12 Quantum Computing (pm830+sjh227)

(*a*) Consider the following Hermitian matrix.

$$\mathrm{H} = \frac{\pi}{2\sqrt{2}} \begin{bmatrix} \sqrt{2}-1 & -1 \\ -1 & \sqrt{2}+1 \end{bmatrix}$$

(*i*) Define what it means for a matrix to be Hermitian. What property do the eigenvalues of Hermitian matrices have? [2 marks]

(*ii*) Verify that $\begin{bmatrix} 1 \\ \sqrt{2}-1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -\sqrt{2}-1 \end{bmatrix}$ are eigenvectors of H and find its eigenvalues. [4 marks]

(*iii*) Find the eigenvectors of the matrix $e^{-i\mathrm{H}t}$, where t is a positive real number. [1 mark]

(*iv*) What kind of matrix is $e^{-i\mathrm{H}t}$? [1 mark]

(*b*) Quantum states may be perfectly distinguished if they are orthogonal. Let $|\psi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$, $|\phi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$ and $|\omega\rangle = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(*i*) Show that $|\psi\rangle$ and $|\phi\rangle$ are orthogonal, and that $|\psi\rangle$ and $|\omega\rangle$ are not orthogonal. [3 marks]

(*ii*) Find a unitary matrix which, when applied to the states $|\psi\rangle$ and $|\phi\rangle$, allows them to be perfectly distinguished by measurement in the computational basis. [2 marks]

(*iii*) If one attempts to distinguish the non-orthogonal states $|\psi\rangle$ and $|\omega\rangle$, what is the probability of correctly inferring the state if the optimal strategy is used? [2 marks]

(*iv*) Give two measurement bases to distinguish $|\psi\rangle$ and $|\omega\rangle$ such that in each basis one measurement outcome has the property that the state is known with certainty. In your answer you should indicate the measurement outcome and the state for each measurement basis. [5 marks]