

8 Algorithms (fms27)

Reminder:

$$f(n) \in \Theta(g(n))$$

$$\iff$$

$$\exists n_0, c_1, c_2 \in \mathbb{R}_{>0} \text{ such that } \forall n > n_0 : 0 < c_1g(n) \leq f(n) \leq c_2g(n).$$

- (a) For the bubblesort algorithm, state its best-case  $\Theta$  complexity and describe, for any given input of arbitrary size  $n$ , one permutation that would trigger this best-case behaviour. Then give the corresponding permutation of  $[0, 1, 2, 3, 4, 5, 6, 7, 7, 7]$ . Then repeat the above for the worst case: state the  $\Theta$  complexity, saying when it is achieved, and exhibit as a concrete example a permutation of the 10 numbers given. [4 marks]
- (b) Repeat Part (a) for the heapsort algorithm. [4 marks]
- (c) Repeat Part (a) for the basic quicksort algorithm, where the pivot is simply chosen as the last element in the range. [4 marks]
- (d) Write clear and efficient pseudocode to eliminate all duplicates from a linked list of  $n$  elements, without changing the order of the remaining elements. Then derive and justify its  $\Theta$  complexity. [4 marks]
- (e) [*Hint:* The  $\Omega$  notation, like the  $\Theta$  notation, is typically used to describe the asymptotic behaviour of a worst-case cost function  $f(n)$ . When we say, by extension, that a certain task has a complexity bound of  $\Omega(g(n))$ , we mean that this bound applies to the worst-case cost function of every possible algorithm that could solve that task.]

Give a formula for  $f(n) \in \Omega(g(n))$ , in a format similar to that of the Reminder above, and briefly explain it. Then derive, with a clear justification, a tight  $\Omega$  complexity bound for the task of eliminating all duplicates from a linked list of  $n$  elements. [4 marks]