Visualising Bayesian Probability in the Kalahari

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Abstract  
We describe an approach to interactive visualisation of conditional probability within a Bayesian framework, developed to explore the potential for end-user probabilistic programming languages to be used in educational contexts. We are specifically interested in a setting where there are both educational challenges and distinctive approaches to probabilistic reasoning, working with the Ju|’hoan people who live around the town of Tsumkwe, in the Namibian region of the Kalahari.

1. Introduction  
This paper builds on proposals previously presented at the Psychology of Programming Interest Group (PPIG) in 2019 (Blackwell et al. 2019), setting out an agenda for research in the usability of probabilistic programming languages (PPLs). That paper suggested a number of avenues of enquiry, including investigations that might lead to end-user PPLs, PPLs for educational settings (either to teach basics of machine learning, or basic principles of probability), and a suggestion that research in this field might benefit from a “furthest first” strategy, in which usability research focuses from the outset, on those who are most excluded from educational or professional opportunities.

The research reported in this paper focuses on the last of those questions, and has been conducted in collaboration with a team based in Namibia, including field researchers from the Ju|’hoan people living in and near the town of Tsumkwe, located in the Kalahari desert, where all the authors have also carried out fieldwork for varying lengths of time. We report on experiments with different ways of representing and thinking about probability, in ways compatible with PPLs, and in relation to the specific context and needs of the Ju|’hoan people.

The findings of this work relate to questions in three main areas. In education, our questions relate in particular to teaching of probability and statistics. In human-computer interaction, our questions relate especially to intelligent user interfaces or human-centred AI, as they might be applied within HCI for Development. Finally, in critical AI studies, our questions especially take into account indigenous knowledge practices and decolonising perspectives. Some of those questions will be discussed in far greater detail elsewhere (Bidwell et al. 2022). In this presentation for PPIG, we focus on the use of interactive tools, especially the way that those tools might offer insights for the creation of novel programming technologies in future.

However, we also take the opportunity of engaging with the PPIG community to challenge conventional distinctions between using and programming computers. This considers the ways that some design elements of PPLs may also be the key to empowering those who interact with AI systems, for example in explaining the system behaviour, making it more accountable, or offering potential for more sophisticated or flexible operation and control.

1 Following Covid-19 disruption, it has not been possible to review this manuscript with all authors in advance of the workshop. A future revision will be published once communication among the authors has been able to resume. If reading a version of the paper with this footnote, please contact the first author (afb21@cam.ac.uk) to request the final revision.
As reported in previous research at the PPIG, these capabilities require that system users have the opportunity to engage with their tasks not only at the level of an immediate action (direct manipulation), but also as an abstract formulation that might affect the system behaviour in a range of situations (programming). In keeping with the Cognitive Dimensions of Notations framework and its derivatives (Green 1989, Hadhrawi et al 2017), we recognise that abstract formulation is dependent on notational representation, in order that users are able to construct, reason about, and interact with descriptions of what might happen in the future or in other places (Blackwell 2002).

The main focus of this paper is therefore on the study of interactive notational representations that might in future be used as a starting point to describe machine learning system abstractions in ways that are relevant to the Ju‘hoan people. We also discuss further, at the end of this paper, the important questions of how work of this kind should be considered in relation to other development goals, to the needs and priorities expressed by Ju‘hoan people themselves, and to the problems of conducting research alongside legacies of colonialism. Those questions are much larger than the ones addressed in this paper, and not as familiar to the PPIG audience, so cannot be the main focus of our discussion here. Nevertheless, they should be treated very seriously by anyone considering building on this research, not least the present authors.

2. Background

2.1. Machine learning and (Artificial Intelligence) AI in development settings

A great deal of public attention is being paid to the use of machine learning methods in deployment of computer systems. As with previous generations of computer technology, many of the fundamental questions underlying study in the psychology of programming, also apply to these new methods. These include: understanding the underlying nature and capability of such systems, in order to shape and inform social expectations of how they are applied; empowering end-users to configure, adapt or create such systems, rather than allowing users only to consume functionality created by others; and developing new curriculum materials that will allow young researchers, creators and activists to acquire and advance beyond previous sociotechnical assumptions.

In addition to these established concerns in the psychology of programming, the distinctive system architectures and business environments of contemporary machine learning technologies, introduce new challenges. These arise from i) the very large scale of computation and data collection in deep learning architectures, such that major advances are generally produced by multinational corporations rather than within any national jurisdiction; ii) the economic and labour implications of ghost work (Irani 2015) and surveillance capitalism (Zuboff 2019) that acquire intelligent work from humans, at low or no cost and with limited consent, to be passed off as original intelligent behaviour by machines; and iii) the way that much of the knowledge infrastructure around AI builds on historical legacies that already disadvantage lower-income countries and excluded populations, bringing the only-too-likely outcome that such disadvantages will be further reinforced by future deployment of AI systems.

In presenting this problematic background, we do not of course want to claim that our own small project is going to solve any of these large problems, but rather to illustrate the urgency of looking at AI from perspectives outside of its current framing, and not to be afraid of inspecting even rather foundational concepts, as we will be doing in the remainder of this work-in-progress paper.

2.2. The Kalahari context

The traditional life of the Ju‘hoan people is based on hunting and gathering, with sophisticated data science practices essential to survival and established over millennia (Liebenberg 2013). Hunter-gatherer livelihood is fundamentally dependent on observation of evidence and judgment of likelihood, in the daily decisions of where game might be found, how to avoid dangerous animals, and locating water and edible plants. Similarly, the economy of Tsumkwe in Namibia, is very substantially reliant on data about local wildlife, including the infrastructure that supports state-funded conservation activities, and the tracking and guiding skills that are essential to local wildlife tourism.

Our field research, to be described in more detail elsewhere (Bidwell et al 2022), had been guided by three objectives. The first was to ensure that future machine-learning technologies should consider the needs of, and offer advantages to, communities such as the Ju‘hoan, who are far away from the world
centres of AI research. Our second objective was to consider the ways that school curricula contributing to understanding and appropriation of such technologies will be made accessible to those communities. The third objective was to ensure that the research itself both responded to perspectives in local Namibian schools and, importantly, extended a community-based collaborative relationship established over the past six years between the university-based and Ju|’hoan authors (e.g. https://www.apc.org/en/huinom-project).

2.3. Bayesian statistics and PPLs
None of the present authors are professional statisticians, and we are aware that this paper touches on questions that are the subject of substantial and continuing debates in the history and philosophy of science and mathematics. Many PPL researchers have designed their languages to reflect specific philosophical perspectives in relation to the status and application of statistical models, and these were still the subject of debate at the time this project was conceived (Blackwell et al. 2019). While it is probably fair to report a consensus view among PPL researchers that their languages reflect a Bayesian standpoint, and while the first recognisable PPL implemented Bayesian inference (Spiegelhalter et al 1996) this may not be true of all PPLs. Our own starting point for enquiry has assumed a small number of linked elements that are common both to the design of PPLs and to many approaches to teaching Bayesian statistics:

• The acknowledgement that prior expectations can be expressed mathematically
• The quantification of uncertainty
• The use of calculation to update expectations on the basis of observations
• The value of conditional probability in relating observations to cause and effect reasoning
• The use of Monte Carlo or other simulations to explore likely outcomes in complex models

Our design work as reported below, and the field explorations within which it has been developed, address each of these points to some degree, although in ways that are rather different to conventional formulations in the PPL community.

3. Representational tools for foundations of Bayesian probability
It is notable that the above principles of Bayesian statistics are seldom taught at high school level, even in wealthy countries. University teaching in technology disciplines is changing rapidly to recognise the many advantages of Bayesian over traditional “frequentist” statistics, and it seems likely that secondary school curricula will follow suit over the next 10-20 years. Therefore, in consultation with members of an international mathematics curriculum research project², we asked why it should be the case that curriculum innovations are generally trialled first in wealthy countries, and only gradually disseminated and published as textbooks and assessment standards for countries like Namibia in the Global South. We wanted to explore whether new curriculum content, potentially supported by interactive PPL tools, could be designed first for a context such as the Tsumkwe secondary school, where we were able (although interrupted by the Covid-19 pandemic) to work with the Principal, the head of Mathematics, and several Ju|’hoan students.

Our starting point for practical investigation was a specific device presented in the book Teaching Probability, written by Jenny Gage and David Spiegelhalter, and published by our collaborators at the Cambridge Mathematics project (Gage & Spiegelhalter, 2016). It is important to note that this book is not specifically focused on the teaching of Bayesian probability or PPLs, despite Spiegelhalter’s other research contributions in those fields, but is described by the authors as presenting a hybrid between Bayesian and established curriculum approaches.

Both Gage and Spiegelhalter had experience of teaching in Africa (though at more advanced levels than our own project), and described lesson plans and teaching aids that seemed widely accessible. In particular, we focused on the use of “spinners” as a classroom tool - a simple disc, with differently coloured sectors, and a bent paper clip that is spun around the end of a pen held at the centre of the

² Cambridge Mathematics is an organisation rethinking support for curriculum design in mathematics. https://www.cambridgemaths.org
circle. When the paper clip stops, the colour it is pointing to can be recorded as an observation. Flicking the paper clip to spin multiple times will demonstrate over time that long run observed frequency of where it lands will be in proportion to the relative size of the sectors, and that the likelihood of landing in a particular sector will intuitively be in proportion to the visible size of that sector.

Figure 1 – Cardboard spinners with different-sized shaded regions, showing a bent paperclip being flicked to spin around a ballpoint pen

We made spinners out of cardboard and paperclips as seen in Figure 1, and used these in a range of experimental trials with teachers, students, and field research co-authors. In consultation with representatives of the local community, we discussed examples of probabilities, likelihood judgments and observations that might traditionally be taught for hunting and gathering, and then explored how those traditional practices might be related to representations of likelihood in terms of sector sizes and observed frequencies. Because of our interest in supporting mathematical calculations in Bayesian terms, many of these experiments focused on conditional probabilities, on the way that observed outcomes might modify future expectations, and discussion of how the sector sizes should be drawn if other spinners were to be made that corresponded to prior expectations of the likelihood of some event.

At this point in our research, it became necessary for authors HA and AB to leave Namibia, in response to the pandemic. We therefore explored the potential for the physical apparatus of the spinners, which combines both representational (sector proportions) and simulation (outcome observation) properties, to be replaced with a mobile phone application. Although smartphones are a relatively costly appliance in this low-income setting, commodity Android-based smartphones are available in Tsumkwe, and also in a number of Ju’hoan villages, following an earlier community deployment initiative reported elsewhere (Bidwell et al 2022)

The spinner app, implemented in JavaScript and CSS for portability across a wide range of devices and browsers, renders a number of spinners representing a binary choice with two sectors of different likelihood as we had already used (Figure 2). The spinning paperclip is replaced by an animated rotating needle that spins when it is “flicked” by tapping the touch screen. A simple physical model of rotational momentum and friction results in a realistic simulation of the physical device, which stops at random locations in a manner that convincingly emulates the random behaviour of the paperclip and cardboard.
Figure 2 – JavaScript spinner app, shown after the spinners have been ‘flicked’ 11 times

The spinner app visually records the outcome of each trial in two ways, as seen in Figure 2. The first is rows of tally marks, rendered in colours that can be related to the sectors of the spinner. The tally marks accumulate over time, (with individual marks becoming smaller as necessary to fit on the page), so that the area covered by the accumulated marks of each colour corresponds to the observed outcome frequencies, and can be visually compared to the proportions of the spinner sectors. The second record of outcomes is a pie chart, corresponding to the relative proportion of each outcome. In the long run, the observed proportion of outcome frequencies will approach the relative size of the sectors in the original spinner. Comparison of the pie chart sectors to the spinner sectors, as they are updated and converge toward this long-run expectation, offers an intuitive model of long-run observation in relation to single trial likelihoods.

At the time of writing, these devices have provided an apparatus within which all the core elements of Bayesian probability can be discussed in mathematical terms, and in relation to traditional ways of reasoning. The design approach of presenting a PPL model in terms of simulated spinners for each conditioned variable is compatible with the more powerful end-user PPL BayesDB that is under active development by Mansinghka et al (2015), and our own use of pie charts has been adopted by the designers of BayesDB. We hope that we will find further opportunities to extend our design into more expressive relational models, as informed by ongoing collaboration with PPL developers, and in Namibia and Tsumkwe.

4. Discussion

At this point in our research, we have confirmed that the spinner app does operate acceptably on Android devices that are available in Tsumkwe. We have also refined the design, considering how colours might be applied, how the spinners might be labelled for exploration of the same scenarios that we had used during our field visit, and relating the representational functions of these different elements.

How does this app relate to our project goals, regarding the design of accessible PPLs that might be used to understand or configure machine learning systems? In our field discussions, we have related this work to data science, in relation to the data collection practices of conservation rangers in Tsumkwe, and also to decision processes, in discussion of traditional education and gathering practices.
That work is ongoing, but there are key points of theoretical connection to current research in PPLs for end-user and educational applications.

Firstly, while we adopted the spinners as appropriate teaching aids for use in a low-resource context, the action of the spinners turns out to be an intuitive representation of the Monte Carlo simulation mechanisms that are integrated in many PPLs, including end-user data science tools such as BayesDB. Although AI and machine learning applications are as yet not extensively deployed in Tsumkwe, it was possible to explain the underlying mechanisms of such systems in relation to internal random variables as (notional, invisible) spinners that might operate inside the computer to select probabilistic outcomes, for example when an AI system outputs natural language text based on how likely the individual words are. The role of data in adjusting the likelihood of particular outcomes in a machine learning system can also be understood as arising from observations – in that example, by counting the words that have been seen in previous texts.

In principle, although we did not address this question during our field research, important questions for public understanding of AI, such as data bias in algorithmic decisions, can be directly understood in these terms. Similarly, the role of prior expectation in Bayesian modelling can be understood in relation to the selection of appropriate spinners.

In future research, we plan to explore whether this representation can be extended to express more complex models, including larger numbers of variables, differing numbers of outcomes and continuous variables with nonuniform outcome distributions. These might be supplemented with other representational devices that offer a visual correspondence between outcome distribution and physical simulation, such as the Galton Board that was suggested in the earlier PPIG paper (Blackwell, Church et al., 2019). An example of a prototype application recently implemented by an undergraduate design team in Cambridge can be seen in Figure 3b).

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Figure 3 – Visual simulations demonstrating the interaction of distributions in conditional probability via “cascading” Galton boards. a) whiteboard animation created by Breck Baldwin (screenshot from video available at https://youtu.be/18eLk-fsCI and b) as implemented by Computer Science undergraduates in Team Delta 2021 (screenshot from video available at https://youtu.be/k_ETDeWA4MU)

In anticipating this future work, we wish to emphasise that sketches and prototypes such as these, created in a Western context to support current conventions of describing and representing probability, will not necessarily be appropriate to our field site in the Kalahari. Although the current pandemic has

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3 Other than familiar applications built-in to the Android platform such as predictive text or recommendation facilities in online stores
created additional challenges for our continuing co-design work, the technical experiments reported in this paper can only be considered a starting point for mutual exploration with Ju’hoan colleagues, not a product in themselves.

5. Related Work
In addition to recent work on end-user PPLs, and other educational tools for teaching Bayesian probability, this approach can be compared to two technical education projects familiar to the PPIG audience: the work of Andrea diSessa and colleagues on the use of programmable simulations in physics education, and the work of Ken Kahn and colleagues on the ToonTalk language. Both of these systems share a concern with supporting intuitive reasoning that is linked to mathematical formalism via visual representation of computational processes.

Ken Kahn uses the phrase “animated programming” to make it clear that intuition relating to dynamic behaviour is supported by presenting a programming environment in which elements can be seen moving (Morgado & Kahn 2008). ToonTalk is also a formally specified programming environment, and it supports an interactive transition between the intuitive physical animation and larger-scale computation.

In the metaphorical ToonTalk world, agents can be watched carrying out individual computations as operations on physical objects, for example when Bammer the mouse uses his hammer to add two numbers by knocking their blocks together. The physical space that Bammer and other characters work in is represented as the inside of a house within which the programmer/player can observe the computation being carried out. When you step outside the house, the work continues, but speeded up to normal computation speed – the program is running, but invisibly following the same algorithmic steps that can still be seen whenever you might choose to go back into the house and observe them.

Andy diSessa’s keynote address at PPIG 2018 described the use of programmable simulations, not to teach principles of computation (as in ToonTalk), but to teach basic principles of physics, supporting intuition about the way that different measurable quantities change during the observed behaviour of a typical classroom physics demonstration such as observing acceleration under force (e.g. Sherin, diSessa & Hammer, 1993). diSessa proposed that understanding the computation of those quantities over time offered superior intuition to the conventional algebraic representation using calculus notation.

In programming with physical simulations of probabilistic behaviour, we are similarly presenting an intuition of fundamental principles (here, relating expectation, observation and likelihood) in relation to animated mechanical representations of the same underlying principles. In further developments of this approach, we plan to explore whether simulation of local conditional relations can be scaled up to more complex probabilistic programs, in a manner analogous to stepping outside the ToonTalk house, while maintaining the underlying intuition.

We note in passing that many other educational programming systems, also familiar from previous presenters and keynote speakers at PPIG, have been designed specifically to help students program their own physical, biological or environmental simulations. These include systems such as AgentSheets (Repenning et al., 2000), Squeak etoys / Scratch (Steinmetz, 2002; Maloney et al., 2004), and KidSim / Cocoa (Cypher & Smith, 1995). However, we draw a distinction between the use of visual programming tools to teach the structure of a different domain (which has undoubted education value), and the use of diagrammatic correspondence between the structure of the simulation and notational conventions in order to support mathematical intuitions.

As in the previous section, we note once again that this related work has all been developed in US and UK contexts, and reflects the educational needs and curriculum requirements of those countries. Furthermore, any assumption about the cognitive underpinnings of “intuition” should be treated with great care if it presumes that WEIRD (Western, Educated, Industrial, Rich Developed (Henrich, Heine & Norenzayan, 2010)) habits of thought and ways of teaching are universal. To the extent that diagrammatic representations might potentially rely on embodied conceptual metaphor (Johnson, 2013), we must take care that future designs consider the reality of cognitive variations across cultural and historical divides (Lloyd, 2007), a perspective from which there has been very little attention in current research literature.
6. Conclusions
This work in progress report, continues the research agenda that was originally set out at PPIG 2019 (Blackwell Church et al., 2019), and more briefly discussed in an unpublished panel contribution at PPIG 2020, addressing the topic of Software for Indigenous Communities (presented by the first author together with Jason Lewis – see Lewis et al. 2018, Running Wolf et al 2020) during the conference session on Community Computing.

Although our fieldwork is currently suspended as a consequence of the Covid-19 pandemic, we would welcome feedback on this work from the PPIG community. We are keen to continue our educational experiments, especially the potential for new curriculum perspectives to be influenced by scientific traditions outside the usual Western assumptions. Theories of probability are particularly bound up in ways of seeing the world, as can be seen by the recent debates between Bayesian and frequentist traditions even in Western curricula.

The future of machine learning systems, in which computational tools increasingly operate on probabilistic principles inferred from training data rather than explicitly specified deterministic algorithms, is having many consequences in all countries of the world. Interactive representations that help people from many backgrounds to understand such systems, and even more importantly, to influence, modify, adapt and customise them, could be a valuable contribution to international development, inclusion and equity.

We have set out a direction of travel, with a work in progress report on grounds that are relatively familiar to PPIG. The work reported here is, however, a very small contribution by comparison to much larger and more important questions. National policy in Namibia, and elsewhere in Southern Africa, is seriously concerned with preparation for the opportunities of the Fourth Industrial Revolution, including large national educational initiatives with similar curriculum goals to our own, such as Data Science Nigeria (Adekanmbi, 2020). Despite national policy priorities, serious questions are raised by local researchers with regard to potential inequities arising from these policies (Adams, 2021a).

Furthermore, research with local communities must pay close attention to their own priorities, and much of our discussion in Tsumkwe was concerned with educational needs and practical problems beyond those discussed in this paper. When considering these broader questions in development settings and in relation to indigenous knowledge practices, future research must not only ensure that work with local communities is not extractive, but collaborative, and ideally co-constructed. In working across such divides, it is essential to remain aware of the colonial legacies embedded in AI technologies and research (Adams 2021b), and the need for research methods that acknowledge and advance beyond these where possible (Awori, Bidwell et al, 2016). The CHI community is becoming increasingly alert to the activism that is inherent in designing and deploying software tools, as others in the PPIG community are aware (e.g. Leal et al 2021), and we would not want to give the impression that any of this is easy. Nevertheless, even in such a specialist setting such as PPIG, there are opportunities to step aside from old assumptions and value other kinds of knowledge.

7. Acknowledgements
We are grateful for the time and assistance we received from all those we met in Tsumkwe, especially staff and students at Tsumkwe secondary school, and Chief Bobo and staff of the Ju’hoansi Tribal Authority. Thanks to Bruce Parcher, Candi Miller, David Spiegelhalter, and Breck Baldwin for advice that has contributed to this research, and to PPIG reviewers for detailed and insightful feedback. This project was supported by a grant from the Alborada Foundation.

8. References


