Discrete Mathematics

Supervision 10

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17. On regular languages

1. Why can't the automaton $Star(M)$ used in step (iv) of the proof of part (a) of Kleene's Theorem be constructed by simply taking M, making its start state the only accepting state and adding new ε-transitions back from each old accepting state to its start state?

2. Construct an NFA$^{\varepsilon}$ M satisfying $L(M) = L((\epsilon|b)^* aab^*)$ using Kleene's construction.

3. Show that any finite set of strings is a regular language.

4. Use the construction given in the proof of part (b) of Kleene's Theorem to find a regular expression for the DFA M whose state set is $\{0, 1, 2\}$, whose start state is 0, whose only accepting state is 2, whose alphabet of input symbols is $\{a, b\}$, and whose next-state function is given by the following table.

δ	a	b
0	1	2
1	2	1
2	2	1

5. If $M = (Q, \Sigma, \Delta, s, F)$ is an NFA, let $Not(M)$ be the NFA $(Q, \Sigma, \Delta, s, Q \setminus F)$ obtained from M by interchanging the role of accepting and nonaccepting states. Give an example of an alphabet Σ and an NFA M with set of input symbols Σ such that $\{u \in \Sigma^* \mid u \notin L(M)\}$ is *not* the same as $L(Not(M))$.

6. Let $r = (a|b)^* ab(a|b)^*$. Find a regular expression that is equivalent to the complement for r over the alphabet $\{a, b\}$ with the property $L(\sim r) = \{u \in \{a, b\}^* \mid u \notin L(r)\}$.

7. Given DFAs $M_i = (Q_i, \Sigma, \delta_i, s_i, F_i)$ for $i = 1, 2$, let $And(M_1, M_2)$ be the DFA

$$(Q_1 \times Q_2, \Sigma, \delta, (s_1, s_2), F_1 \times F_2)$$

where $\delta : (Q_1 \times Q_2) \times \Sigma \to (Q_1 \times Q_2)$ is given by

$$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$$

for all $q_1 \in Q_1, q_2 \in Q_2$ and $a \in \Sigma$. Show that $L(And(M_1, M_2)) = L(M_1) \cap L(M_2)$.

18. On the Pumping Lemma

1. Briefly summarise the proof of the Pumping Lemma in your own words.

2. Consider the language $L \triangleq \{c^m a^n b^n \mid m \geq 1 \wedge n \geq 0\} \cup \{a^m b^n \mid m, n \geq 0\}$. The notes show that L has the pumping lemma property. Show that there is no DFA M which accepts L.

   Hint: argue by contradiction. If there were such an M, consider the DFA M' with the same states as M, with alphabet of input symbols just consisting of a and b, with transitions all those of M which are labelled by a or b, with start state $\delta_M(s_M, c)$ where s_M is the start state of M, and with the same accepting states as M. Show that the language accepted by M' has to be $\{ a^n b^n \mid n \geq 0 \}$ and deduce that no such M can exist.