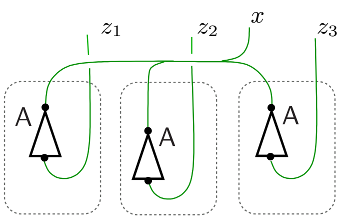
Solutions to the Exercises for Lecture I on

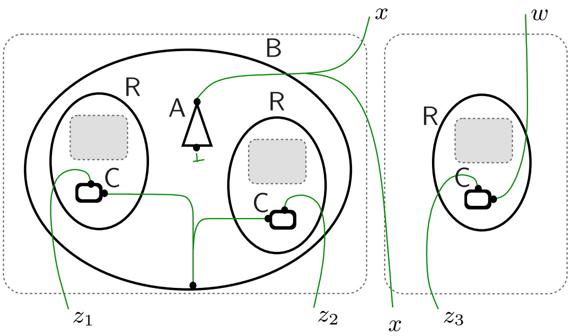
Bigraphs: a Model for Mobile Agents

S1

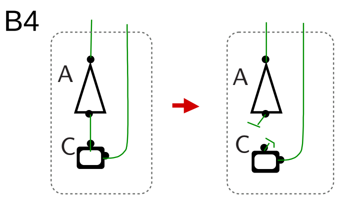
bigraph $D : \epsilon \rightarrow \langle 3, \{x z_1 z_2 z_3\}\rangle$

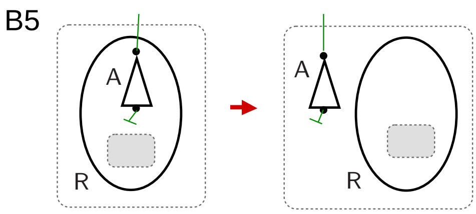


bigraph $F : \langle 3, \{x z_1 z_2 z_3\}\rangle \rightarrow \langle 2, \{xw\}\rangle$



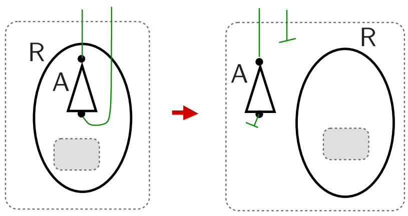
S2 (a)





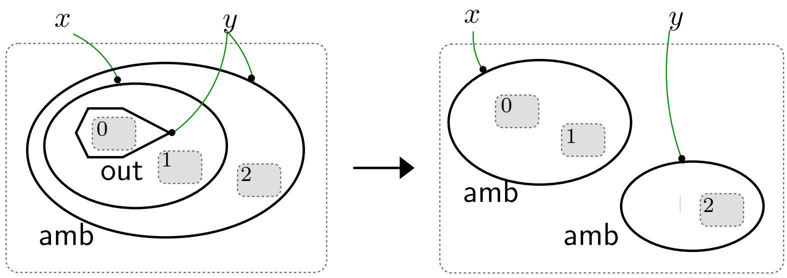
(b) With B1–B5 there are at least the following invariants:
the structure of buildings and rooms is unchanged;
each room contains a single computer, linked to the infrastructure of its building;
each computer is linked to at most one agent, who is in the same room;
there are exactly five agents;
there is at most one conference call in progress;
an agent who leaves a conference call never rejoins it.

B6

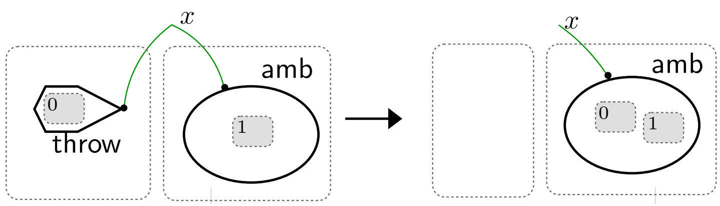


(c) When B4 and B5 are replaced by B6 all the above hold, and also:
an agent cannot unlink from a computer without leaving the room.

S3



$$\mathsf{amb}_y.(\mathsf{amb}_x.(\mathsf{out}_y.d_0 \mid d_1) \mid d_2) \longrightarrow \mathsf{amb}_x.(d_0 \mid d_1) \mid \mathsf{amb}_y.d_2$$



$$\mathsf{throw}_x.d_0 \parallel \mathsf{amb}_x.d_1 \longrightarrow 1 \parallel \mathsf{amb}_x.(d_0 \mid d_1)$$

Solutions to the Exercises for Lecture II on

Bigraphs: a Model for Mobile Agents

S4 Let $F : I \to J$, where $I = \langle m, X\rangle$ and $J = \langle n, Y\rangle$. To prove id_J is a left identity:

$$\begin{array}{rcl} \mathsf{id}_J \circ F & = & \langle \mathsf{id}_n, \mathsf{id}_Y\rangle \circ \langle F^{\mathsf{P}}, F^{\mathsf{L}}\rangle \\ & = & \langle \mathsf{id}_n \circ F^{\mathsf{P}}, \mathsf{id}_Y \circ F^{\mathsf{L}}\rangle \\ & = & \langle F^{\mathsf{P}}, F^{\mathsf{L}}\rangle \\ & = & F\ . \end{array}$$

Proof that id_I is a right identity is similar. To prove composition associative:

$$\begin{array}{rcl} H \circ (G \circ F) & = & \langle H^{\mathsf{P}}, H^{\mathsf{L}}\rangle \circ \langle G^{\mathsf{P}} \circ F^{\mathsf{P}}, G^{\mathsf{L}} \circ F^{\mathsf{L}}\rangle \\ & = & \langle H^{\mathsf{P}} \circ (G^{\mathsf{P}} \circ F^{\mathsf{P}}), H^{\mathsf{L}} \circ (G^{\mathsf{L}} \circ F^{\mathsf{L}})\rangle \\ & = & \langle (H^{\mathsf{P}} \circ G^{\mathsf{P}}) \circ F^{\mathsf{P}}, (H^{\mathsf{L}} \circ G^{\mathsf{L}}) \circ F^{\mathsf{L}}\rangle \\ & & \cdots\cdots \\ & = & (H \circ G) \circ F\ . \end{array}$$

S5 Any bigraph $G : I \to \epsilon$ has an empty place graph, since a non-empty place graph implies at least one root. Also, in the link graph $G^{\mathsf{L}} : X \to \emptyset$ of G, every link is an edge. But if G has empty support then it has no edges, so $X = \emptyset$, $I = \epsilon$ and $G = \mathsf{id}_\epsilon$.

S6 A linking is just a map from inner names to outer names and edges. So a substitution σ from X to Y is just a tensor product of elementary substitutions

$$\sigma \stackrel{\text{def}}{=} y_0/X_0 \otimes \cdots \otimes y_{n-1}/X_{n-1},\ \text{where } X = X_0 \uplus \cdots \uplus X_{n-1} \text{ and } Y = \{\vec{y}\}\ .$$

Now partition Y into $Z = \{y_0 \cdots y_{k-1}\}$ and $W = \{y_k \cdots y_{n-1}\}$. We get any linking λ by setting $/W \stackrel{\text{def}}{=} /y_k \otimes \cdots \otimes /y_{n-1}$, and forming

$$\lambda \stackrel{\text{def}}{=} (\mathsf{id}_Z \otimes /W) \circ \sigma\ .$$

This use of composition is the only way to form a closed link between at least two points.

S7 The expression G can be specialised to the four quoted cases by setting:

(1) $C_1 = \mathsf{id}$ and $I = \epsilon$,
(2) $I = \epsilon$ and $C_0 = \mathsf{id}$,
(3) $C_1 = \mathsf{id}_{K \otimes I}$ and $C_0 = \mathsf{id}_J \otimes C$ (for $F : J \to K$), and
(4) $C_1 = \gamma_{K,I}$ and $C_0 = (\mathsf{id}_J \otimes C) \circ \gamma_{I,J}$.

To show that $g = C \circ a$ implies that a occurs in g, take $F = a$, $I = \epsilon$, $C_0 = \mathsf{id}_\epsilon$. For the converse, assume that $g = C \circ (a \otimes \mathsf{id}_I) \circ C'$; we must find D such that $g = D \circ a$.

Indeed, since a is ground we have $g = C \circ (a \otimes C')$, and since g is ground we have C' ground, say $C' = b$; the result then follows by taking $D = C \circ (\mathsf{id} \otimes b)$.

If E occurs in F and F occurs in G then we have

$$\begin{array}{rcl} F & = & C_1 \circ (E \otimes \mathsf{id}_I) \circ C_0 \text{ and} \\ G & = & D_1 \circ (F \otimes \mathsf{id}_J) \circ D_0 \;, \end{array}$$

So one can deduce 'E occurs in G', i.e. $G = B_1 \circ (E \otimes \mathsf{id}_K) \circ B_0$, by setting $K = I \otimes J$, $B_1 = D_1 \circ (C_1 \otimes \mathsf{id}_J)$ and $B_0 = (C_0 \otimes \mathsf{id}_J) \circ D_0$.

S8 The formation rule Φ for stratified sorting constrains only place graphs, so we can ignore link graphs when checking it. And since a place graph is a forest of trees, if it is augmented with sorts then the forest satisfies Φ iff each tree does.

To see that an identity satisfies Φ, note that a place interface m augmented with sorts is essentially a sequence $\theta_0 \cdots \theta_{m-1}$ of sorts. So in an identity id_I augmented with sorts, each tree whose root has sort θ has just one child—i.e. a site—with sort θ. This clearly satisfies Φ. A similar argument applies to symmetries.

Now suppose that each of F and G, augmented with sorts, satisfies Φ. Each tree of a *tensor product* $F \otimes G$ is just a tree of either F or G, so clearly $F \otimes G$ satisfies Φ. Each tree of a *composition* $G \circ F$ is a tree of G in which each site $i : \theta$ is replaced by some tree of F whose root (with sort θ) is removed. Now every place in $G \circ F$ is either a root or node of G or a node of F; the appropriate condition of Φ can be checked in these two cases separately.

Without sorts in interfaces, a composition $G \circ F$ would be possible in which the sort of a child v of a root i of F need bear no relation to the sort of the parent u of the corresponding site i in G. But u is the parent of v in $G \circ F$, thus the composition may violate Φ.

S9 Extend structural congruence by making the comma that assembles a system both commutative and associative.

Extend the stratified sorting by adding a sort sy for systems and a nullary control $\mathsf{cell} : (\mathsf{sy}, 0)$, with the condition that the children of a sy-node have sort pr. Also add a unary control $\mathsf{throw} : (\mathsf{ch}, 1)$, like send but for distributed communication.

For the translation, we need a third translation function $\mathcal{S}_X[\cdot]$ from systems to $\epsilon \to \langle 1 : \mathsf{sy}, X \rangle$, with the definition

$$\mathcal{S}_X[(\!|P|\!)] = \mathsf{cell}.\mathcal{A}_X[P] \qquad \text{and} \qquad \mathcal{S}_X[S,T] = \mathcal{S}_X[S] \mid \mathcal{S}_X[T] \;.$$

Solutions to the Exercises for Lecture III on Bigraphs: a Model for Mobile Agents

S10 To allow ordinary reactions within a cell to occur in the surrounding system we need rules that allow cells and systems to propagate reaction:

$$\frac{P \longrightarrow P'}{(\!|P|\!) \longrightarrow (\!|P'|\!)} \qquad \frac{S \longrightarrow S'}{S, T \longrightarrow S', T}$$

We also need axiom that allow 'thrown' reactions to occur either within a cell or between cells:

$$(\overline{\overline{x}}.P + A) \mid (x.Q + B) \longrightarrow P \mid Q$$

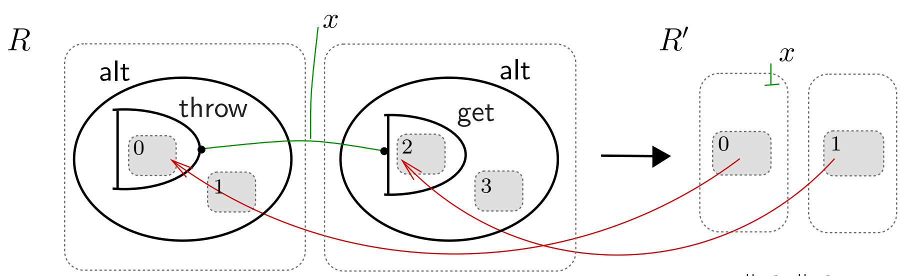
$$(\!|\nu Z((\overline{\overline{x}}.P + A) \mid P')|\!),\ (\!|\nu W((x.Q + B) \mid Q')|\!) \longrightarrow (\!|P \mid P'|\!),\ (\!|Q \mid Q'|\!)$$

where in the second case $x \notin Z \cup W$, and either P' or Q' or both may be absent. The second axiom can be reduced to simpler ones, but only if we allow reactions to be deduced from labelled transitions, rather than defined independently.

S11 $R = \mathsf{alt.}(\mathsf{send}_x.d_0 \mid d_1) \mid \mathsf{alt.}(\mathsf{get}_x.d_2 \mid d_3) \qquad R' = x \mid d_0 \mid d_2.$

$R : \langle \mathsf{p\,a\,p\,a}, \emptyset \rangle \to \langle \mathsf{p}, x \rangle \qquad R' : \langle \mathsf{p\,p}, \emptyset \rangle \to \langle \mathsf{p}, x \rangle$
$d : \epsilon \to \langle \mathsf{p\,a\,p\,a}, Y \rangle \qquad r, r' : \epsilon \to \langle \mathsf{p}, x \uplus Y \rangle$.

S12 The dynamic signature must be extended to assign active status to the control cell. Then we need just one rule for thrown communications:



$x \parallel d_0 \parallel d_1$

$\mathsf{alt.}\,(\mathsf{throw}_x.d_0 \mid d_1) \parallel \mathsf{alt.}\,(\mathsf{get}_x.d_2 \mid d_3)$

The only differences from the previous rule are that throw replaces send, and that using two distinct regions (expressed algebraically by using $\parallel$ in place of $\mid$) we allow the two factors of the rule to be placed in possibly distinct cells. Unlike the second axiom in Solution S10, the rule need not mention the surrounding cell(s).

S13 P and Q have the same *traces*, i.e. they can both perform just two action sequences, $x\,y$ and $x\,z$. But P is deterministic; it can only start by doing x, and then it will do either y or z according to whether the environment (or observer) does $\overline{y}$ or $\overline{z}$. On the other hand, Q is non-deterministic; in doing an x it will randomly choose the first, or the second, alternative. If it chooses the first, and then the environment offers only $\overline{z}$, then there is deadlock.

S14 Consider just one case (the others are similar or easier); suppose $\mu = \tau$ and $P_1 \,|\, Q \xrightarrow{\tau} R_1$ occurs because $P_1 \xrightarrow{\overline{x}} P_1'$ and $Q \xrightarrow{x} Q'$, for some P_1' and Q', and $R_1 = P_1' \,|\, Q'$. Then, because $P_1 \sim P_2$, there exists P_2' such that $P_2 \xrightarrow{\overline{x}} P_2'$, with $P_1' \sim P_2'$. But then, by then same inference rule, we deduce that $P_2 \,|\, Q \xrightarrow{\tau} R_2$, where $R_2 = P_2' \,|\, Q'$. Since $P_1' \sim P_2'$, the pair (R_1, R_2) is in the relation $\mathcal{S}$.

When we have found such a pair for every way the transition of $P_1 \,|\, Q$ can occur, then we have proved that $\mathcal{S}$ is a bisimulation.

S15 In the first case, let u be the shared left-hand send-node. If the get-nodes were shared, say v, then v would be linked to u in $D \circ r$. But this is impossible in $L \circ a$ because the link to v is closed.

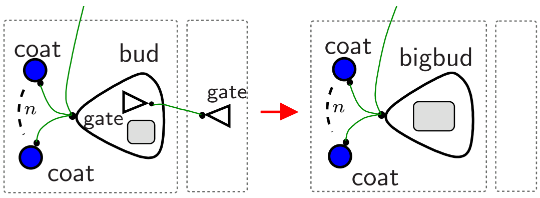
In the second case, let u denote a's left-hand send-node and r's right-hand send-node; then a and r must also share their other send-node—call it u'. If the get-nodes were *not* shared then u' would be linked to two distinct get-nodes in $D \circ r$. But this is impossible in $L \circ a$ because the link to u' in a is closed.

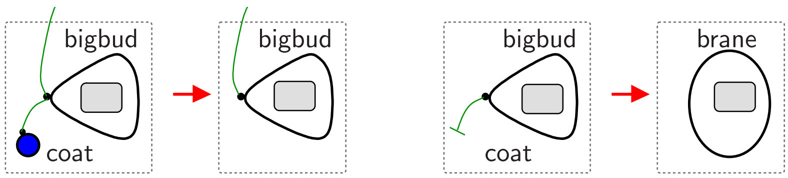
A third possibility as that a and r share no nodes. Then we can achieve $L \circ a = D \circ r$ by defining the bound $(L, D) \stackrel{\text{def}}{=} (\mathsf{id} \otimes r, a \otimes \mathsf{id})$.

Solutions to the Exercises for Lecture IV on Bigraphs: a Model for Mobile Agents

S16 One way is to add a new control ‘bigbud’; a bud in this state allows the coat proteins to be shed before a bud becomes a fully fledged brane. This can be done by extending the set of reaction rules as follows:

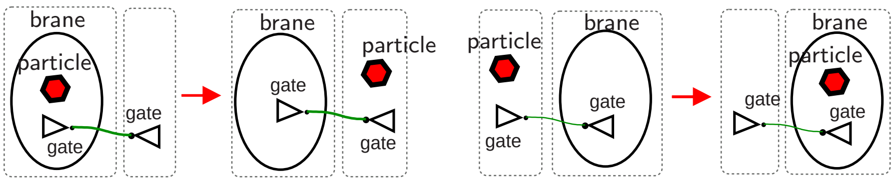
bud fission
(amended)





S17 These two rules are for migration of a particle into, and out of, a bud.

particle migration (amended)



S18 The probability that all but the i^{th} reaction occur at or after t is $\prod_{j \neq i} e^{-\rho_j t}$, which can be expressed as $e^{-\overline{\rho}_i t}$, where $\overline{\rho}_i = \sum_{j \neq i} \rho_j$.

The probability that the i^{th} reaction occurs in the interval $(t, t+\delta t)$ is $-(\frac{d}{dt} e^{-\rho_i t})\delta t$, i.e. $\rho_i e^{-\rho_i t} \delta t$.

Multiplying these two and integrating from 0 to ∞ gives the required result:

$$\int_0^\infty \rho_i e^{-\rho_i t} e^{-\overline{\rho}_i t} dt = \int_0^\infty \rho_i e^{-\rho t} dt = \rho_i / \rho \, .$$

Solutions to the Exercises for Lecture V on Bigraphs: a Model for Mobile Agents

S19

(a) It may be useful for a node to change its control through a reaction. In membrane budding, for example, we may like to say that the bud formed by a membrane is tracked to its parent membrane, and that the new membrane which it eventually becomes also has the same tag.

(b) We have $r = R.(d_0 \otimes \cdots \otimes d_{m-1})$ and $r' = R'.(d'_0 \otimes \cdots \otimes d'_{n-1})$ where $d'_j = d_{\eta(j)}$ for each $j \in n$. So we would extend the tracking map to

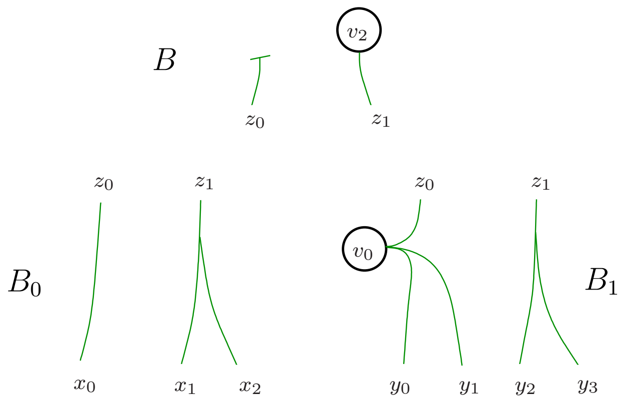
$$\tau \cup \mathsf{Id}_{|d_{\eta(0)}|} \cup \cdots \cup \mathsf{Id}_{|d_{\eta(n-1)}|} \ .$$

(c) We have $(g, g') = (D.r, D.r')$, and we can assume that everything in $|D|$ preserves its identity. So we can extend the tracking map by adjoining $\mathsf{Id}_{|D|}$.

(d) For a reaction sequence $g \longrightarrow\!\!\triangleright\, g_1 \longrightarrow\!\!\triangleright \cdots\cdots \longrightarrow\!\!\triangleright\, g_n = g'$, we can naturally compose the tracking maps of the individual reactions.

(e) Causal structure is often defined on *events*, rather than on components. But it makes some sense to say that an individual component owes its existence to certain past component(s). In this case it would be natural for tracking maps to be relations, not just partial maps.

S20 The RPO is as follows. Note that no links need be closed except by B.



S21 Let $\langle \vec{C}, C\rangle$ be any bound for $\vec{A}$ relative to $\vec{D}$. For a triple $\langle \vec{B}, B\rangle$ to be an RPO we must prove that there exists a unique bigraph E such that

$$E \circ B_i = C_i \ (i = 0, 1) \text{ and } C \circ E = B \ . \qquad (*)$$

For this purpose, first consider the place graph RPO $(\vec{B}^{\mathsf{P}}, B^{\mathsf{P}})$. We easily see that $(\vec{C}^{\mathsf{P}}, C^{\mathsf{P}})$ is a bound for $\vec{A}^{\mathsf{P}}$ relative to $\vec{D}^{\mathsf{P}}$; hence there exists a unique place graph E^{P} such that $E^{\mathsf{P}} \circ B_i^{\mathsf{P}} = C_i^{\mathsf{P}}$ $(i = 0, 1)$ and $C^{\mathsf{P}} \circ E^{\mathsf{P}} = B^{\mathsf{P}}$. Now considering the link graph RPO $(\vec{B}^{\mathsf{L}}, B^{\mathsf{L}})$, a unique link graph E^{L} exists with an analogous property in the link graphs.

Also E^{P} and E^{L} have the same node-set. To see this, note that $|C^{\mathsf{P}}| \uplus |E^{\mathsf{P}}| = |B^{\mathsf{P}}|$, and similarly $|C^{\mathsf{L}}| \uplus |E^{\mathsf{L}}| = |B^{\mathsf{L}}|$. We already know that the combinations $C = \langle C^{\mathsf{P}}, C^{\mathsf{L}} \rangle$ and $B = \langle B^{\mathsf{P}}, B^{\mathsf{L}} \rangle$ exist. Hence $E \stackrel{\text{def}}{=} \langle E^{\mathsf{P}}, E^{\mathsf{L}} \rangle$ also exists.

It is easy to check that E has the property $(*)$. To see that it is unique with this property, suppose that some bigraph F has the property. Then, by considering its constituents and the given place graph and link graph RPOs, we find by the uniqueness of E^{P} and E^{L} that $F^{\mathsf{P}} = E^{\mathsf{P}}$ and $F^{\mathsf{L}} = E^{\mathsf{L}}$; hence $F = E$. This completes the proof.

S22

(1) We have $b'_0 = E_0 \circ r'_0$. Also, the result of the transition of a_0 is $a'_0 = D_0 \circ r'_0$. So $b'_0 = E \circ a'_0$. Now the result of the transition of b_1 is $b'_1 = E_1 \circ r'_1 = E \circ a'_1$. Thus, with $a'_0 \sim a'_1$ and taking $C' = E$, we have what is required.

(2) We know E_0 is active, by the assumed transition of $C \circ a_0$. Hence D_0 is active, and E is active at $\tilde{\imath} = \mathsf{width}(D_0)(m_0)$, where m_0 is the width of the outer face of r_0. Now, from the existence of the transition of a_1 we have D_1 active. Also $\tilde{\imath} = \mathsf{width}(D_1)(m_1)$, where m_1 is the width of the outer face of r_1. Since also E is active at $\tilde{\imath}$, we deduce that $E_1 = E \circ D_1$ is active, as required.