PET: Optimizing Tensor Programs with Partially Equivalent Transformations and Automated Corrections

KH. Wang, J. Zhai, M. Gao, Z. Ma, S. Tang, L. Zheng, Y. Li, K. Rong, Y. Chen, and Z. Jia

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Background: Tensor Program

A linear algebra operator (e.g., convolution, matrix mul) or a non-linear activation (e.g., relu, sigmoid)

A tensor (i.e., n-dimensional array)

Background: Tensor Program Transformations

Background: Current Systems Consider only Fully Equivalent Transformations

\[ \forall p. \ Y[p] = Z[p] \]

- **Fully Equivalent Transformations**
  - Pro: preserve functionality
  - Con: miss optimization opportunities

\[ \exists p. \ Y[p] \neq Z[p] \]

- **Partially Equivalent Transformations**
  - Pro: better performance
    - Faster ML operators
    - More efficient tensor layouts
    - Hardware-specific optimizations
  - Con: potential accuracy loss

Current Systems Consider only Fully Equivalent Transformations

\[ \forall p. \ Y[p] = Z[p] \]

\[ \exists p. \ Y[p] \neq Z[p] \]

Is it possible to exploit partially equivalent transformations to improve performance while preserving equivalence?

**Fully Equivalent Transformations**

**Pro:** preserve functionality

**Con:*** miss optimization opportunities

**Partially Equivalent Transformations**

**Pro:** better performance
- Faster ML operators
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- Hardware-specific optimizations

**Con:** potential accuracy loss

Motivating Example

Input Program

Partially Equivalent Transformation

Correcting Results

Incorrect results

Motivating Example

Input Program

Correcting Results

- Transformation and correction lead to $1.2x$ speedup for ResNet-18
- Correction preserves end-to-end equivalence

PET

- **First tensor program optimizer** with partially equivalent transformations
- **Larger optimization space** by combining fully and partially equivalent transformations
- **Better performance**: outperform existing optimizers by up to **2.5x**
- **Correctness**: automated corrections to preserve end-to-end equivalence

Key Challenges

1. How to generate partially equivalent transformations?
   - Superoptimization

2. How to correct them?
   - Multi-linearity of DNN computations

Mutant Generator

Superoptimization adapted from TASO\(^1\)

Enumerate all possible programs up to a fixed size using available operators

1. TASO: Optimizing Deep Learning Computation with Automated Generation of Graph Substitutions. SOSP’19.

Mutant Generator

Superoptimization adapted from TASO\textsuperscript{1}

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Challenges: Examine Transformations

1. Which part of the computation is not equivalent?
2. How to correct the results?

A Strawman Approach

- **Step 1**: Explicitly consider all output positions (m positions)

- **Step 2**: For each position $p$, examine all possible inputs (n inputs)

Require $O(m \times n)$ examinations, but both m and n are too large to explicitly enumerate

Multi-Linear Tensor Program (MLTP)

- A program $f$ is multi-linear if the output is linear to all inputs
  - $f(I_1, \ldots, X, \ldots, I_n) + f(I_1, \ldots, Y, \ldots, I_n) = f(I_1, \ldots, X + Y, \ldots, I_n)$
  - $\alpha \cdot f(I_1, \ldots, X, \ldots, I_n) = f(I_1, \ldots, \alpha \cdot X, \ldots, I_n)$

- DNN computation = MLTP + non-linear activations

No Need to Enumerate All Output Positions

Group all output positions with an identical summation interval into a region.

*Theorem 1*: For two MLTPs $f$ and $g$, if $f = g$ for $O(1)$ positions in a region, then $f = g$ for all positions in the region.

Only need to examine $O(1)$ positions for each region.

**Complexity**: $O(m \times n) \rightarrow O(n)$

\[
\text{conv}(c, h, w) = \sum_{d=0}^{D-1} \sum_{x=-1}^{1} \sum_{y=-1}^{1} I_1(d, h + x, w + y) \times I_2(d, c, x, y)
\]

Summation interval

*Proof details available in the paper*

No Need to Consider All Possible Inputs

Examine equivalence for a single position is still challenging

*Theorem 2:* If $\exists I. f[I][p] \neq g[I][p]$, then the probability that $f$ and $g$ give identical results on $t$ random integer inputs is $(\frac{1}{2^{31}})^t$

Run $t$ random tests for each position $p$

**Complexity:** $O(n) \rightarrow O(t) = O(1)$

*Proof details available in the paper*
Mutant Corrector

**Goal:** quickly and efficiently correcting the outputs of a mutant program
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**Step 1:** recomputate the incorrect outputs using the original program

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**Step 1:** recompute the incorrect outputs using the original program

**Step 2:** opportunistically fuse correction kernels with other operators

Correction introduces less than 1% overhead

Source: Zhihao Jia. (2021)
Program Optimizer

- **Beam search**
- Optimizing a DNN architecture takes less than 30 minutes

Other optimizations:
- Operator fusion
- Constant folding
- Redundancy elimination

Source: Zhihao Jia. (2021)
End-to-end Inference Performance (Nvidia V100 GPU)

PET outperforms existing optimizers by 1.2-2.5x by combining fully and partially equivalent transformations

More Evaluation in Paper

1. A case study on tensor-, operator-, and graph-level optimizations discovered by PET

2. Both fully and partially equivalent transformations are critical to performance

3. PET consistently outperforms existing optimizers on various backends (cuDNN/cuBLAS, TVM, Ansof)

4. Partially equivalent transformations w/ corrections can directly benefit existing optimizers

PET

- A tensor program optimizer with partially equivalent transformations and automated corrections

- **Larger optimization space** by combining fully and partially equivalent transformations

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Source: Zhihao Jia. (2021)
Criticism - pros/cons

• While PET outperforms rule-based optimizers, it only discovers transformations between expressions that can be constructed using only the predefined operators.

• PET attempts to save human effort in DNN optimization by searching for optimised transformations given a set of operators. PET then introduces inequivalent transformations and correction mechanisms to find even more optimizations.

• Existing attempts to improve a DNN’s tensor algebra expression only address expressions representable by a fixed set of predefined operators (e.g. matrix multiplication), leaving out possible optimization opportunities between general expressions.

• We can improve the design to explore a much larger search space of general expressions. By deriving tensor algebra expressions, we can broaden the search space from predefined operator representable (POR) expressions to general expressions.