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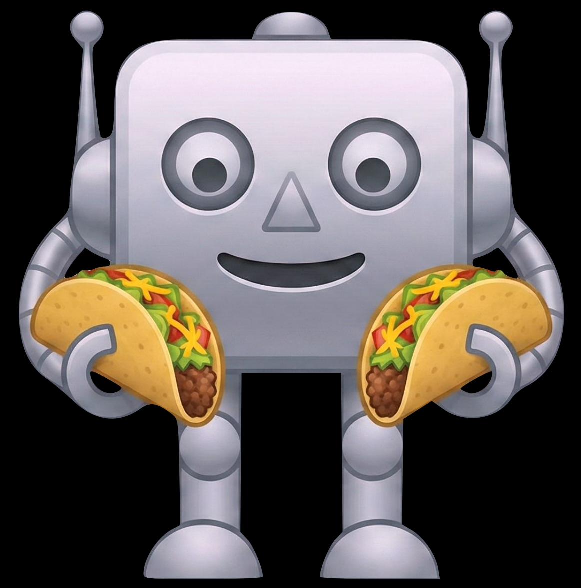

TACO

The Tensor Algebra Compiler

Fredrik Kjolstad, Shoaib Kamil, Stephen Chou, David Lugato,
and Saman Amarasinghe

26 November 2025 • University of Cambridge • Cambridge, UK

Carl Seifert
`cs2331@cam.ac.uk`

Tensors!

Tensors

❗ But: data often sparse

Tensors

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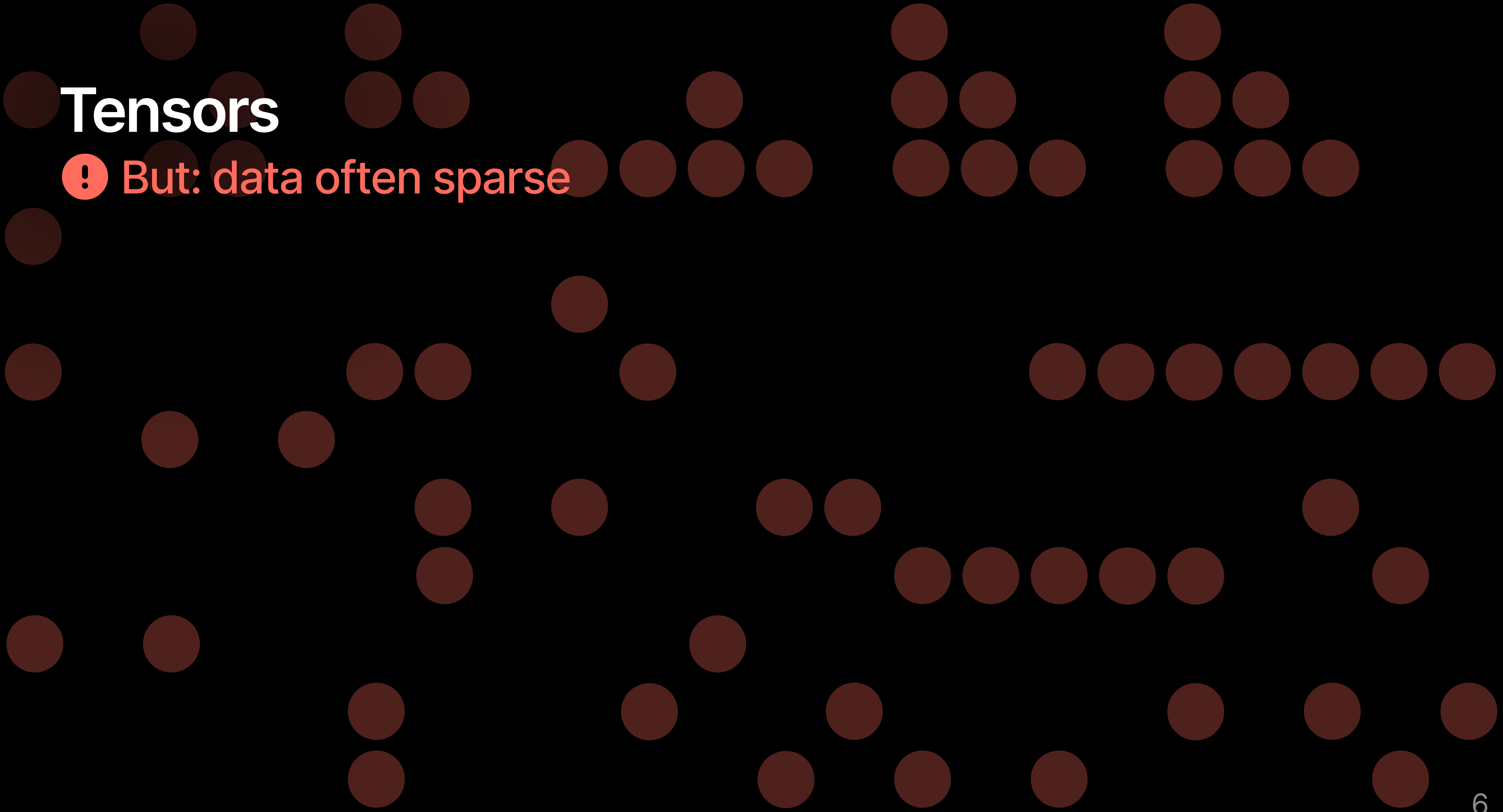

Tensors

❗ But: data often sparse

99.99999998%

Zeros in Amazon Review Tensor 2017

Tensors

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- Many storage formats

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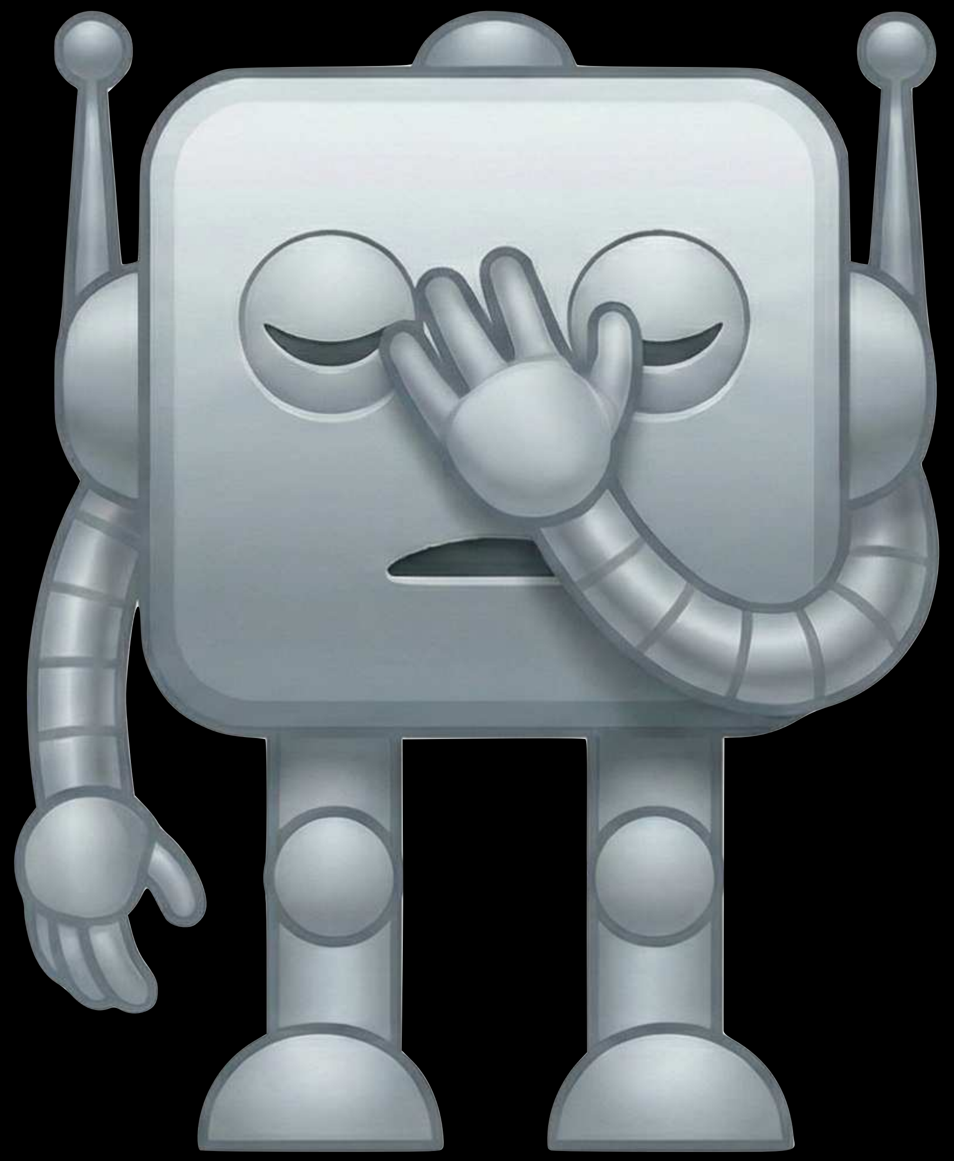

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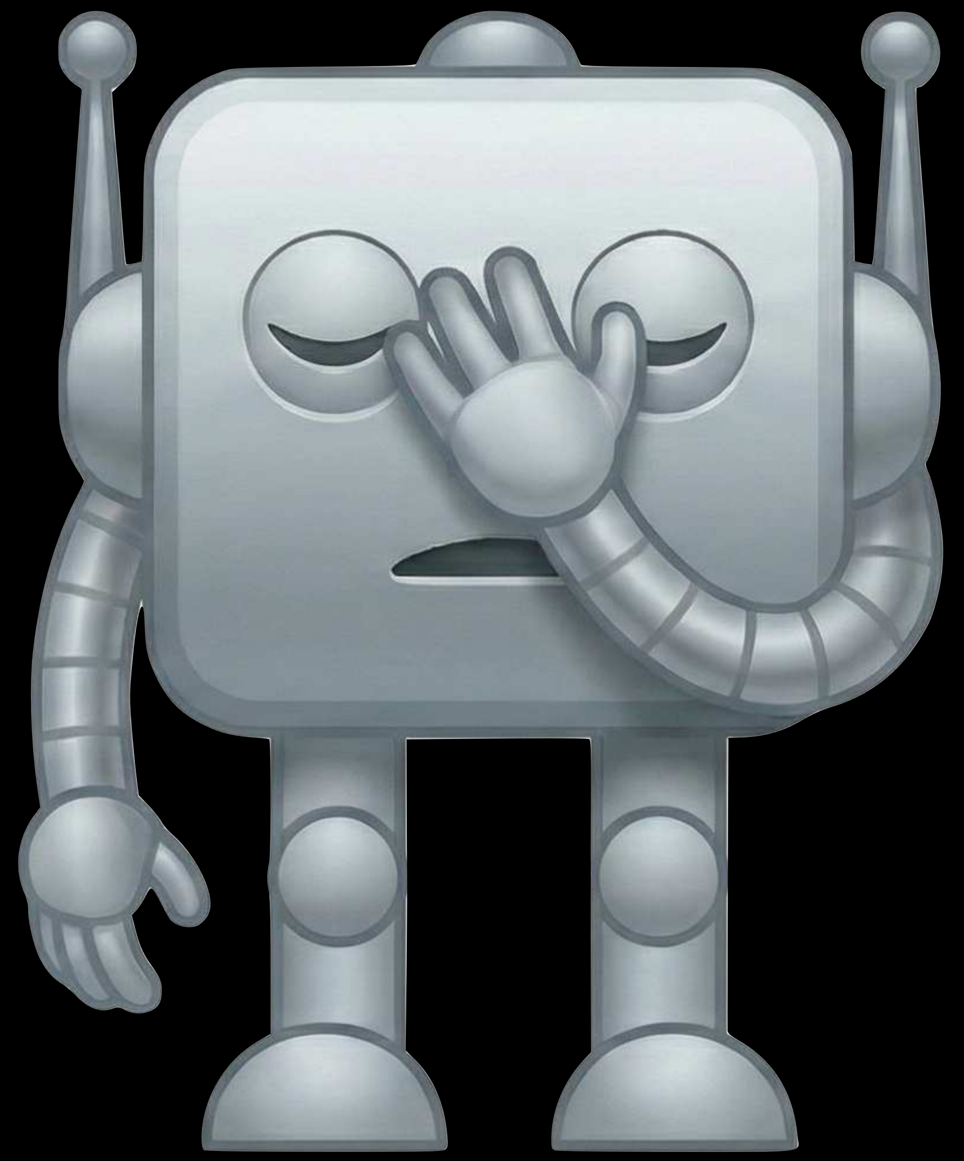
- Many operations



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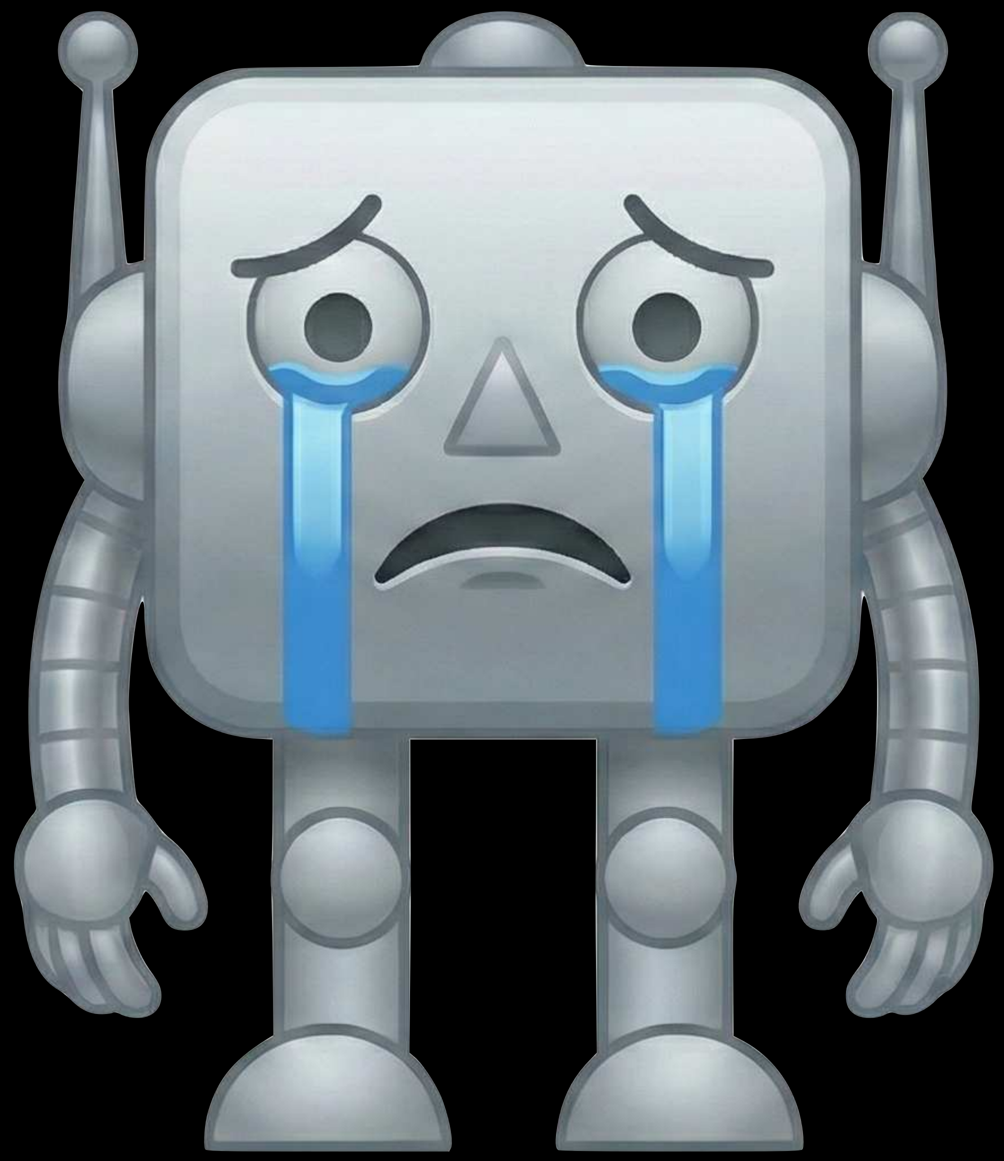

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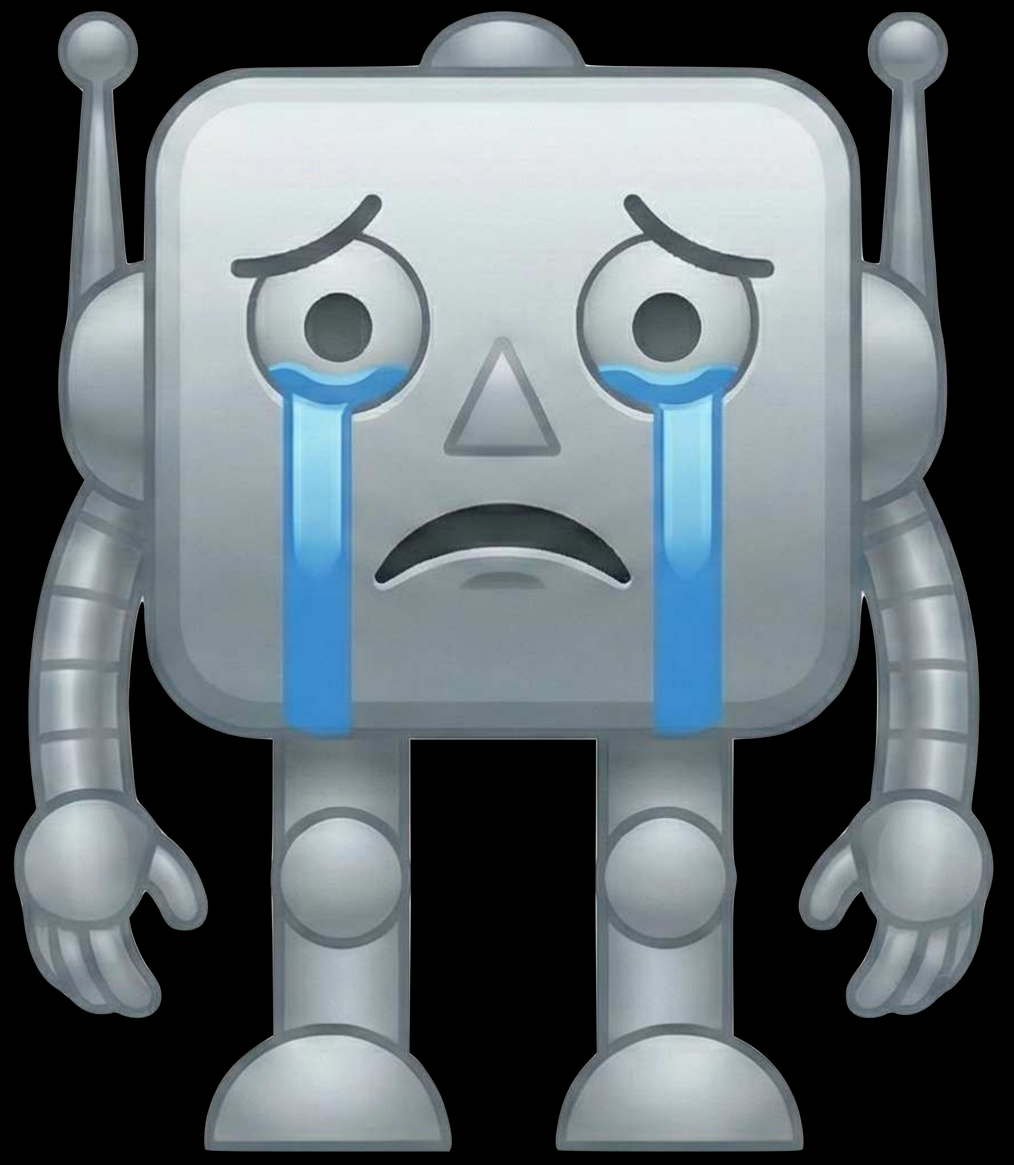
- ❗ But: data often sparse
- ❗ Many storage formats
- ❗ Many operations
- ❗ Many versions of operations, hand-optimised kernels



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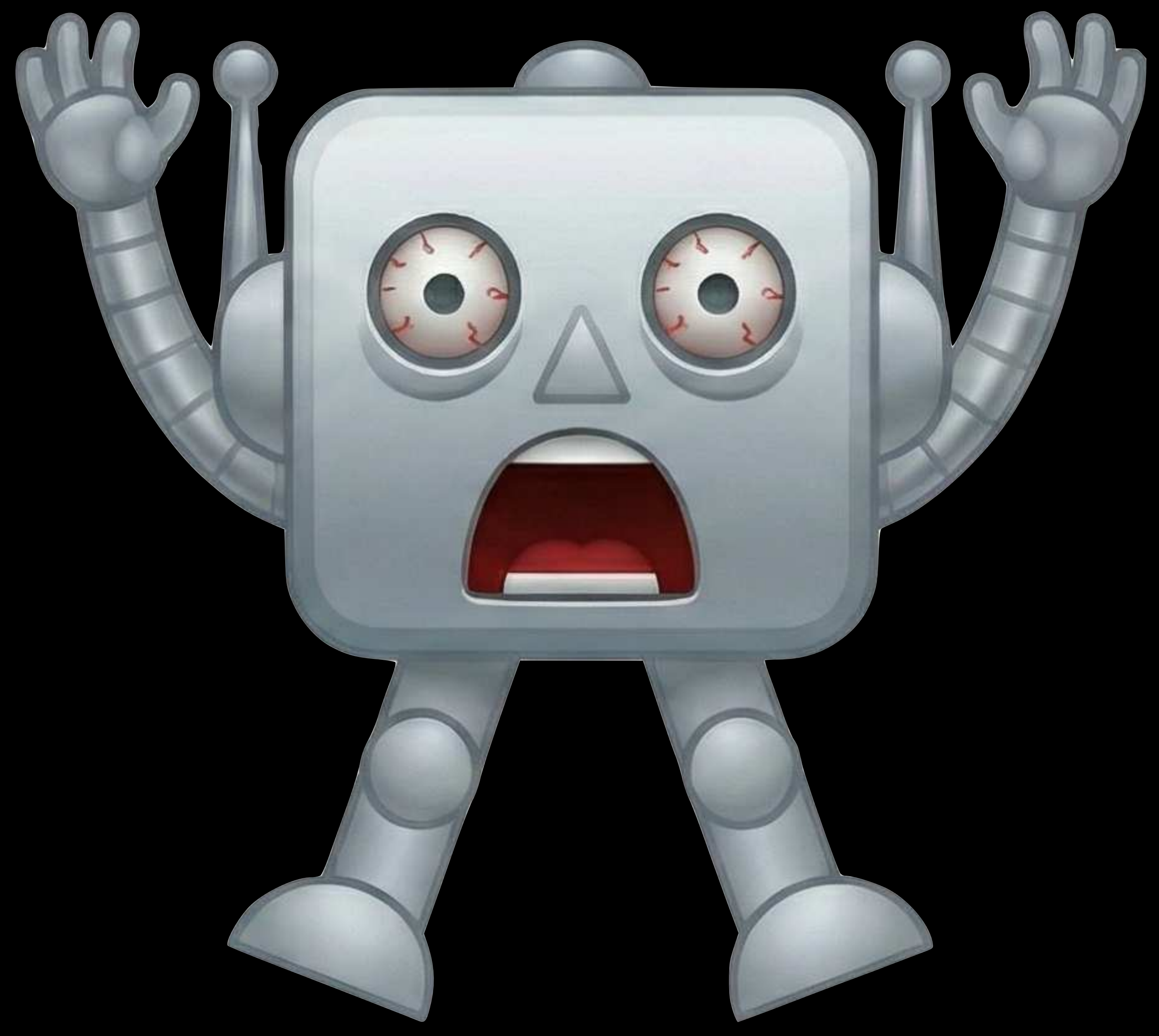

Goals

- Many storage formats
- Many different operations
- Auto-generated kernels for any expression

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The Tensor Algebra Compiler

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$$A_{ij} = B_{ijk} \cdot c_k$$

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SPARSE

SPARSE SPARSE SPARSE

DENSE DENSE

TACO

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Expression, format

TACO

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DENSE DENSE

Expression, format → Compiler

TACO

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A: i DENSE, j SPARSE

B: i DENSE, j SPARSE, k SPARSE

c: k SPARSE

C

Expression, format → Compiler → C code

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The Tensor Algebra Compiler

```
Format csr({ Dense, Sparse });
Tensor<double> A({ 64, 42 }, csr);
IndexVar i, j, k;
// --snip--
A(i,j) = B(i,j,k) * c(k);
```

C

Expression, format → Compiler → C code

1

2

3

1 Sparse Storage Formats

Store only sparse indices + boundaries

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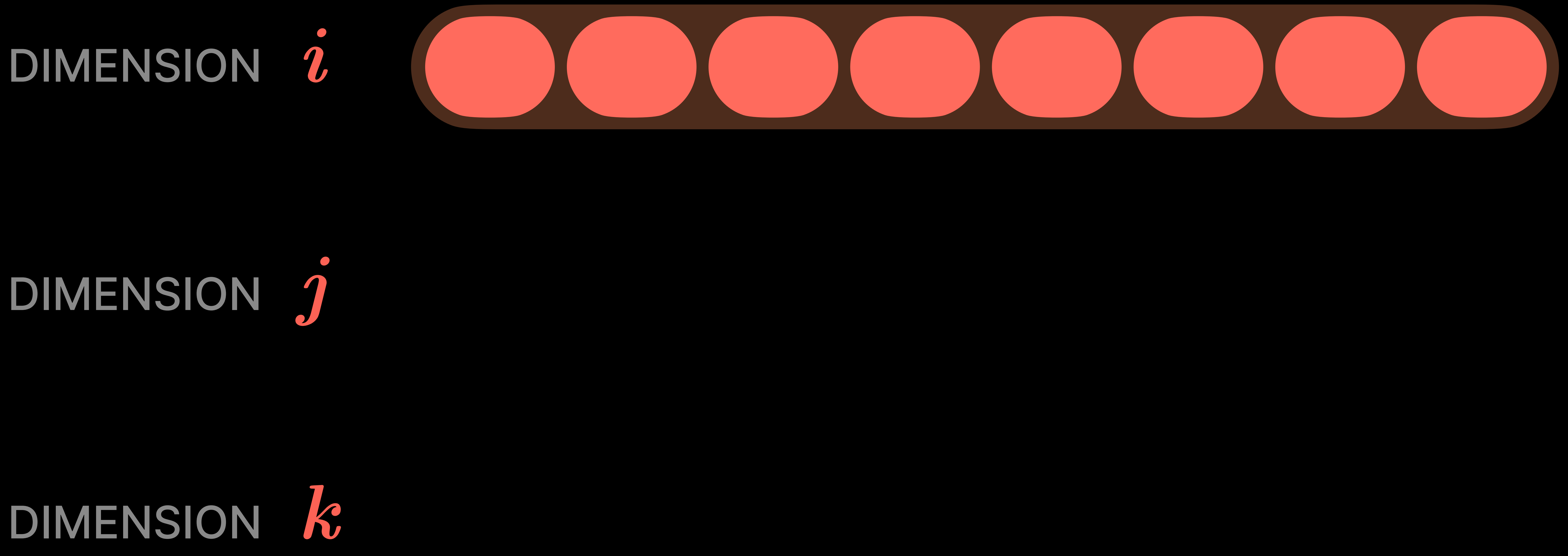

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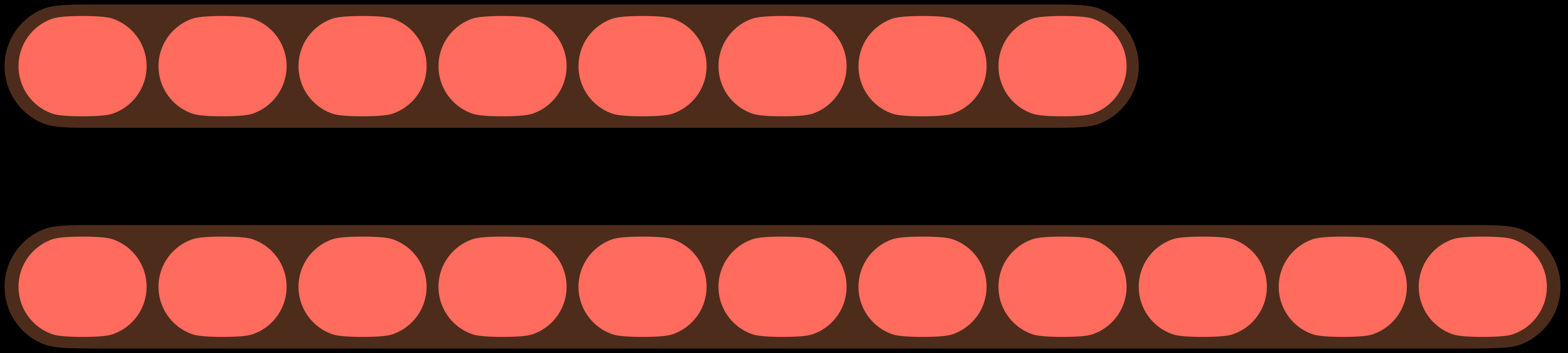

DIMENSION k

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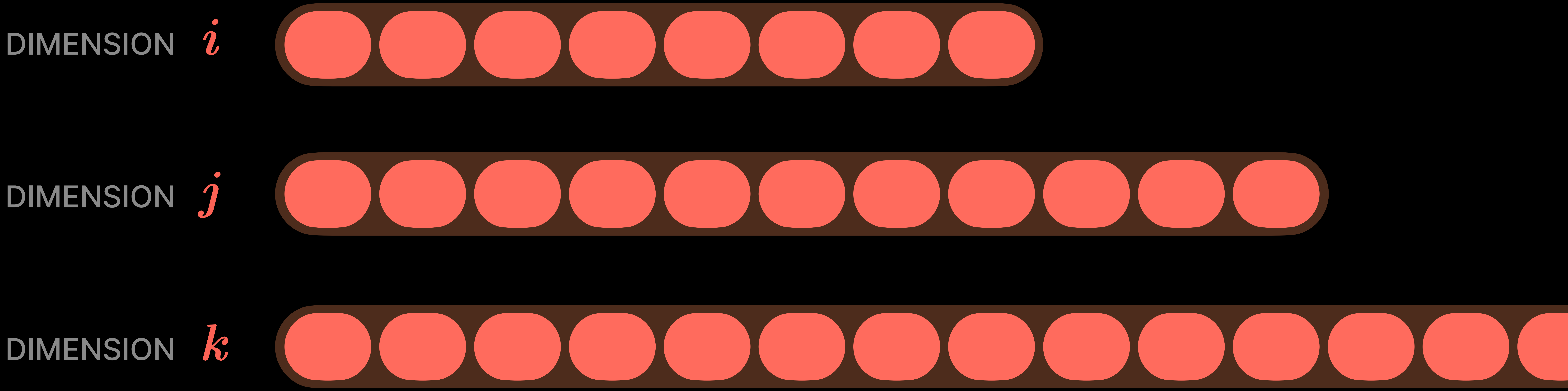

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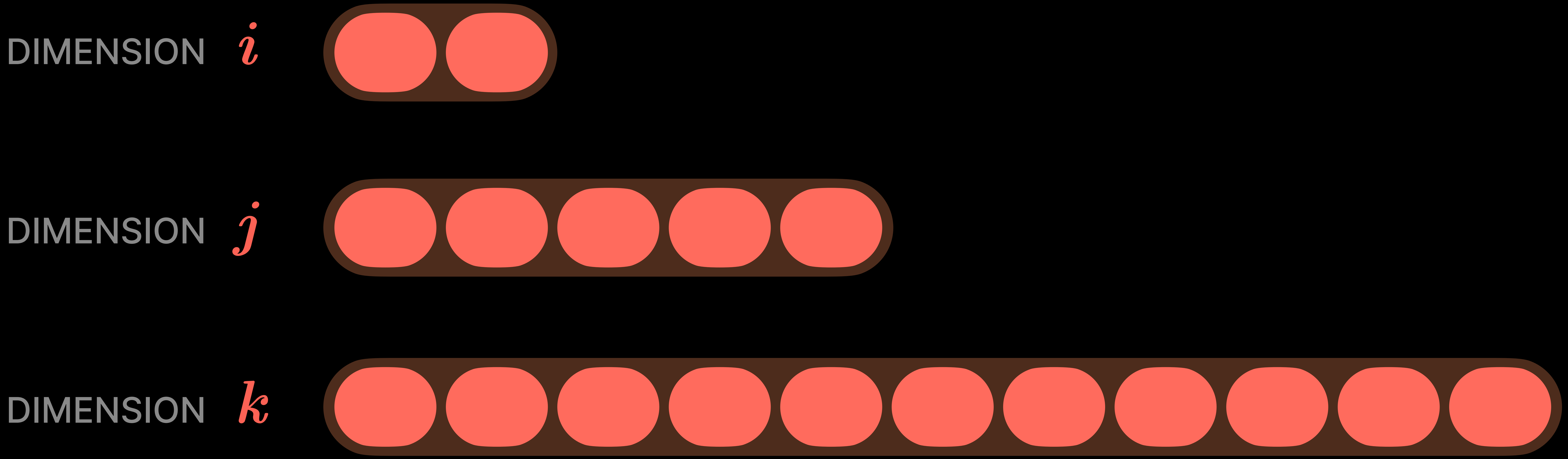

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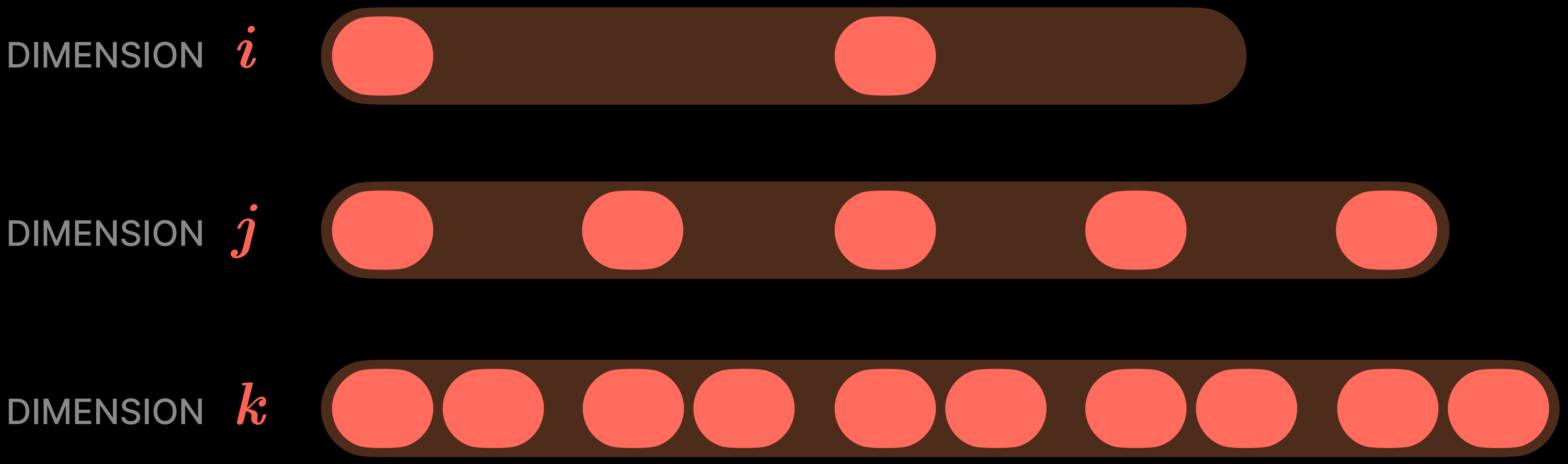

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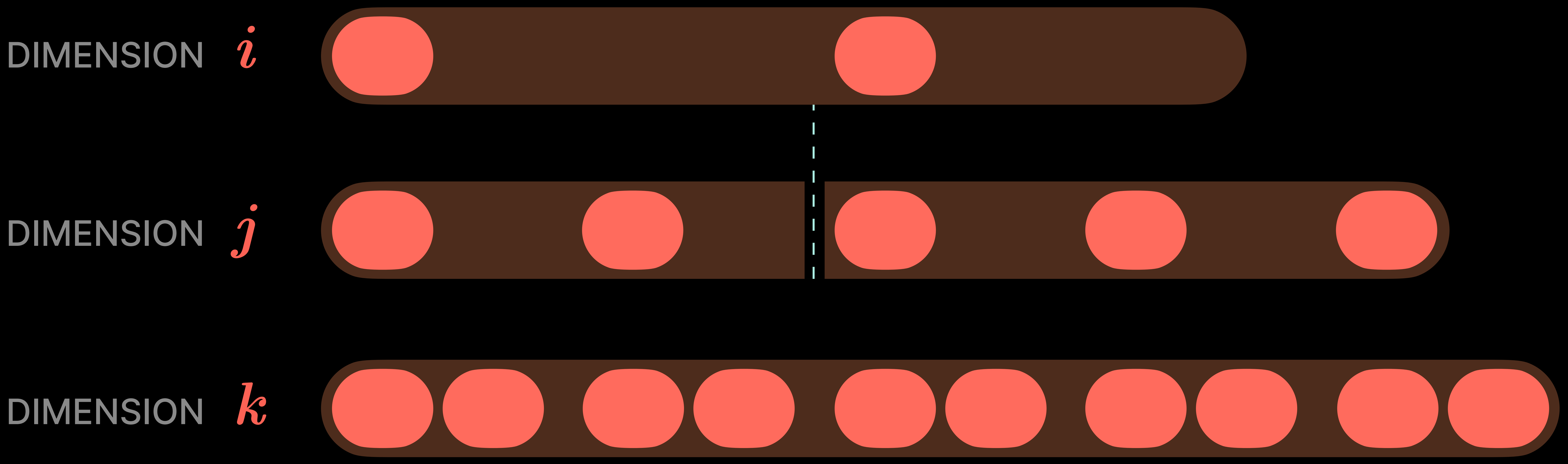

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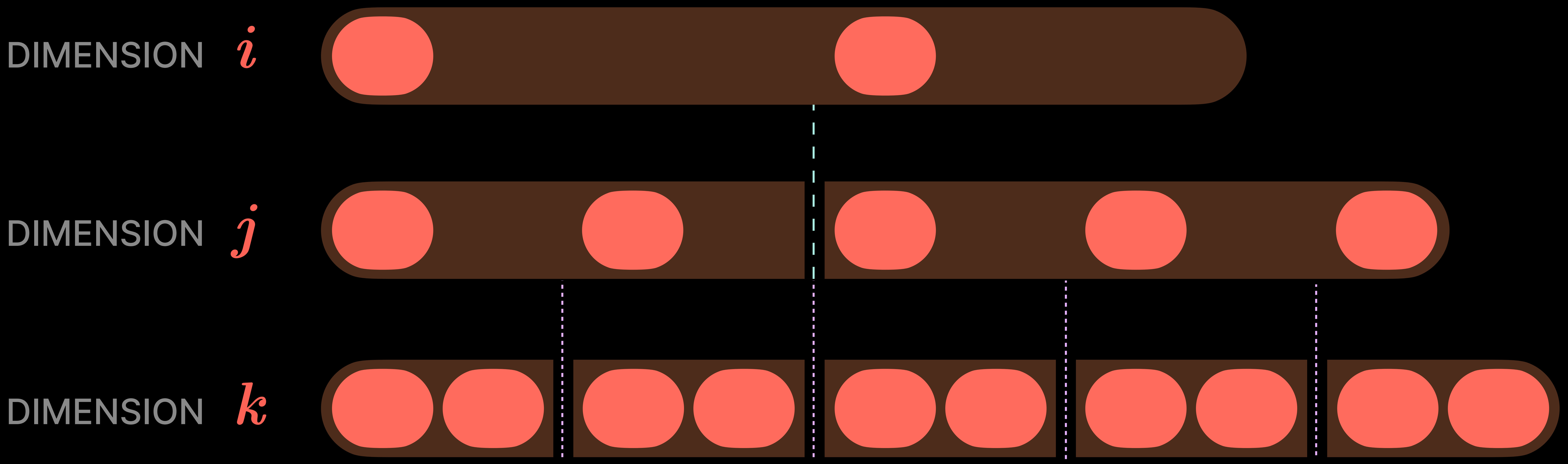

1 2 3

❷ Iteration Graphs

Represent co-iterable indices

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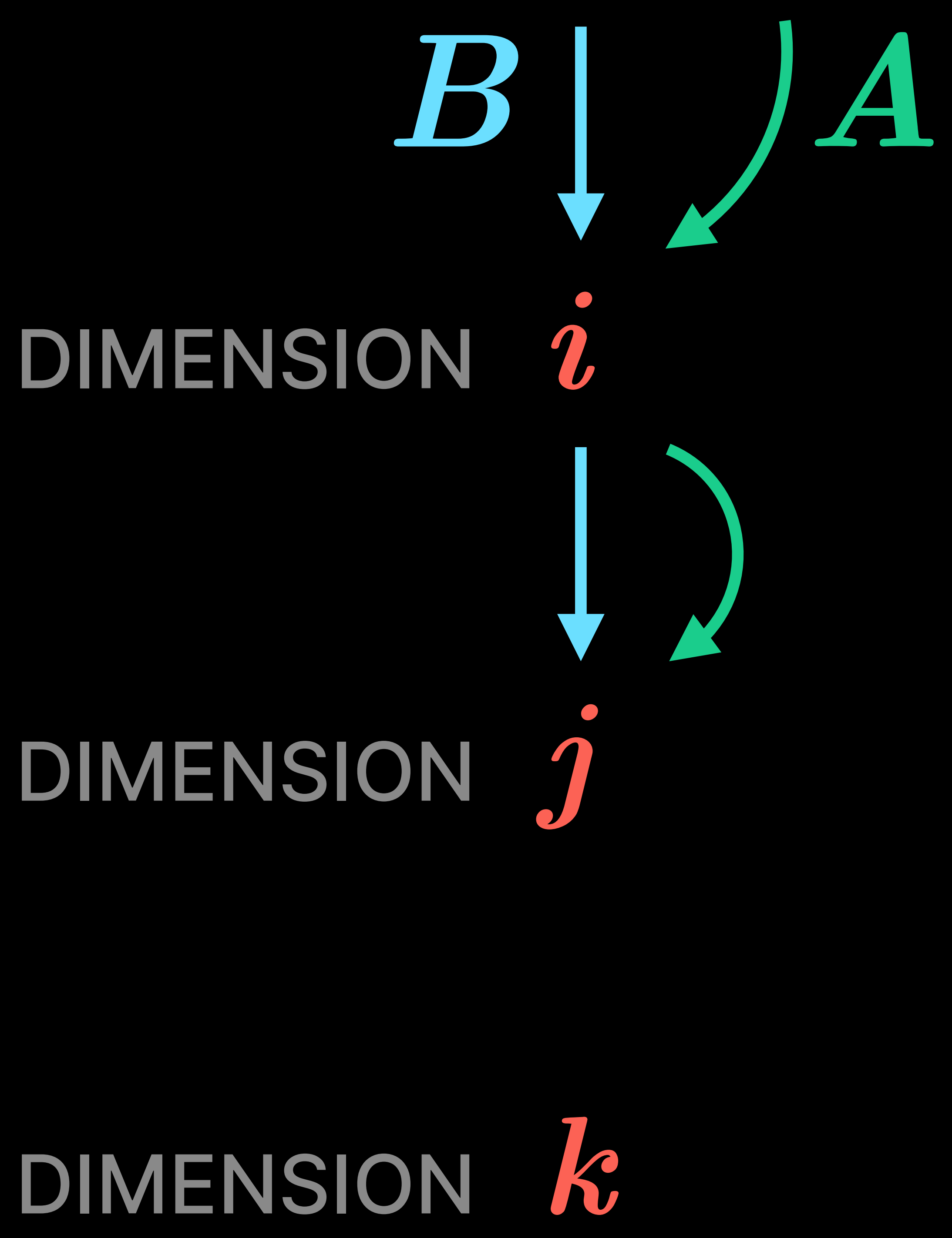

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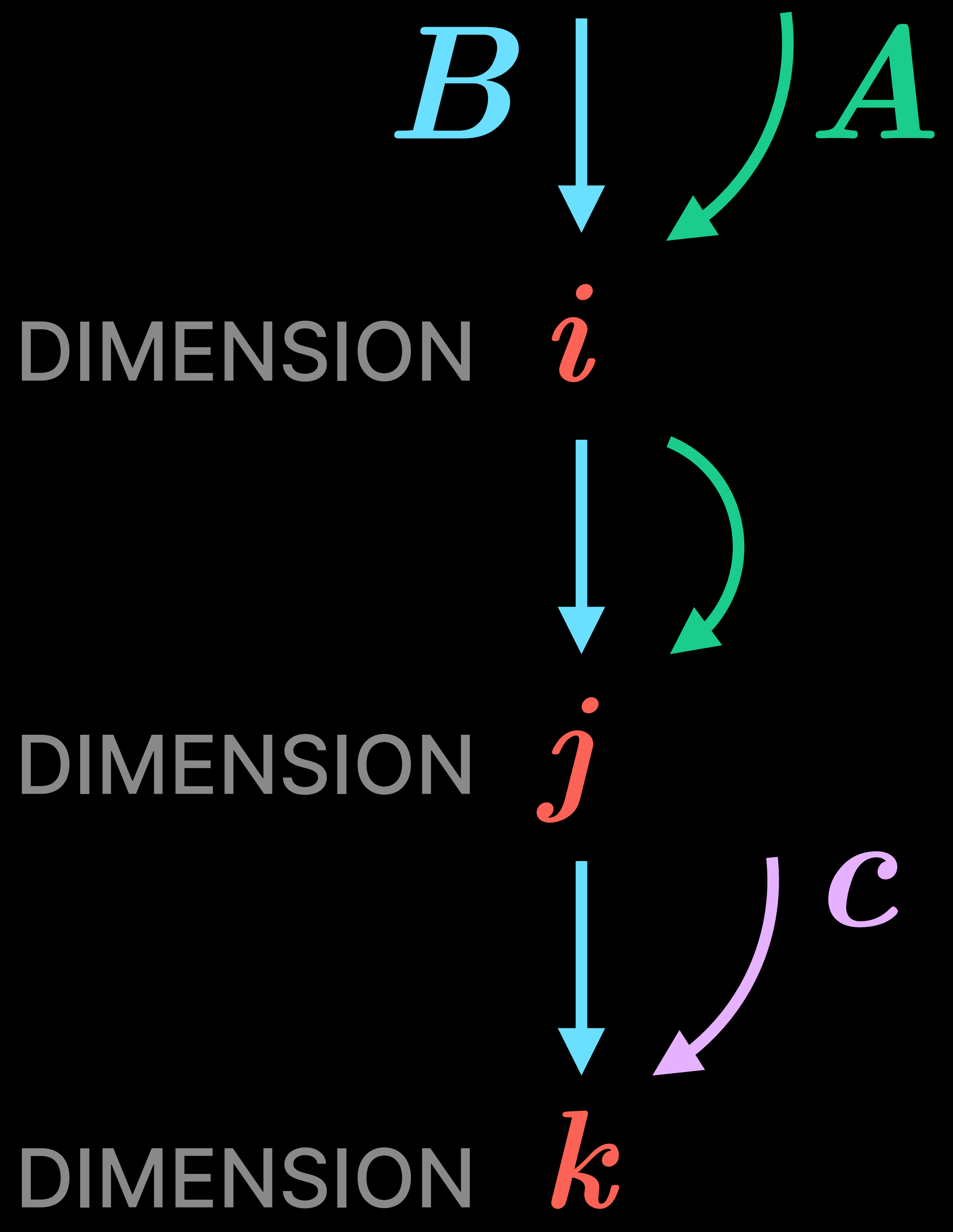

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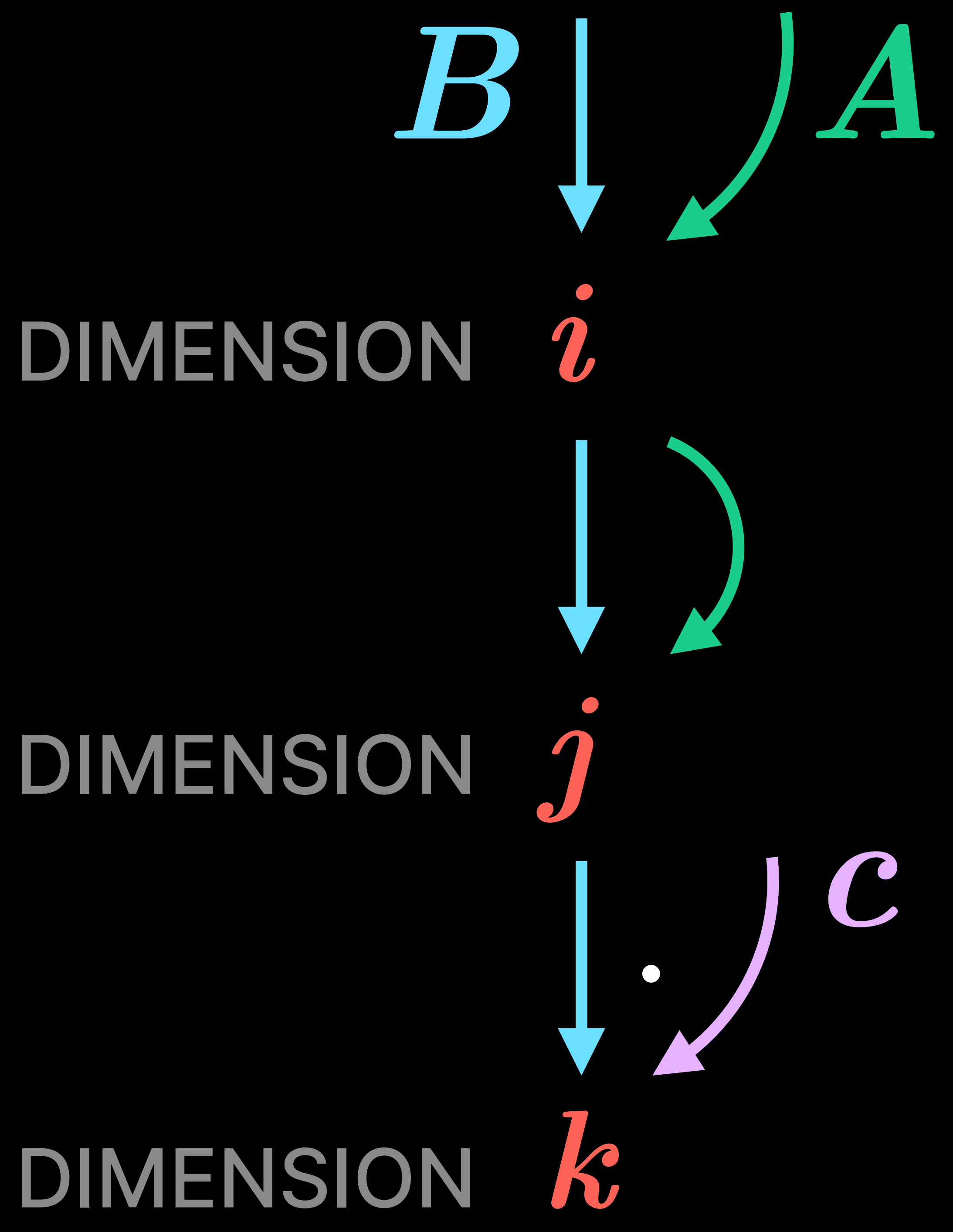

1

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3 Merge Lattices

Basis for optimisation of co-iteration over indices (two-way merge)

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$$\begin{tikzcd}
\boxed{\mathbf{b} + \mathbf{c} \quad \textit{iff} \quad \boldsymbol{b} \wedge \boldsymbol{c}} \arrow[d, "\neg \boldsymbol{c}"'] \\
\boxed{\text{just } \mathbf{b}}
\end{tikzcd}$$

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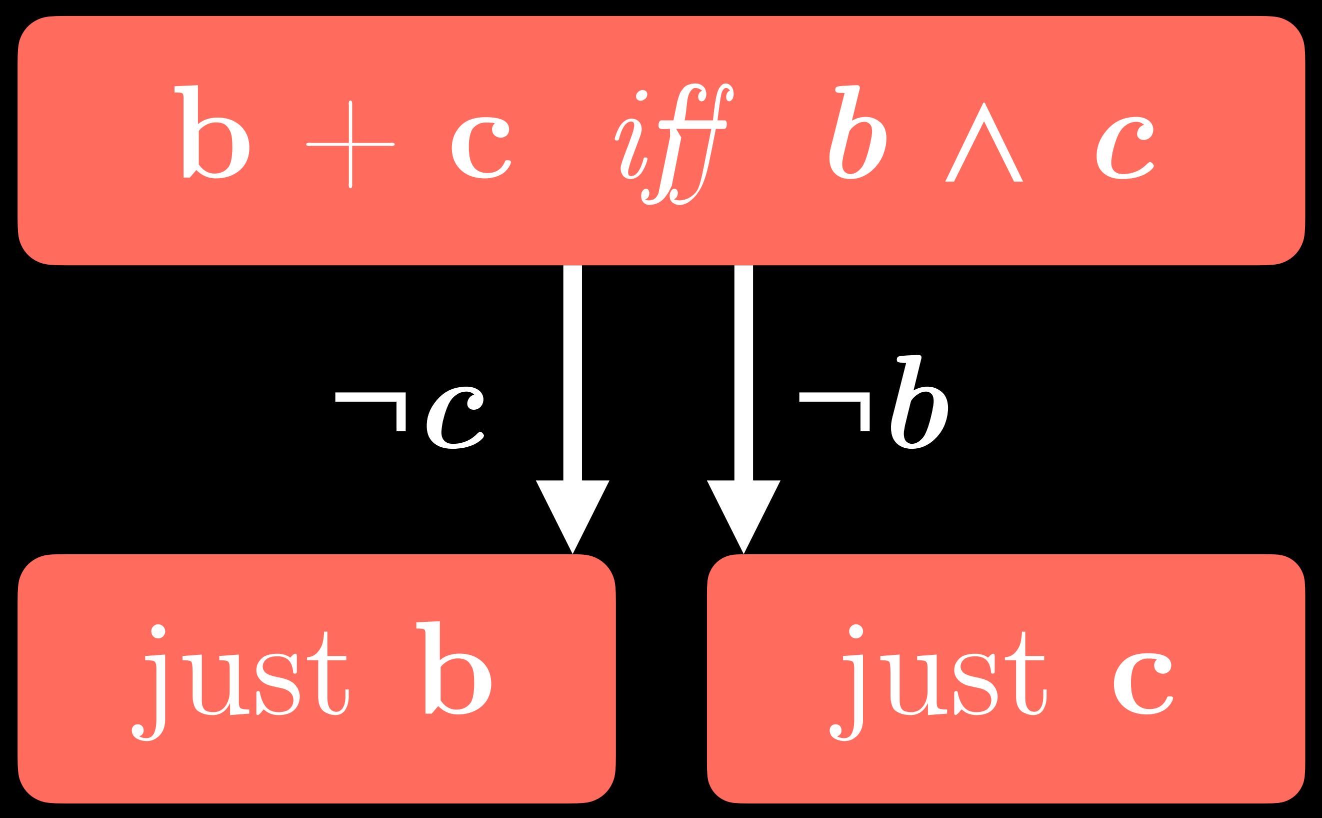

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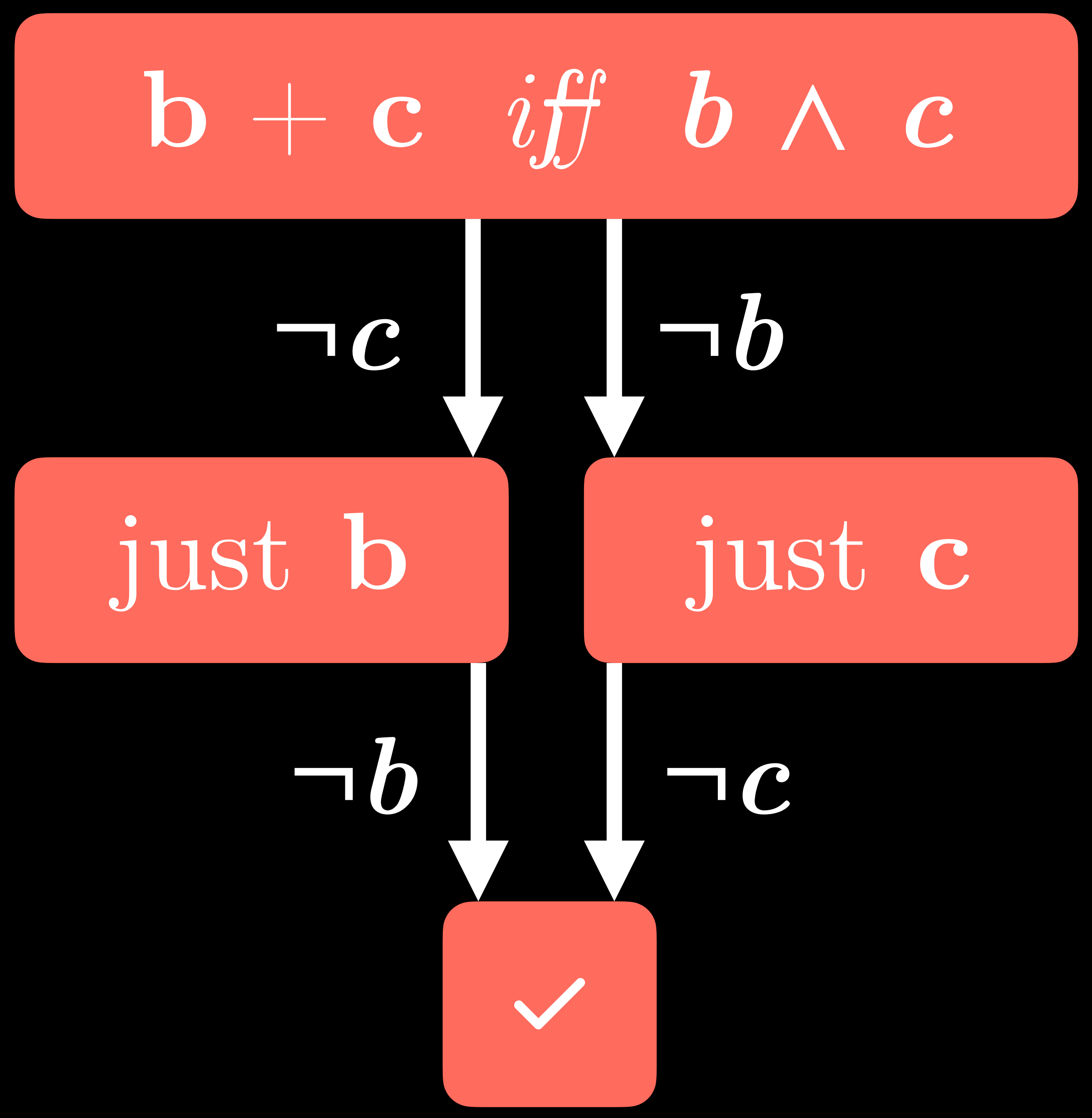

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→ Optimise by eliminating points/loops

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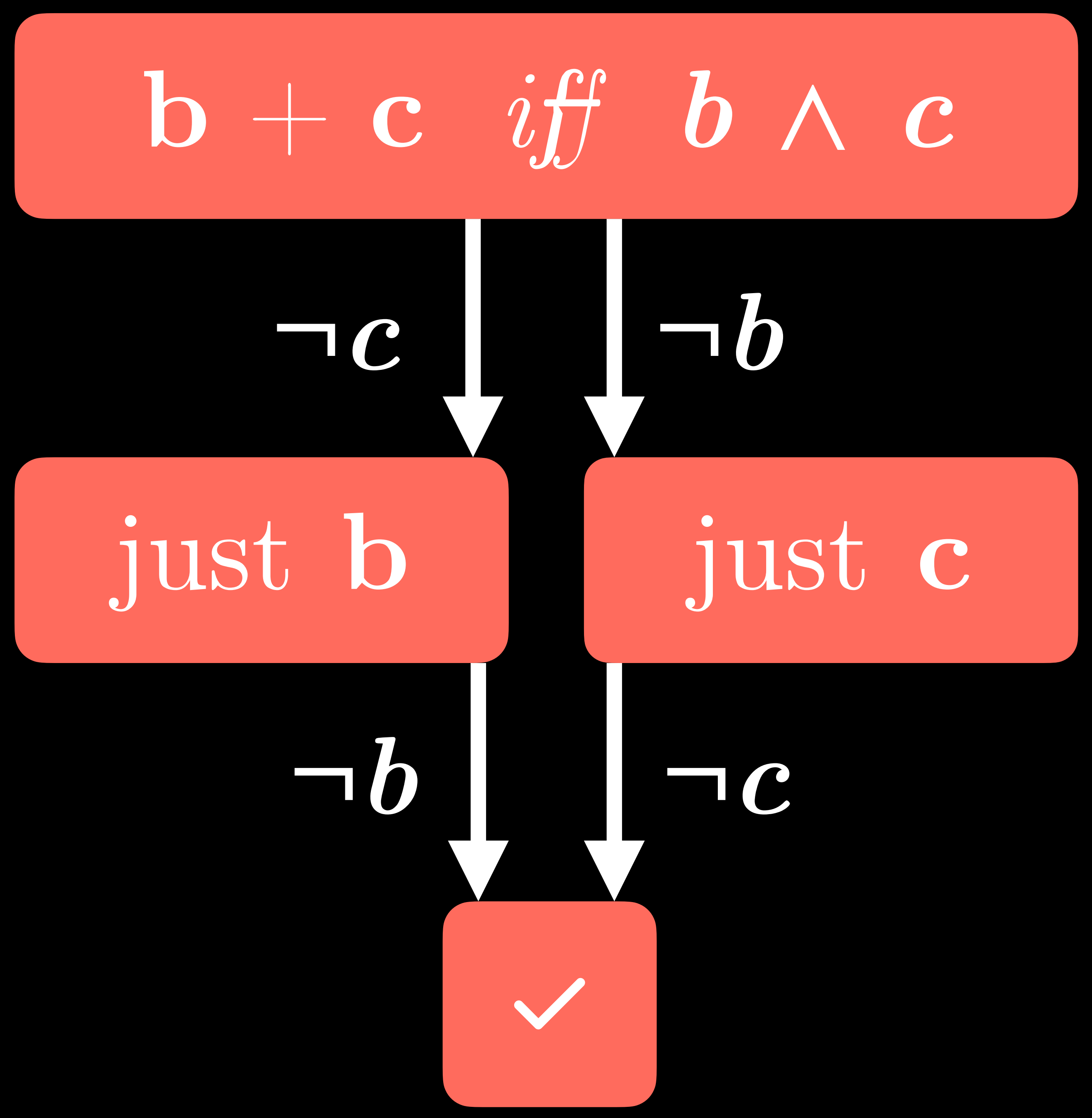

1 2 3

Evaluation

Sparse matrix–vector multiplication

$a = Bc$

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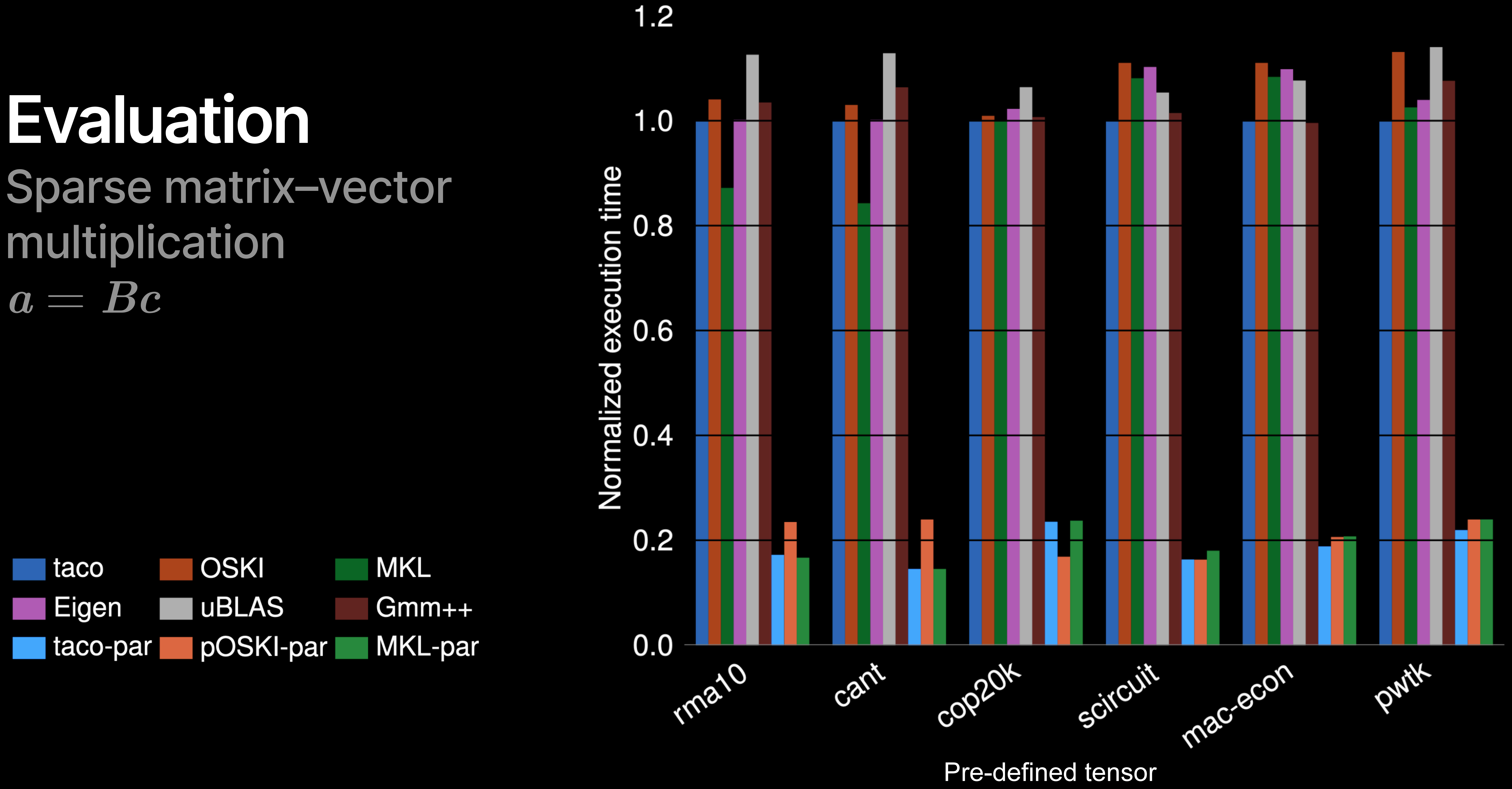

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- Further measurements: no ideal tensor format

Conclusion and Thoughts

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  parallel execution mode, (transpose/format*), conversion when merging;
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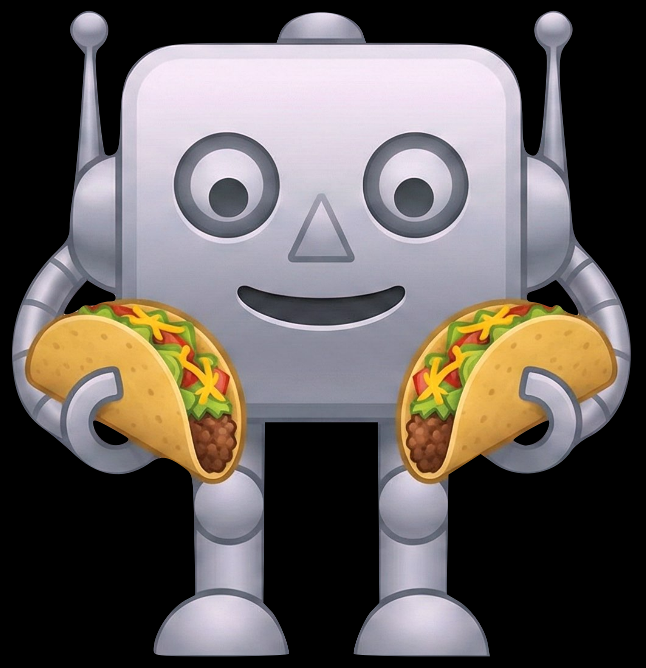
- Future work
  dimension formats (crd*), /temporary dense workspaces*[1]), JIT

[1] F. Kjolstad, W. Ahrens, S. Kamil and S. Amarasinghe, "Tensor Algebra Compilation with Workspaces," 2019 IEEE/ACM International Symposium on Code Generation and Optimization (CGO), Washington, DC, USA, 2019, pp. 180-192, doi: 10.1109/CGO.2019.8661185.



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