7 Further Graphics (aco41)

A height-field is defined as a surface in 3D over the xy-plane such that for each x and y coordinate, the z coordinate is given by $h(x, y)$. For a particular height-field, assume the partial derivatives $\frac{\partial h}{\partial x}(0,0) = \frac{\partial h}{\partial y}(0,0) = 0$.

(*a*) Write an implicit function for the height field. [2 marks]

(*b*) What is the surface normal at $x = 0, y = 0$? [2 marks]

(*c*) Write an implicit function for the tangent plane at $x = 0, y = 0$. [2 marks]

(*d*) Recall that normal curvature at a surface point for a direction is defined as the curvature of the curve formed by intersecting the surface with a particular plane. The plane intersects the surface at $x = 0, y = 0$, and is given in the direction $\mathbf{d} = [d_x, d_y, d_z]$, where $\mathbf{d}$ lies on the plane tangent to the surface. Write an explicit equation for the points on the plane used to compute the curvature. [6 marks]

(*e*) Write an expression for the curve given by the intersection of the above plane with the surface. [4 marks]

(*f*) For $\mathbf{d} = [1, 0, 0]$, derive the condition on the height-field such that we can use the expression $\frac{\partial^2 \mathbf{c}(t)}{\partial t^2} = \kappa(t)\mathbf{n}(t)$ to derive the curvature $\kappa(t)$ of the curve $\mathbf{c}$ in the previous question. [4 marks]