Looking at the λ-calculus as a theory of "programs", we are naturally lead to define what is, in a λ-term, a "stable relevant minimal information" directly observable in the term. This is the token of information which cannot be altered by further reductions but can only be added upon.
If one organizes all the stable relevant minimal information which can be obtained from a computation and orders it according to the order it is produced, then it is quite natural to obtain a tree representation of the information implicit in the original term. There are many tree representations in the literature, depending on the possible notions of stable relevant minimal information, the most commonly used being Berarducci trees [2], Lévy-Longo trees [10], Böhm trees [1], Böhm trees modulo finite or infinite η-expansions (Böhm trees modulo infinite η-expansions are in one-one correspondence with Nakajima trees [9]).

There exist precise correspondences between the tree representations of λ-terms and the local structures of certain topological λ-models, as shown in [11, 1, 8, 4].

In this talk we will focus on the relations between the tree representations of the terms and the observational equivalences inside calculi obtained by adding parallel and non-deterministic features to the pure λ-calculus.

In [11] Wadsworth shows that two λ-terms have the same Böhm tree modulo infinite η-expansions iff they are observational equivalent, assuming that the set of values is the set of head normal forms. The same property holds even considering Böhm trees modulo finite η-expansions and normal forms, as proved by Hyland in [7].
Instead, Lévy-Longo trees correspond to observational equivalence with respect to weak head normal forms in suitably enriched versions of the λ-calculus, see [10, 3, 6].
Lastly we will show that by adding to the pure λ-calculus a non-deterministic choice operator and an adequate numeral system we obtain a calculus which internally discriminate λ-terms having different Böhm trees [5].

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