COMPUTER SCIENCE TRIPOS Part II – 2025 – Paper 9

11 Quantum Computing (pm830)

Quantum error correction.

(a) Define the 3 qubit bit flip code. What is the logical state encoded in this code? [2 marks]

(b) What is a logical X operator for this code? What are the different possible logical Z operators for this code? [3 marks]

(c) Prove or disprove: when a general single qubit state $a|0\rangle + b|1\rangle$ is encoded using the 3 qubit bit flip code, we will violate the no cloning theorem because the encoding circuit of the code will copy the qubit's state onto two others. [2 marks]

(d) The following defines a 4-qubit error detecting code that encodes the state of two data qubits using four physical qubits. The codewords corresponding to the four basis states of the two data qubits are:

$$|00\rangle \rightarrow (1/\sqrt{2})(|0000\rangle + |1111\rangle),$$

$$|01\rangle \rightarrow (1/\sqrt{2})(|1100\rangle + |0011\rangle),$$

$$|10\rangle \rightarrow (1/\sqrt{2})(|1010\rangle + |0101\rangle),$$

$$|11\rangle \rightarrow (1/\sqrt{2})(|0110\rangle + |1001\rangle).$$

Using the codewords corresponding to $|00\rangle$ and $|01\rangle$ basis states, show why this code can detect a single qubit bit flip, but cannot correct it. [3 marks]

(e) For the 4-qubit code in (d), using the codewords corresponding to $|00\rangle$ and $|01\rangle$ basis states, show why this code can detect a single qubit phase flip, but cannot correct it. [2 marks]

(f) For the 4-qubit code in (d), design a circuit that can detect a single qubit bit flip error in the codeword qubits. Use only 1 ancilla qubit to perform error detection. [4 marks]

(g) For the 4-qubit code in (d), design a circuit that can detect a single qubit phase flip error in the codeword qubits. Use only 1 ancilla qubit to perform error detection. [4 marks]