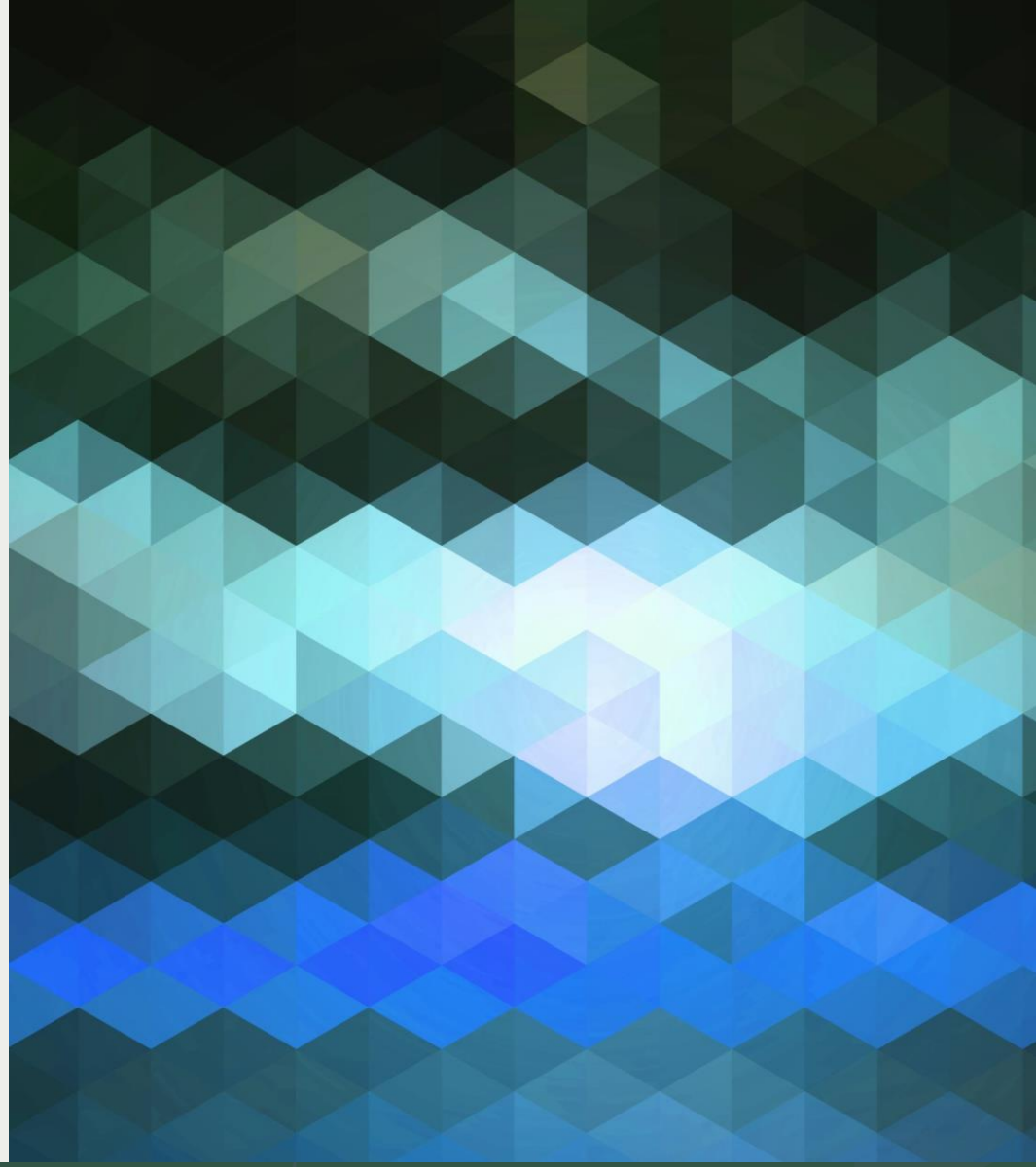
Bayesian Optimisation with a Probabilistic Model

(in Python)

Ross Tooley



Motivation

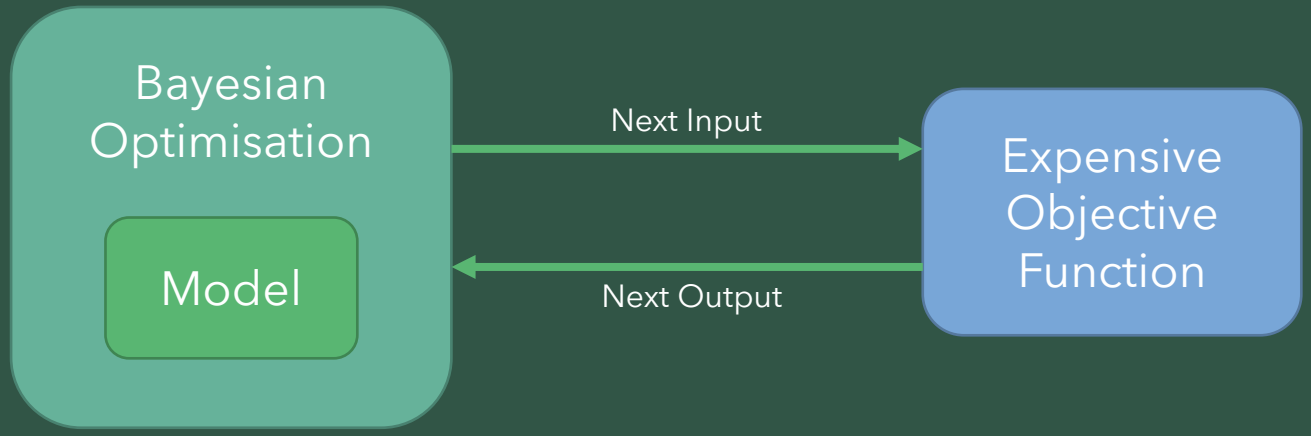
Inspired by ProBO

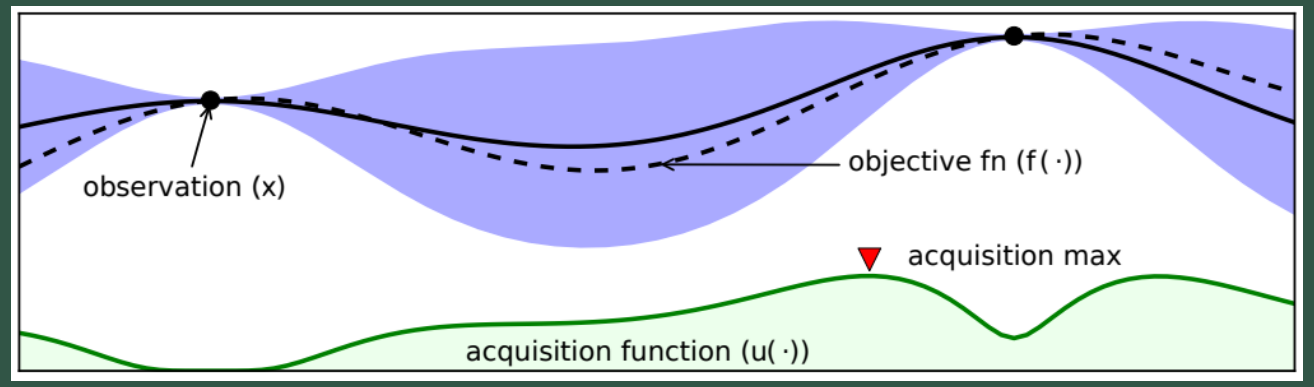
ProBO: Versatile Bayesian Optimization Using Any Probabilistic Programming Language

Coming soon. Read the paper here.

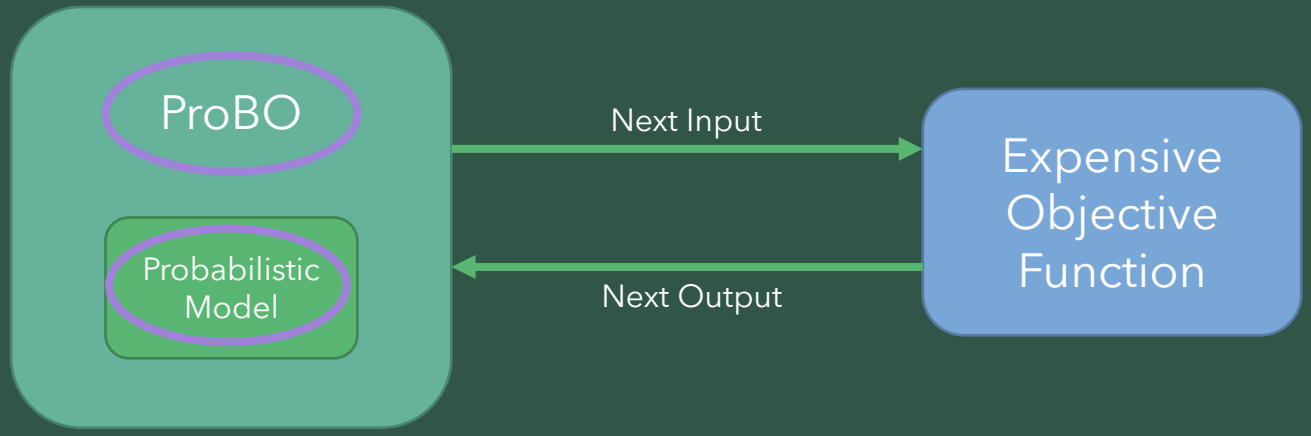
For questions, please email willie@cs.cmu.edu.

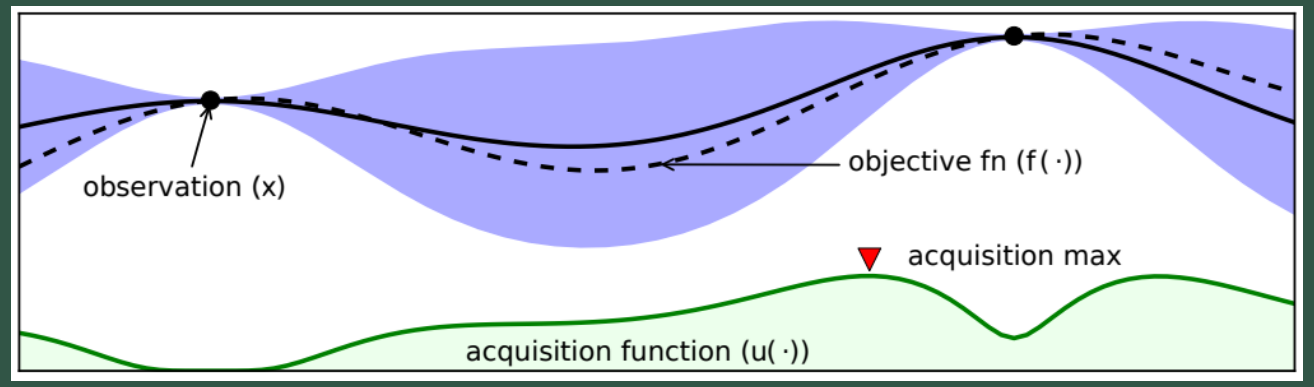
BO Recap


Bayesian Optimisation
Model
Next Input
Next Output
Expensive Objective Function


observation (x)
objective fn (f(·))
acquisition max
acquisition function (u(·))

ProBO Recap


ProBO
Probabilistic Model
Next Input
Next Output
Expensive Objective Function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ProBO Recap

Algorithm 1 ProBO($\mathcal{D}_0, \texttt{inf}, \texttt{gen}$)

for $n = 1, \ldots, N$ **do**
  $\texttt{post} \leftarrow \texttt{inf}(\mathcal{D}_{n-1})$
  $x_n \leftarrow \operatorname{argmin}_{x \in \mathcal{X}} a(x, \texttt{post}, \texttt{gen})$
  $y_n \sim s(x_n)$
  $\mathcal{D}_n \leftarrow \mathcal{D}_{n-1} \cup (x_n, y_n)$
Return $\mathcal{D}_N$.

Algorithm 2 $a_{\mathrm{EI}}(x, \texttt{post}, \texttt{gen})$ ▷ EI

for $m = 1, \ldots, M$ **do**
  $z_m \leftarrow \texttt{post}(s_m)$
  $y_m \leftarrow \texttt{gen}(x, z_m, s_m)$
$f_{\min} \leftarrow \min_{y \in \mathcal{D}} f(y)$
Return $\sum_{m=1}^{M} \mathbb{1}\left[f(y_m) \leq f_{\min}\right] (f_{\min} - f(y_m))$

ProBO Recap

Algorithm 1 ProBO($\mathcal{D}_0, \texttt{inf}, \texttt{gen}$)

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3: $\quad x_n \leftarrow \operatorname{argmin}_{x \in \mathcal{X}} a(x, \texttt{post}, \texttt{gen})$
4: $\quad y_n \sim s(x_n)$
5: $\quad \mathcal{D}_n \leftarrow \mathcal{D}_{n-1} \cup (x_n, y_n)$
6: Return $\mathcal{D}_N$.

Algorithm 2 $a_{\mathrm{EI}}(x, \texttt{post}, \texttt{gen})$ ▷ EI

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5: Return $\sum_{m=1}^{M} \mathbb{1}\left[f(y_m) \leq f_{\min}\right] (f_{\min} - f(y_m))$

Plan: implement pseudocode in Python

Enter BoTorch

Enter BoTorch

Enter BoTorch... and Pyro

Enter BoTorch... and Pyro

New Plan:

1. Implement ProBO in Python
2. Implement in BoTorch + Pyro
3. Compare

Work so far

(very little)

Some BoTorch + Pyro code

```python
class BO_Model(Model):
    def probabilistic_model(marriage=None, age=None, divorce=None):
        a = numpyro.sample('a', dist.Normal(0., 0.2))
        M, A = 0., 0.
        if marriage is not None:
            bM = numpyro.sample('bM', dist.Normal(0., 0.5))
            M = bM * marriage
        if age is not None:
            bA = numpyro.sample('bA', dist.Normal(0., 0.5))
            A = bA * age
        sigma = numpyro.sample('sigma', dist.Exponential(1.))
        mu = a + M + A
        numpyro.sample('obs', dist.Normal(mu, sigma), obs=divorce)

    def condition_on_observations(X, Y):
        # Start from this source of randomness.
        rng_key = random.PRNGKey(0)
        rng_key, rng_key_ = random.split(rng_key)
        # Num samples
        num_warmup, num_samples = 1000, 2000
        # Run NUTS.
        kernel = NUTS(probabilistic_model)
        mcmc = MCMC(kernel, num_warmup, num_samples)
        mcmc.run(rng_key_, marriage=X, divorce=X)
        mcmc.print_summary()
        self.samples_1 = mcmc.get_samples()

    def posterior(X):
        rng_key, rng_key_ = random.split(rng_key)
        predictive = Predictive(probabilistic_model, self.samples_1)
        predictions = predictive(rng_key_, marriage=X)['obs']
        return lambda () -> np.random.choice(predictions)
```

Pyro model

Updating model

Sampling from model

BoTorch model

Work to do

(everything else)

Q&A

Thanks for listening!