Summary of examinable material

Large Deviations and Queues—Lent 2005—Damon Wischik

§1. Queues. Standard queueing model. Lindley recursion & queue size function.

§2. Cramér's theorem. *Principle of the largest term (Q1).* Statement of Cramér's theorem, proof of upper bound.

§3. An LDP for queue size. Statement and outline of proof of LDP for queue size. Watermark plots, and idea of effective bandwidth.

§4. Abstract large deviations. Definition of LDP and rate function. Rare events occur in the most likely way lemma. Uniqueness of rate function.

§5. Contraction principle, and extended example. Contraction principle. LDP for geometric and exponential random variables.

§6. LDP tools. Model and rate functions for Gärtner-Ellis theorem, Schilder's theorem, Mogulskii's theorem; heuristic proofs for Schilder's theorem and Mogulskii's theorem. Application of Schilder's theorem. LDP for empirical distribution. Exponential tightness, statement of inverse contraction principle. LDP for product spaces. Model and rate function for Dawson-Gärtner theorem. *Hurstiness (Q5). Restriction of LDP (Q4).*

§7. The queue size function. The spaces $\mathcal{D}_\mu$ and $\mathcal{C}_\mu$ and the norm $||\cdot||$. Continuity of queue size function. Relationship between discrete-time and continuous-time versions.

§8. Large-buffer limit. Outline of proof of sample path LDP for traffic. Application to queues. *Variational calculation of the rate function.*

You should appreciate the use of the contraction principle, and the idea of the principle of the largest term. You should be able to heuristically derive rate functions.

Items in italics refer to exercises.