

10 Discrete Mathematics (fms27)

Let $\Sigma = \{0, 1\}$; $A = \{\epsilon, 011, 011111, 011011\}$; $B = \{1, 1111\}$.

- (a) Let $L_{(a)}$ be the subset of Σ^* defined by the following rules. Refer to these rules by the numbers 0 to 3 when producing a derivation.

$$\frac{}{01} \quad \frac{10x}{x01x} \quad \frac{x10}{01xx} \quad \frac{x1}{0}.$$

- (i) Give a derivation for the shortest string in $L_{(a)}$. [1 mark]
- (ii) Give a derivation for the longest string in $L_{(a)}$. [1 mark]
- (iii) Is $L_{(a)}$ regular? [1 mark]
- (iv) Prove your answer to part (a)(iii). [1 mark]
- (b) Produce a regular expression that recognises at least all the strings in A . [Note: half marks if longer than 6 characters.] [2 marks]
- (c) Produce a regular expression r that recognises at least all the strings in A but none of the ones in B . [Note: half marks if longer than 9 characters.] [2 marks]
- (d) Produce a regular expression that recognises all the strings in A and no others. [Note: half marks if longer than 16 characters.] [4 marks]
- (e) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(e)} = \{s \in \Sigma^* | s \text{ has an equal number of occurrences of the substrings } 01 \text{ and } 10 \text{ (overlaps allowed)}\}$, or prove it cannot be done. [Note: state diagrams that are not DFAs will earn no marks.] [4 marks]
- (f) Build the state diagram of a Deterministic Finite Automaton with at most 5 states that recognises $L_{(f)} = \{s \in \Sigma^* | s \text{ has an equal number of occurrences of the substrings } 01 \text{ and } 10 \text{ (overlaps not allowed)}\}$, or prove it cannot be done. [Note: state diagrams that are not DFAs will earn no marks.] [4 marks]