Coursework 2

Network Performance—DJW—2005

[Answers are due at noon on Friday 16 December. Explain your answers carefully. The majority of marks are for correct and well-reasoned answers; only a small number of marks are for the programming. This coursework is graded, and is worth 7.5% of your final grade for the course.]

The Copenhagen Telephone Company wishes to offer video calls as well as normal voice calls. A normal voice call takes up one circuit, whereas a video call takes up four. The Company wishes to know what the blocking probability is for each type of call.

Suppose that voice calls arrive at a rate of 2 calls per minute, and that video calls do likewise, and that all calls last an average of 1 minute. Suppose also that call arrivals are a Poisson process, and that call durations have an Exponential distribution. Consider a single link consisting of 12 circuits.

1. Let (T_t, V_t) be the number of voice calls and the number of video calls respectively, at time t. What is the state space, i.e. what are the possible values that the pair (T_t, V_t) can take? In which of these states would a newly arriving voice call be blocked? In which would a newly arriving video call be blocked?

2. Write a simulator for this system. Use your simulator to measure the blocking probabilities for voice and video calls. Report confidence intervals. Explain how your program works, and how you calculated the confidence intervals. Also, for one simulation run, plot a graph which shows the total number of occupied circuits, and the fraction of occupied circuits which are taken up by video calls, as a function of time.

3. Write a program to set up the Q matrix for the Markov process (T_t, V_t) and to calculate the equilibrium distribution. What is the equilibrium probability that an incoming voice call is blocked? (i.e. compute the sum of equilibrium probabilities of all the states in which an incoming voice call would be blocked). What about video calls?

It has been proposed that, in order to reduce the blocking probability for video calls, the Company should introduce a *trunk reservation* scheme which operates as follows. Let C be the number of circuits available, and pick some

trunk reservation parameter r. When a new voice call arrives, admit it only if this would leave at least $C - r$ circuits free. When a new video call arrives, admit it whenever there are enough free circuits.

4. Suppose now that the link consists of 50 circuits, and that the arrival rates of each type of call are 8 per minute. Calculate the blocking probabilities for each type of call, for a range of trunk reservation parameters.

Customers are willing to pay six times as much for a video call as for a voice call. What is the best choice of trunk reservation parameter?

Note. You can generate an exponential random variable as follows. Let U be a random variable, uniformly distributed in $[0, 1]$. Most programming languages provide a primitive for generating such a random variable, e.g. `java.lang.Math.random()` *or* `random.random()` *in Python. Now let $X = -\log(U)/\lambda$. Then X is an exponential random variable with rate λ.*