1995 Paper 5 Question 10

Foundations of Functional Programming

$$\begin{array}{lrcl} \text{Let} & A & \equiv & \lambda x\ y.\ y\ (x\ x\ y) \\ & \Theta & \equiv & A\ A \\ & suc & \equiv & \lambda n\ f\ x.\ f\ (n\ f\ x) \\ & true & \equiv & \lambda x\ y.\ x \\ & false & \equiv & \lambda x\ y.\ y \end{array}$$

Reduce each of the following λ-terms to normal form (if possible) and to head normal form (hnf) (if possible).

$$\begin{array}{llll} \Theta & suc & \Theta & (suc\ x) \\ \Theta & true & \Theta & false \\ \Theta & (\lambda x.\ x\ x) & \Theta & (\lambda x.\ f\ x\ x) \end{array}$$

[12 marks]

If M has no hnf then $M[N/x]$ has no hnf, for all N. Use this fact to show the following:

If M has no hnf then $M\ N$ has no hnf, for all N. [8 marks]