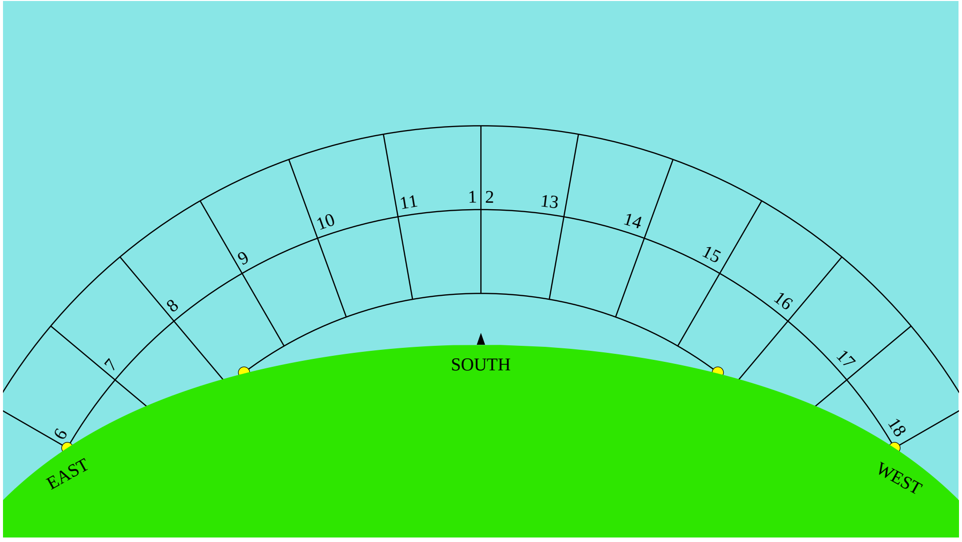

Fig. 4 — Equal Hours on the Celestial Sphere

In Fig. 4, hour-lines for common hours are shown from 6h to 18h and it is clear that, at the equinoxes, sunrise is at 06:00 and sunset is at 18:00. Likewise, at the winter solstice, sunrise is about 08:15 and sunset is about 15:45.

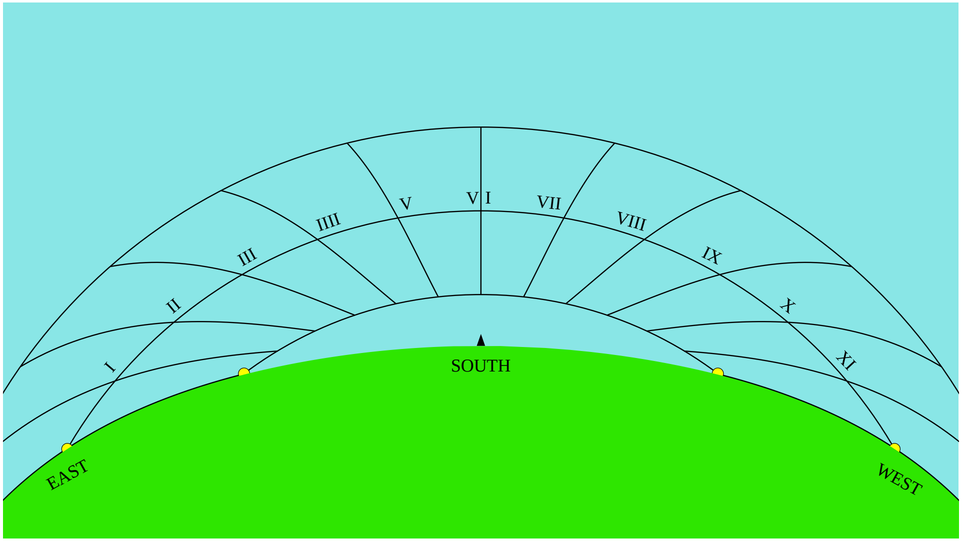

Fig. 5 — Unequal Hours on the Celestial Sphere

In Fig. 5, hour-lines for unequal hours are shown. These hour-lines divide the paths followed by the sun from sunrise to sunset into twelve equal parts. The one-twelfth divisions at the summer solstice are a little over twice as long as those at the winter solstice.

In Fig. 4, the equinoctial arc, the horizon and all the common hour-lines are great circles on the celestial sphere. With a gnomonic projection, great circles project into straight lines. Accordingly, the equinoctial line, the horizon line and the common hour-lines are all straight lines on an ordinary plane dial

In Fig. 5, the equinoctial arc and the horizon are great circles but, of the hour-lines, only those for sunrise, midday and sunset are great circles. The sunrise and sunset hour-lines coincide with the horizon and the midday hour-line coincides with the common hour-line for noon.

The other unequal hour-lines are not great circles or small circles. They are curious curves which, over the short lengths that run from the winter solstice arc to the summer solstice arc, are approximations to great circles.

In Fig. 4, the common hour-lines radiate from a point beneath the figure. This point is the south-celestial pole which is just over 52° below the horizon. This angle corresponds to the latitude of Cambridge. On a conventional sundial, this point projects into the root of the polar-oriented gnomon.

In Fig. 5, the unequal hour-lines, if extended, would converge on the triangular marker on the south horizon. This marker projects into the root of a horizontal gnomon.

The Traditional Design of Unequal-Hours Dial

Fig. 6 shows the traditional design of unequal-hours dial implemented on a circular disc which is assumed to be vertical and direct south-facing. The 13 hour-lines run from sunrise (the horizontal line on the left) to sunset (the horizontal line on the right).

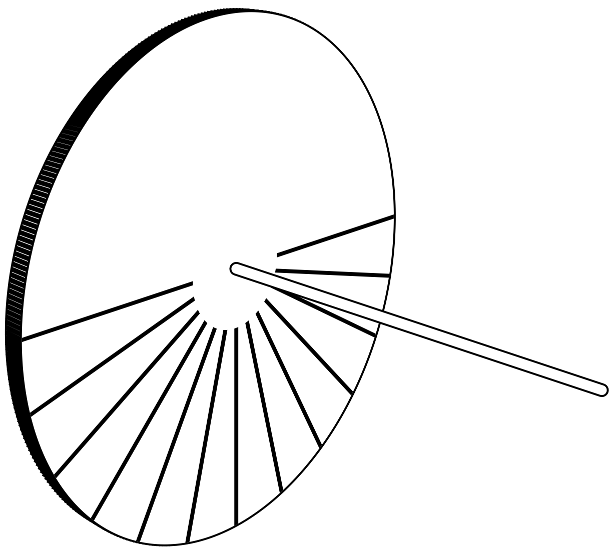

Fig. 6 — Traditional Design of Unequal-Hours Dial

The gnomon is horizontal and aligned north–south. More pedantically, the gnomon lies at the intersection of a horizontal plane and the meridian plane. Accordingly, when the sun is on the horizon there is a horizontal shadow so the sunrise and sunset hour-lines are sound. When the sun crosses the meridian there is a vertical shadow so the midday hour-line is sound too.

The other hour-lines are naïvely placed at 15° intervals and unfortunately equal angular intervals don't translate into equal intervals of time. This design is therefore largely flawed but it is this design that I wanted to tweak so as to make it as gnomonically respectable as possible.

A Minimalist Unequal-Hours Dial

Fig. 7 show the same dial with all the unsound hour-lines discarded but with an academic midnight hour-line added. Additionally, the gnomon is now shown extended through to the north side of the dial. This minimalist sundial divides the daylight period into two equal parts, morning and afternoon, and, in a theoretical way, divides the night into two equal parts too.

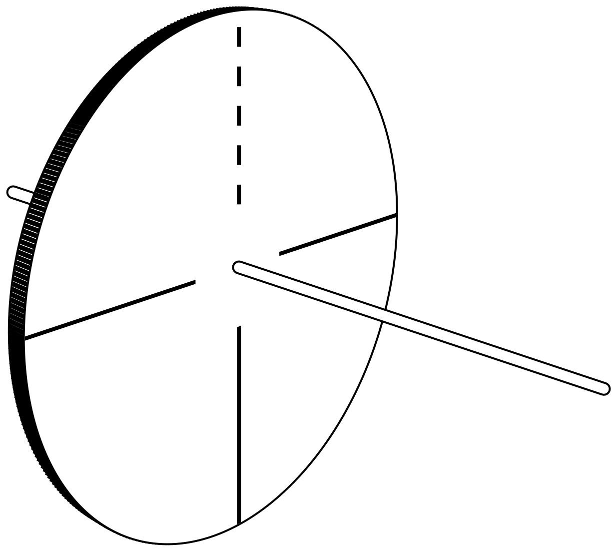

Fig. 7 — Minimalist Unequal-Hours Dial

This is close to being a universal design because it works at almost any latitude, at any time of year, and the dial can have almost any orientation. The only hard constraints are that the gnomon must be horizontal and aligned north–south.

There is one other detail that can be illustrated by considering what happens at sunrise. In winter, the sun rises to the south of due east so the shadow clearly falls on the sunrise hour-line. In summer, the sun rises to the north of due east and so shines on the north side of the dial. It is to attend to this difficulty that the gnomon is extended through to the north side.

The shadow then still falls on the sunrise hour-line which is now imagined to run through the thickness of the dial. One may further imagine that the dial is made of some translucent material so that when the shadow falls on the north side it can be seen from the south side.

If one imagines the Earth itself to be translucent, then the midnight hour-line makes sense. In British latitudes the sun is always due north at midnight and below the horizon. The shadow of the north end of the gnomon will fall on the midnight hour-line.

In operation, the performance of this dial can readily be illustrated by imagining a time in high summer when, in the latitude of Cambridge, there are 16 common hours of day and 8 common hours of night.

At sunrise, the shadow falls on the sunrise hour-line. It then sweeps round the lower half of the dial at a very leisurely pace. It takes 16 hours to reach the sunset hour-line.

Then, continuing to imagine a translucent Earth, the shadow continues sweeping round the upper half of the dial but at a much faster pace, taking only 8 hours to get back to the sunrise hour-line.

The average speed of the sweeping shadow during the night is twice its average speed during the day. In the depths of winter it can be the other way round with the average speed during the night being half the average speed during the day.

This changing of speed takes place continuously throughout the year quite automatically. No moving parts are required other than the Earth itself. To the uninitiated the simple geometry gives rise to an almost magical result.

Unfortunately, the speed of the sweeping shadow does not change instantaneously at sunrise and sunset. The speed changes throughout the day and night. Fig. 8 plots the angular velocity of the sweeping shadow (in degrees per hour) in the depths of winter. The plot starts at sunrise and, after sunset, the night time region is shown shaded.

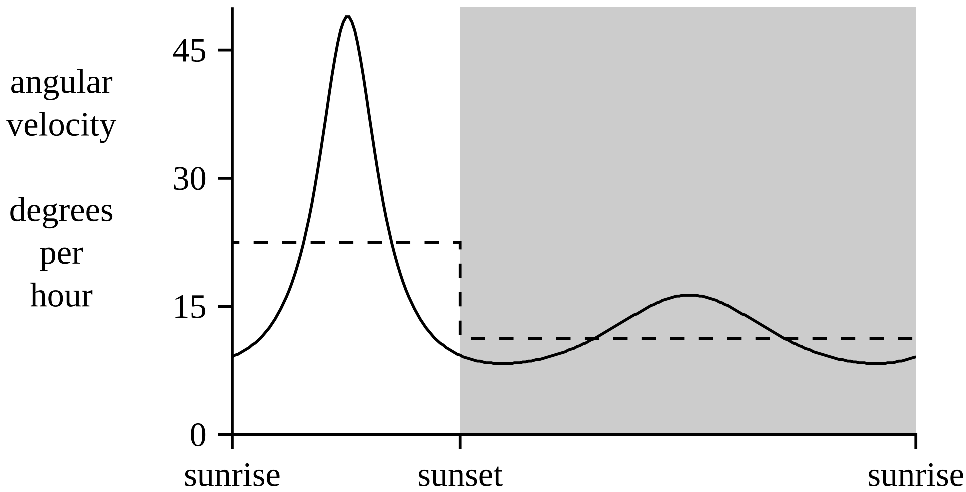

Fig. 8 — Angular Velocity versus Time of Day

If the sunrise and sunset labels are interchanged and the shaded region is shrunk and moved appropriately, the plot shows how the angular velocity changes in high summer.

The broken line in the plot shows the ideal profile. The angular velocity would be a constant 45/2 degrees per hour during the day (eight common hours) and a constant 45/4 degrees per hour during the night (sixteen common hours).

There is evidence[3] that some early verge and foliot clocks indicated unequal hours and followed this ideal profile. The clock-keeper had to move the weights on the foliot at sunrise and sunset each day.

It is the variability in the speed of the sweeping shadow that rules out having hour-lines at 15° intervals. Placing the intermediate hour-lines requires further analysis…

Hour-lines for Mid-Morning and Mid-Afternoon

In the traditional design, the hour-lines for mid-morning and mid-afternoon are in the 45° positions, shown in Fig. 9 as the new heavy black lines.

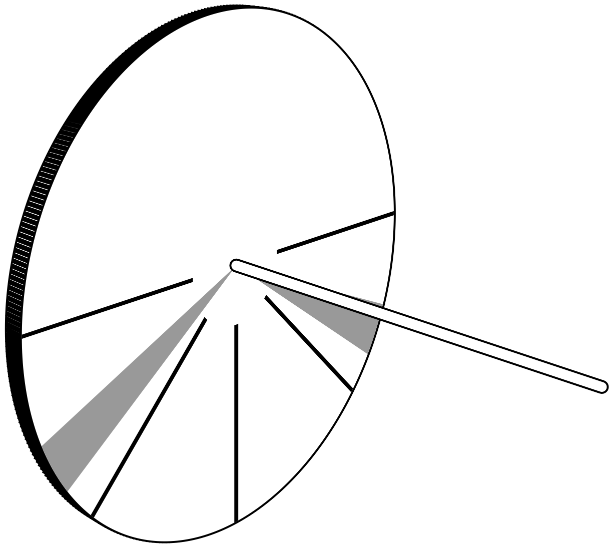

Fig. 9 — Mid-Morning and Mid-Afternoon Errors

The actual position of the shadow in mid-morning and mid-afternoon depends on the solar declination and the latitude and the orientation of the dial.

Assuming a vertical direct south-facing dial in the latitude of Cambridge, the range of directions in which the shadow can fall in mid-morning and mid-afternoon are shown in Fig. 9 as two grey regions each with an angular spread of about 12.9°.

Interestingly, there is no time of year when the shadow in mid-morning or mid-afternoon falls on, or anywhere near, the 45° hour-lines.

The hour-lines can, of course, be shifted into the grey regions and placed, perhaps, at the mean angles of the shadows at the two times. They will be close to, but not coincident with, the centre-lines of the grey regions. Using the mean minimises the root-mean-square angular error.

Moving the hour-lines into the grey regions improves the accuracy but does nothing to improve the precision. That requires reducing the angular spread of the grey regions.

Improving the Precision

Short of moving nearer to the equator, there is no way of reducing the angular spread of the grey regions without incurring a cost: relaxing one of the hard constraints on the orientation of the gnomon.

Nothing is achieved by moving the gnomon out of the meridian plane but it is fruitful to consider the effect of dipping the gnomon slightly downwards towards the south.

Up to some critical angle of dip, this has the beneficial effect of reducing the angular spread of the grey regions but, beyond the critical angle, the spread increases again. Unfortunately, by moving the gnomon out of the horizontal plane, the directions of the shadows at sunrise and sunset will depend on the time of year.

The critical angle of dip depends on the latitude. It is 0° at the equator and increases to around 9.2° at 45° N after which it falls off again. In the latitude of Cambridge the angle is about 8.9° and Fig. 10 shows the consequences of dipping the gnomon by this critical angle.

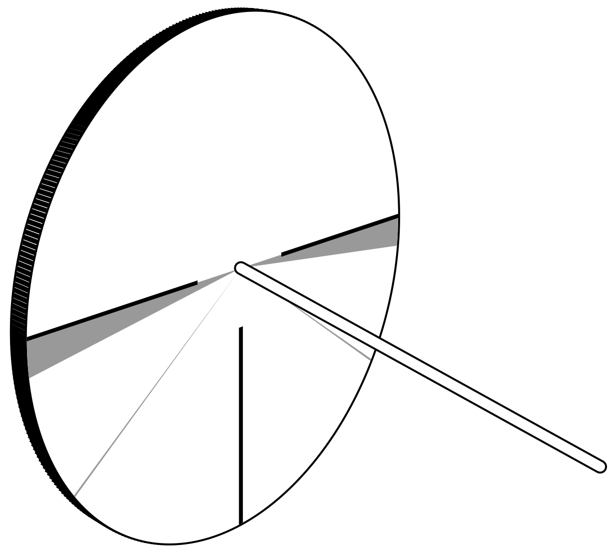

Fig. 10 — Critical Angle of Dip

The angular spread of the mid-morning and mid-afternoon grey regions has been reduced to about 0.5° but new grey regions have been introduced at sunrise and at sunset. In calculating these regions, solar declinations for which the sun shines on the north side of the dial at sunrise and sunset have been ignored.

These new regions have an angular spread of about 7° and deciding whether or not it is worth incurring this cost is outside the realm of mathematics.

To most people, sunrise and sunset are periods of time rather than instants and these new grey regions hint at this. On a more pragmatic note, in its intended site, the sundial does not get a clear view of the horizon in any direction so the gnomon will never cast a shadow at sunrise or sunset. Little is gained by worrying too much about these new grey regions.

The other Intermediate Hour-Lines

Figs 11 and 12 show the grey regions associated with each of the 13 unequal-hours hour-lines in two different circumstances. In Fig. 11 the gnomon is horizontal and in Fig. 12 the gnomon is dipped at the critical angle.

The midday hour-line has zero angular spread in both cases because the gnomon is in the meridian plane. The sunrise and sunset hour-lines have zero angular spread provided that the gnomon is horizontal. This is true in Fig. 11 but not in Fig. 12.

The differences between the two figures are stark. When the gnomon is horizontal, many grey regions are so wide that they are almost in contact with their neighbours. When the gnomon has the critical angle of dip, most of the grey regions are significantly narrower.

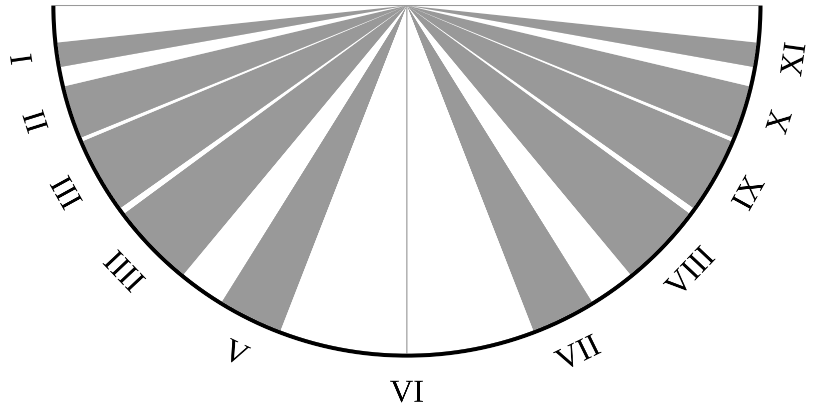

Fig. 11 — Errors with a Horizontal Gnomon

The grey regions for sunrise and sunset are unlabelled. The others are labelled I to XI. There is a case for labelling the 12 gaps instead. The gaps represent the hours whereas each hour-line represents the end of some hour. It was decided that labelling the gaps would have been a step too far for modern users!

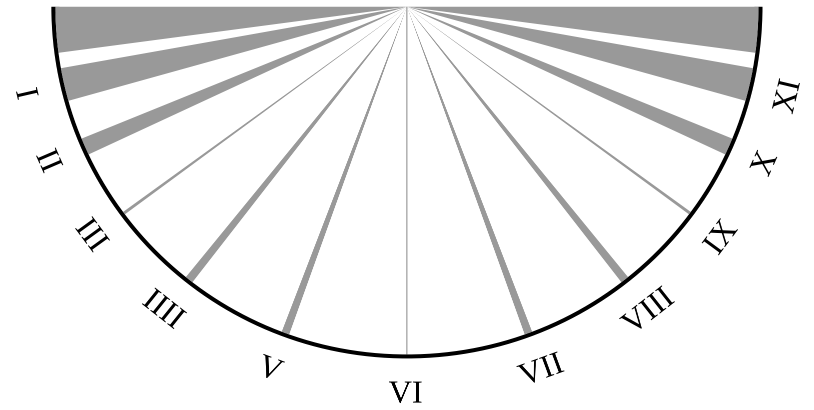

Fig. 12 — Errors with a Dipped Gnomon

The critical angle of dip minimises the angular spread of the grey regions associated with hour-lines III and IX. By inspection, it increases the angular spread at I and XI slightly, though much less than the increases at sunrise and sunset.

Some readers may consider backing off from the critical angle and deliberately degrading the improvement to hour-lines III and IX with a view to reducing the errors at I and IX. This thought was rejected on the grounds that hour-lines III and IX, as quarter-day markers, have particular importance.

On a historical note, hours III, VI and IX are nominally associated with the Christian offices of Terce, Sext and None which provides another reason for paying particular attention to the hour-lines which mark the ends of these hours.

The Celestial Sphere Revisited

The margins of the grey regions are all straight lines which radiate from a common centre. Viewed as a gnomonic projection, these straight lines are all mappings of great circles which radiate from a common point on the celestial sphere.

Fig. 13 is an augmented version of the celestial sphere as illustrated in Fig. 5. The black spot is the common point; it is 8.9° below the south horizon and maps onto the common centre in Fig. 12. The margins of the grey regions which envelop the (odd-numbered) hour-lines are great circles which radiate from the black spot. Subject to a caveat discussed in the next section, these margins map into the margins of the grey regions in Fig. 12.

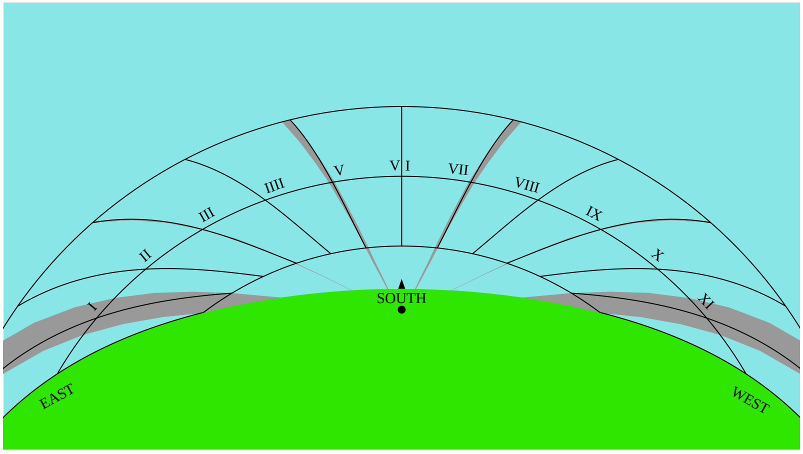

Fig. 13 — Error Regions Mapped to the Celestial Sphere

The hour-lines in Fig. 13 are shown as continuous curves but in any given (common) year the centre of the sun visits only 365 distinct points on each curve. There is a case for viewing each hour-line as 365 separate dots. The dots are much closer together near the ends of each hour-line than near the centre because the solar declination changes slowly at the solstices and rapidly at the equinoxes.

Using this view, each grey region is formed by drawing a great circle from the black spot through each of the 365 dots on the relevant hour-line on the celestial sphere. The margins of each grey region are simply the two extreme cases.

The 365 great circles give rise to a mean great circle whose direction through the black spot is the mean of the 365 directions. This mean great circle maps onto the dial as a straight line and it is this which is used as the hour-line. The hour-lines which overlay the grey regions in Fig. 14 are the mappings of just such mean great circles.

Most of the extreme great circles run from the black spot to the dot at the winter solstice end of the relevant hour-line or from the black spot to the dot at the summer solstice end.

It is now clear that the position of the black spot is where the great circle that best approximates hour-line III intersects the meridian. By symmetry, the great circle that best approximates hour-line IX intersects the meridian at the same point.

It is also clear from Fig. 13 that the grey regions for hour-lines I and XI could be narrowed by raising the black spot and the grey regions for hour-lines V and VII could be narrowed by lowering the black spot. The chosen position is a compromise.

Raising the black spot to the horizon, to the triangular marker, corresponds to having a horizontal gnomon. The grey regions for sunrise, midday and sunset narrow to zero but the grey regions for most other hour-lines broaden.

Fig. 13 purports to represent the celestial sphere and is independent of the orientation of the plane of the dial. Different orientations of the dial alter the appearance of the gnomonic projection but, subject to the caveat below, make no difference to the errors. They can be changed only by altering the angle of dip.

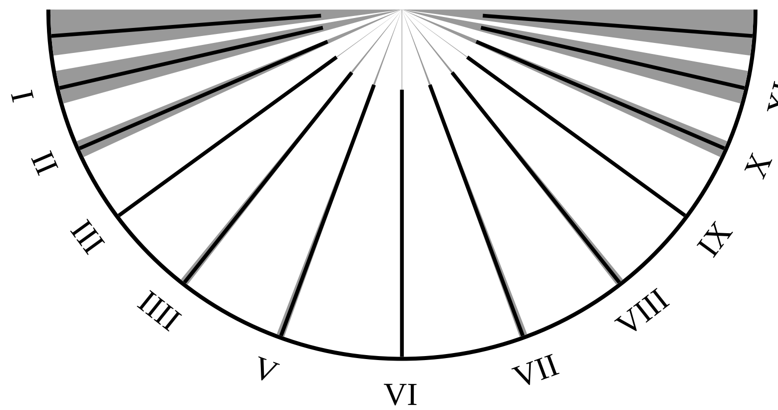

Fig. 14 — Hour-Lines and Associated Errors

Caveat

For certain hour-lines, some dots correspond to positions of the sun where it shines on the wrong side of the dial. It is unnecessary to draw great circles from the black spot through such otiose dots. Accordingly, a particular grey region may be formed from fewer than 365 great circles.

Although ignoring otiose dots reduces the angular spread of a grey region there can be circumstances where this may not be appropriate. The two dials of a dial-pair may have different orientations and, for a given hour-line, the otiose dots may be different for the different dials. This leads to different mean great circles being chosen for mapping into the hour-lines on the dials and the different dials may indicate slightly different times!

With a single dial, as in the present case, this problem does not arise. In Fig. 13 the lower margins of grey regions I and XI run to the summer solstice ends of hour-line I and XI. These ends are beyond due east and due west respectively and therefore behind a dial that is direct south-facing. By ignoring points beyond due east or due west, the lower margins of grey regions I and XI are raised a little and it is the revised margins that are used when mapping into Fig. 12.

For a horizontal dial, Fig. 13 applies without any such modification. For a dial in any other orientation, it is expedient to augment the figure by adding the great circle that corresponds to the plane of the dial. Any dots on the hour-lines outside this plane are ignored.

Outline Design

Fig. 14 shows 13 hour-lines, each with an associated grey region. The margins of the grey regions show the *maximum* errors associated with each hour-line. The *actual* errors on a given day will usually be considerably smaller. If the margins had reflected the root-mean-square errors, the grey regions would be thinner.

As a design for an unequal-hours dial, this has high accuracy and acceptable precision, with particularly high precision at III, VI and IX.

Fig. 15 is identical to Fig. 14 but with the grey regions removed. This, at last, is an outline design for an unequal-hours sundial on a direct south-facing wall at the latitude of Cambridge.

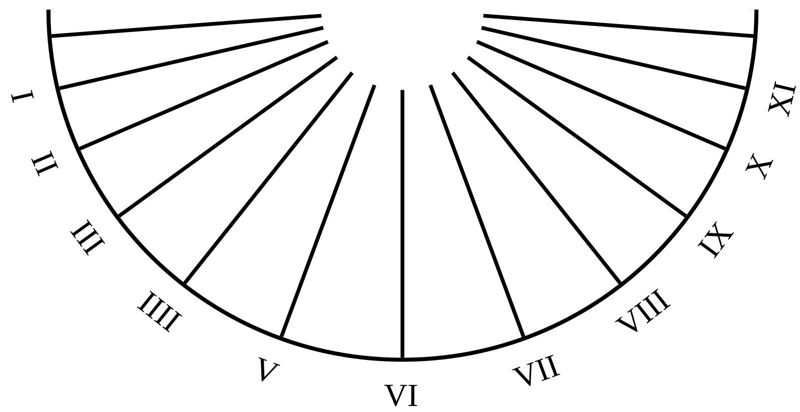

Fig. 15 — Embryonic Design

Fig. 15 is presented as a summary of the underlying mathematics. It is decidedly lacking aesthetic merit and considerable further work is required to turn this design into a crowd-puller. That work was largely undertaken by Annika Larsson, who runs *Inscriptorum*[4], a noted Swedish design workshop. Her work will be the subject of Part 2.

ACKNOWLEDGEMENTS

The black spot in Fig. 13 is placed where the great circle that is the best approximation to hour-line III (or IX) on the celestial sphere intersects the meridian. The term 'best approximation' has not been specified.

A naïve way to approximate each hour-line by a great circle is to choose the great circle that passes through the two end points (the summer solstice end and the winter solstice end). This great circle also passes through the equinoctical point and is a good candidate for the great circle of best fit.

In some early trials, this naïve great circle for hour-line III was used to determine the position of the black spot. The results were reported on the sundial mailing list in a thread entitled *Another Dial and a Puzzle*[5].

Geoff Thurston[6] responded by suggesting that there might be a better approximation and he was right. Using the naïve great circle leads to an angle of dip of 8.6° and, on the dial, the angular spread of the grey regions for hour-lines III and IX is 0.60°. By dipping the gnomon to 8.9° the angular spread reduces to 0.49°. Thanks are therefore due to Geoff Thurston for prompting further exploration.

Thanks are also due to the noted Italian *gnomonista*, Gianni Ferrari, who commented[7] that 'these lines don't have a great gnomonic meaning even if they have a mathematical one'. This comment prompted much careful checking.

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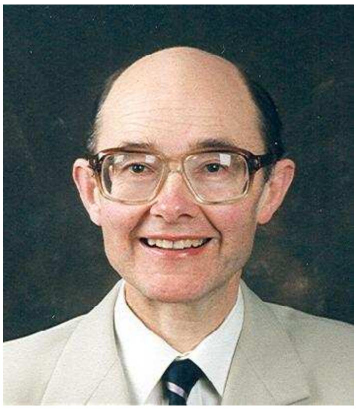

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