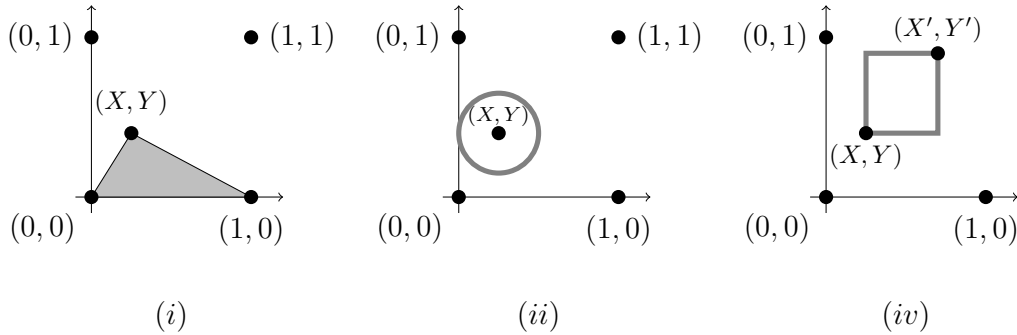


6 Introduction to Probability (mj201+tms41)

- (a) Let $X \sim \text{Uni}(0, 1/2)$ be a uniform continuous random variable. What are $\mathbf{E}[X]$ and $\mathbf{V}[X]$? [3 marks]
- (b) Let $X \sim \text{Uni}(0, 1/2)$ and $Y \sim \text{Uni}(0, 1/2)$ be two independent uniform continuous random variables, and define $Z = \min(X, Y)$.
- (i) What is the cumulative distribution function of Z ? [2 marks]
- (ii) What is $\mathbf{E}[Z]$? [3 marks]
- (c) Let $X \sim \text{Uni}(0, 1)$ and $Y \sim \text{Uni}(0, 1)$ be two independent uniform continuous random variables.



- (i) Consider a random triangle between the three points $(0,0)$, $(1,0)$ and (X,Y) , as illustrated in the figure above. What is the expectation of the area? [2 marks]
- (ii) Now consider a random circle with center (X,Y) such that the circumference is as large as possible but remains within the unit-square $[0,1]^2$ (see figure). What is the expectation of the circumference? [4 marks]
- (iii) Based on your answer from (c)(ii), what can you conclude about the expectation of the area of this circle? [2 marks]
- (iv) Additionally, let $X' \sim \text{Uni}(0, 1)$ and $Y' \sim \text{Uni}(0, 1)$ be two uniform continuous random variables and assume X, Y, X', Y' are mutually independent. Consider a random rectangle with corner points (X, Y) and (X', Y') , which are diagonally opposite. What is the expectation of the circumference? [4 marks]