Equality Saturation For Tensor Graph Superoptimisation [5]

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Gooogle and MIT CSAIL

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Background on tensor graph rewriting and equality saturation

- Tensor graph g and semantics-preserving rewrites R to find lower cost g'.
- Equality saturation [4] is a compiler optimisation technique.

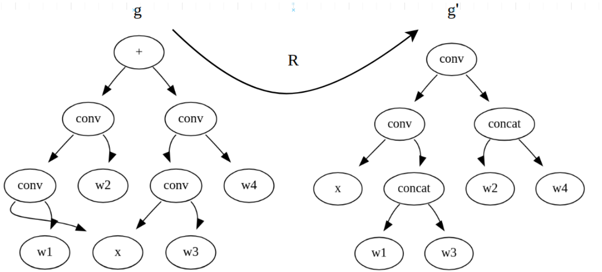

Figure 1: A rewrite useful for NasNet-A [6].
Left: original. Right: optimised.
The dimensions are (`out_channels`, `in_channels`, `height`, `width`) and the operations are `concat(w1, w3, axis=0)`, `concat(w2, w4, axis=1)`

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Why do we need it?

- Runtime speedup of computation.
- Optimisation time speedup.

Features: Equality Saturation (ES)

Problem: The Phase Ordering Challenge

- Traditional graph rewriting is sequential and destructive.

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- Rules apply until saturation (monotonic growth), there is no backtracking.

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An e-graph has e-classes and e-nodes.

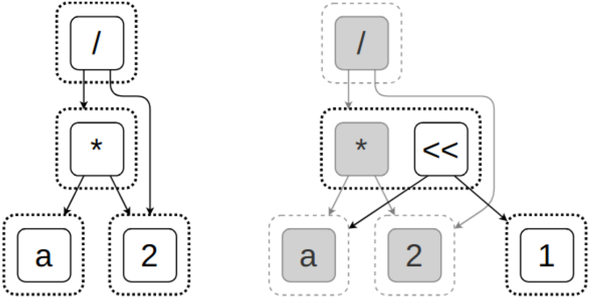

Figure 1. Left: An e-graph representing the term $(a \times 2)/2$. Dotted boxes show e-classes, and arrows connect e-nodes to their e-class children. *Right:* The e-graph after applying the rewrite $x \times 2 \to x \ll 1$. Only a few e-nodes were added (highlighted in white), and the result represents both the initial and rewritten terms.

Features: Extraction

Problem: Greedy Extraction Inefficiency

For each e-class, calculate subtree costs for each e-node, select cheapest e-node. Not globally optimal. No subgraph sharing. Computing subtree costs is an arduous process.

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Approach: Global Integer Linear Program (ILP) Extraction

$$\text{Minimize: } f(x) = \sum_i c_i x_i$$

Subject to:

$$x_i \in \{0, 1\}, \tag{1}$$

$$\sum_{i \in e_0} x_i = 1, \tag{2}$$

$$\forall i, \forall m \in h_i, x_i \leq \sum_{j \in e_m} x_j, \tag{3}$$

$$\forall i, \forall m \in h_i, t_{g(i)} - t_m - \epsilon + A(1 - x_i) \geq 0, \tag{4}$$

$$\forall m, 0 \leq t_m \leq 1, \tag{5}$$

with i nodes, m e-classes, cost c_i, indicator var x_i, children h_i and e-class g_i.

Features: Efficient Cycle Filtering

Problem: Cycles / Deadlock

E-Graphs represent equality ($A = B$) (bidirectional, not DAG). "Check and reject" is inefficient.

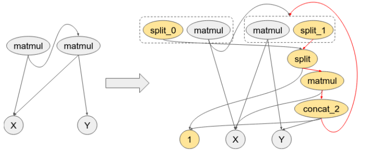
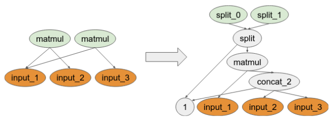


Figure 3. Example on how a valid rewrite can introduce cycles into the e-graph. RHS is the resulting e-graph after applying the rewrite rule from Figure 2 to the LHS. Dotted lines circles the e-classes. We omit the e-classes with a single node for clarity. If the node split_1 is picked in the right e-class, then the resulting graph will have a cycle (indicated by the red edges).



Source: (matmul ?input$_1$?input$_2$), (matmul ?input$_1$?input$_3$)
Target: (split$_0$ (split 1 (matmul ?input$_1$ (concat$_2$ 1 ?input$_2$?input$_3$)))),
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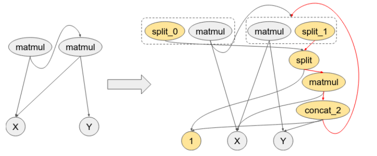

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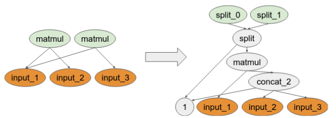

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Approach: ILP Constraints

Topological sort with t_m guarantees acyclic execution.

$$\text{Minimize: } f(x) = \sum_i c_i x_i$$

Subject to:

$$x_i \in \{0, 1\}, \tag{1}$$

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Exploration time (s)	k_{multi}	Vanilla	Efficient
BERT	1	0.18	**0.17**
	2	32.9	**0.89**
NasRNN	1	1.30	**0.08**
	2	2932	**1.47**
NasNet-A	1	3.76	**1.27**
	2	>3600	**8.62**

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Results

Top-line: 16% speedup over TASO [1]. **48x** less time optimising.

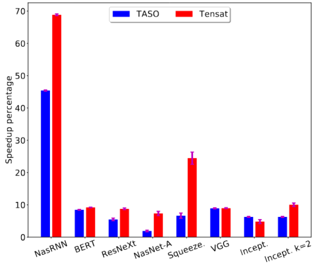

Figure 2: Speedup percentage of the optimised graph, mean and std of 5 runs.

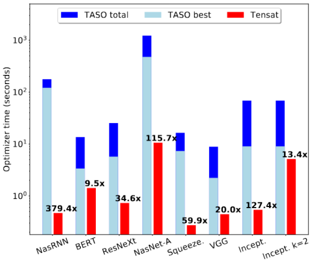

Figure 3: Optimisation time. TASO Best indicates when its found the best result during search.

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- E-graph can expand indefinitely: $x- > x + 0$ (max. 50k iter).
- Blindly applying every rule consumes a lot of memory. Enter Omelette (RL) [3] or MCTS.
- Can they start extracting and exploring at the same time?

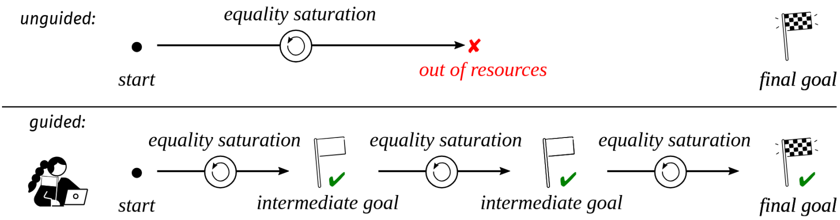

Figure 4: Guided equality saturation [2]

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- I think a future work section would have been useful.
- No demonstration of integration into production. Maybe this is outside the scope of the research paper.

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Q & A