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On the calculation of explicit polymetres

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On the Calculation of Explicit Polymetres

Abstract

Computer scientists take an interest in objects or events that can be counted, grouped, timed and synchronised. The computational problems involved with the interpretation and notation of musical rhythm are therefore of particular interest, as the most complex time-stamped structures yet devised by humankind are to be found in music notation. These problems are brought into focus when considering *explicit polymetric notation*, which is the concurrent use of different time signatures in music notation. While not in common use, this notation can be used to specify complicated cross-rhythms, simple versus compound metres, and unequal note values without the need for tuplet notation. From a computational point of view, explicit polymetric notation is a means of specifying sychronisation relationships amongst multiple time-stamped streams. Human readers of explicit polymetric notation use the time signatures together with the layout of barlines and musical events as clues to determine the performance. However, if the aim is to lay out the notation (such as might be required by an automatic music notation processor), the locations of barlines and musical events will be unknown, and it is necessary to calculate them given only the information conveyed by the time signatures. Similar problems arise when attempting to perform the notation (*i.e.* animate the specification) in real-time. Some problems in the interpretation of explicit polymetric notation are identified, and a solution is proposed. Two different interpretations are distinguished, and methods for their automatic calculation are given. The solution given may be applied to problems which involves the synchronisation or phase adjustment of multiple independent threads of time-stamped objects.

Introduction

Computer scientists generally take an interest in objects or events that can be counted, grouped, timed and synchronised. The computational problems involved with the interpretation and notation of musical rhythm are therefore of particular interest, as the most complex rhythmic structures yet devised by humankind are to be found in music notation. These problems are brought into focus when considering the construction and interpretation of multiple different rhythmic patterns in effect concurrently. The accurate interpretation of such complex rhythmic structures has always been one of the composer's most pressing problems, and how to represent them can be the most perplexing of all compositional procedures. While recognising the validity of a compositional commitment to purposeful ambiguity and temporal approximation, it is likewise imperative to provide a theory for the intelligible, consistent and readily comprehensible interpretation of complex rhythmic patterns when it is the composer's intention to convey such.

In this paper I consider the computational problems of interpreting multiple rhythmic structures governed by different time signatures at the same time. Following the nomenclature of Read (1980), the use of different time signatures in effect simultaneously is called explicit polymetric notation. Although this device is not common, composers have used explicit polymetric notation for hundreds of years with varying interpretations. Interpreting the notation has relied on arbitrary conventions with no consistent mathematical basis, and has in some situations required much effort on the part of the reader in discerning the composer's intention. Such a state of affairs can lead to practical difficulties. For example, in one passage discussed below, the composer has implied the use of two different – but mutually compatible – interpretations of explicit polymetre on the same page. How to read and perform the piece is well known and understood intuitively, but the compatibility of the different interpretations of the notation can only be made explicit computationally in the context of a theory that admits the existence of multiple interpretations in the first place. Being presented with such complexity, the reader requires a consistent account of one or more interpretations to determine precisely how the explicit polymetre is to be understood. Such a theory might take the form of a procedure for determining the starting time and duration of all musical events, and such information is also required for automatic music processing tasks such as calculating the correct layout of notation.

I use the following terminology: a *musical event* is a note or rest with a certain duration to be performed at a certain time (or printed in a certain place); a *note value* refers to the orthographic notation of an event (crotchet, quaver, *etc.*) independent of its temporal properties; *unequal notation* is where the usual duration relationships between note values are modified (for example, where 3 quavers = 1 crotchet). Unequal notes are called irrational notes by some authors, but this term can be misleading: in a mathematical sense, the duration of unequal notes often can be specified by a ratio of integers, so this is properly termed rational, not irrational.

Explicit Polymetric Notation

Read (1980) gives a comprehensive survey of both implicit and explicit polymetric notation. Simultaneous multiple tempi and rhythm amongst one or more musical strands may be implied within a single meter by means of cross-accenting and phrasing, or polymeters may be made explicit by the employment of several different time signatures in effect concurrently. While fully justifying the use of explicit polymetre on the basis of its 'incontestable pragmatism', its use in the 'skilful blending of disparate musical ideas' and its use in representing musical mood (*e.g.* the 'dramatic juxtaposition of emotional opposites'), Read's admirable survey and analysis stops short of containing his observations within a theory that makes explicit the underlying assumptions required for the calculation of timing relationships in passages governed by explicit polymetre. Although to the music analyst such a theory may be so obviously implied by the observations as to not merit further comment, I shall show that it is necessary to distinguish between two different interpretations of polymetric notation in order to fully specify the timing of musical events. Later Read does distinguish two 'polymetric premises', but these concern whether the different time signatures are of greatly differing denominator or whether different tempo indications are specified. Without further detail, such observations, although valid, do not carry the predictive value or computational utility of the theory required here.

I suggest that that there are two main interpretations of polymetric notation, distinguished by whether the intention is

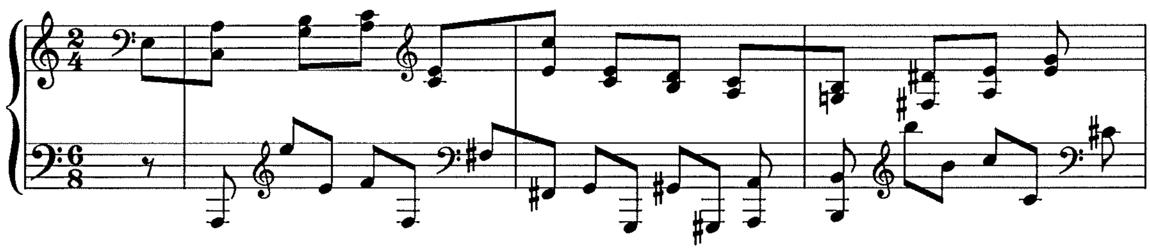
I. to notate unequal note values without using tuplet notation, or

II. to specify cross rhythms using equal note values.

It is important to be able to distinguish which interpretation is being adopted in a musical design, as it turns out that the computational implications of the two interpretations are quite different. I believe that all known uses of explicit polymeter may be explained by one or other of these interpretations. In a few exceptional cases it turns out that different time signatures are being used to notate a piece which is in fact not polymetric or that uniform time signatures could be used without loss of generality.

Interpretation I.

The first interpretation is illustrated by the following extract from Variation V. of Johannes Brahms: *Variazioni*, Op. 35 of 1866 (in all the musical examples that follow, only timing and pitch information is shown; articulation and phrasing marks are suppressed):



Here the intention is to notate a 4-against-6 rhythm, but without using triplet notation for the notes in the lower staff. Instead, opposing time signatures of 2/4 and 6/8 are used with coinciding barlines. The coincidence of barlines together with the spacing of the notes suggests that note values in the upper staff are in a nonunity proportion to the note values on the lower staff. The constant of proportionality x is easily determined from the time signatures by the equation $(2/4)x = (6/8)$, or $x = 1.5$. Thus, a quaver on the upper staff has a duration 1.5 times the duration on the lower staff. This proportional relation is properly termed *sesquialtera*, and it is clear that Interpretation I. pertains. At this point, analysts might rest satisfied, but from the computational viewpoint, two issues are unresolved:

(*a*) The fact that a nonunity constant of proportionality exists is not sufficient to fully specify the performance or layout of this piece. There is the question of whether the durations of notes on the upper staff are to be understood as 1.5 times the note values on the lower staff, or whether the durations on the lower staff are understood to be two-thirds the note values on the upper staff. From a mathematical point of view both statements are equivalent, but from the standpoint of notation and performance these statements have different implications as to layout, tempo and phrasing.

(*b*) If the actual purpose of calculating note durations is to determine the spacing of notes and placement of barlines in the first place, the only available information is the concurrent presence of two or more different time signatures. Although the human reader might use the barlines and note spacing as clues for discerning the timing relationships, the placement of notes and barlines is in fact a *consequence* of a previous assumption about how timing is to be calculated. In the absence of such an assumption (such as the information that Interpretation I. is to be applied in the case of the Brahms example), layout cannot be computed at all.

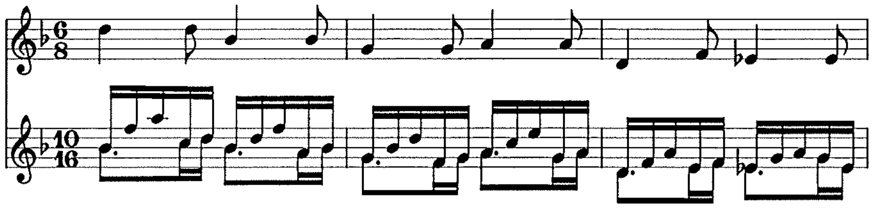
Moreover, the answer to the first issue is not determined simply by the presence of two different time signatures, but by an assumption as to which signature is taken as a reference point. This issue is sharpened particularly when one polymetric scheme is followed by a different one, possibly causing the reference of the underlying 'beat' to change from one part to another.

A further but minor point to consider is that the quaver rest shown in the anacrucis of the lower staff ought to be shown dotted to be entirely consistent with the principle of *sesquialtera*. From the human reader's viewpoint, such a detail is irrelevant because the rest can be assumed as pertaining to the bar, and so the notation of its precise duration is not an issue. However, for the purposes of automatic processing, no such assumptions are necessarily known or uniquely represented solely from the information given in the score, and it would be necessary to specify the correct value of the rest, even if a dot is not shown.

The first interpretation applies also to this extract (bars 56-60) from Paul Hindemith: *Konzert* for trumpet and bassoon with strings (1949):

Here the *sesquialtera* relation is retained over several changes of time signature, but a triplet is also used when the intention is for the rhythms to break the pattern and coincide. Again because the interpretation is clearly notated by time signatures, coinciding bars and explicit triplet notation, this does not present computational problems provided that Interpretation I is assumed. The note values of the instrumental parts are always two-thirds the duration of the note values in the grand staff, regardless of the change of signature, except where unequal notation is used.

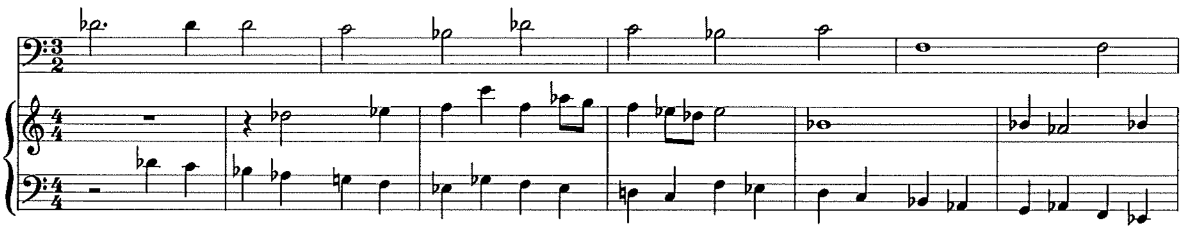
Interpretation I. is of course, not restricted to *sesquialtera* proportions, as is illustrated by this extract from Gabriel Pierné: violin sonata Op. 36:



Although this passage has been used as a textbook example of additive rhythm (Sachs, 1953), it is relevant here as an example of explicit polymetre, where the notation implies that a quaver on the lower staff has a duration 1.2 times the duration of a quaver on the upper staff (that is, $(6/8)\,x = (10/16)$, or $0.75x = 0.625$, or $x = 1.2$).

Interpretation II.

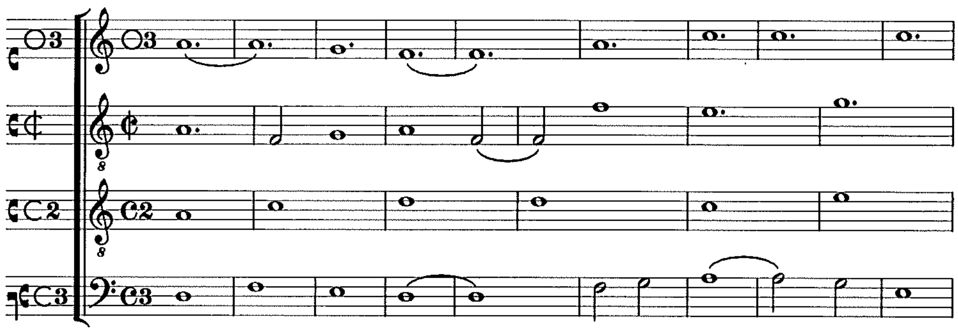
An example of the second interpretation is taken from Hindemith: *Mathis der Maler* (1937):



Although different time signatures are used, it is clear from the positioning of barlines and events that note values are equal thoughout the system (that is, a minim on the first staff is of the same duration as a minim on the grand staff). The interpretation is different from that discussed previously, because any constant of proportionality that may be derived from the time signatures cannot be used to calculate the relative durations of musical events. Here the durations of note values agree in all staves, but because the time signatures specify different numbers of beats per measure, barlines do not coincide until six beats have elapsed (6 is the least common multiple of 2 and 3, the numerators of the time sigatures). Here the intention appears simply to control the timing of primary stresses.

The examples thus far assume one or the other interpretation to establish the correct timing relationship. In Interpretation I, the duration of musical events (notes and rests) is determined by a constant of proportionality derived from the time signature. In the previous examples, barlines have coincided, but this is a consequence of that fact that measures are of equal duration. In Interpretation II, the note values are equal, but barlines are shown not to coincide when the measures are of unequal duration.

From a computational viewpoint, barlines are not strictly necessary. Coincidence of barlines is a consequence, and not a determiner of the interpretation. To illustrate the non-coincidence of barlines even in Interpretation I., consider modern editions of early polyphony, as in the following rendering of the Busnois hymn 'Conditor alme siderum' (colouration not shown, barlines as in Seay, 1971):



Against the perfect longa mensuration of the Cantus the proportional relationships are Altus: *tempus imperfectum diminutum*; Tenor: *tempus imperfectum, proportio dupla*; Bassus: *tempus imperfectum, proportio tripla.* On the assumption of Interpretation I, note values need to be multiplied as follows (relative to the 1.0 of the Cantus): Altus: 9/6 = 1.5; Tenor: (9/4) = 2.25, Bassus: 9/6 = 1.5. In fact there are two distinct Interpretation I schemes operating concurrently here: the Cantus and Bassus form one scheme, and the Altus and Tenor form another. No note values are equal between staves, so Interpretation II clearly does not apply. However, the two schemes are barred 3-to-2 against each other in a way suggestive of an Interpretation II timing relationship. The suggestion is misleading, which demonstrates that barline coincidence may not be used as a general rule to determine which interpretation applies.

A modern monometric setting in 3/4 might gain clarity and sacrifice rhythmic nuance, but from a computational viewpoint it is identical to the polymetric version:

One of the most frequently cited examples of polymetric notation happens to be one in which Interpretations I and II are both identifiable in the same passage. The following is an extract from the finale to Act I of W.A. Mozart: *Don Giovanni*:

The events governed by the 2/4 and 3/4 signatures are to be read in Interpretation II (equal note values, with differing beats per bar), while the events governed by the 3/8 are to be read in interpretation I (unequal note values) in order to implement an alternative to notating triplets in 3/4. An indication of the problems involved with calculating the layout of this passage is illustrated by the 'raw' note list before the timing information has been calculated:

Implementation

Calliope (Clocksin, 1994) is a computer program for the automatic layout of musical notation. Calliope uses a 'what you see is what you get' style of interaction, and works at the conceptual level of the user's musical intentions. In particular, most layout and formatting tasks are done automatically according to conventional notational practices. This frees the user from making detailed graphical adjustments, reduces the amount of time required to input and edit a score, and reduces the amount of training required to use the system. Calliope runs under the NeXTStep system and was used for the preparation of the musical notation in this paper.

Calliope is unique among music processing programs in providing the means for automatic layout of scores that use explicit polymetric notation. In keeping with the design intentions of Calliope, only the minimum of extra specification is required. The first version of the implementation took this principle to extremes by deducing note value proportions automatically from the given time signatures. For example, if two signatures were 6/8 and 10/16, Calliope would first consider the signatures as ratios and nominate the larger signature as the reference (here 6/8 > 10/16), then find the constant of proportionality relative to the reference (here 1.2) and then multiply the durations of events governed by the smaller signature by the constant of proportionality. Although this rule could be operated without any additional information and gave correct answers much of the time, it had three shortcomings: (a) the choice of reference signature was arbitrary and might not reflect musical sense, (b) it ignored the possibility of Interpretation II (in which the constant of proportionality must equal 1.0 regardless of time signature), and (c) could not handle the situation of three or more signatures in which there is more than one reference.

The current implementation addresses these shortcomings and uses the minimum of extra specification. It is up to the user to give two pieces of information: (*a*) whether a time signature is to be used to specify non-equal note values, and (*b*) which of two formatting algorithms is to be used. One algorithm forces barlines to coincide, and the other makes barlines coincide only when they fall at the same time. Which algorithm to use may be specified on a system-by-system basis. These two pieces of information provide a means not only for notating Interpretations I and II, but are flexible enough to permit the notation of both Interpretations I and II on different staves in the same system, as is required in the Mozart passage.

With each time signature is associated a *proportionality ratio* which is 1:1 by default, but which may be redefined to any ratio. This value is specified as a pair of integers (a ratio) to avoid the use of repeating decimals (such as 1/3, which is the repeating decimal 0.333...). It is the user's responsibility to determine which interpretation is used and what the ratios should be. For example, in the Mozart passage, the user simply sets the proportionality ratios of the two 3/8 signatures to 2/3, and sets the system to use the non-coinciding bars algorithm.

The ratio is not visible to the reader of the score, but may be inspected and changed by the notator.

Before any operation requiring timing information (namely, formatting and playback), Calliope automatically multiplies the value of each event governed by the time signature by the proportionality ratio. This general method serves to implement all uses of explicit polymetre, and is also useful in specifying the layout and performance of white mensural notation (Aldrich, 1966), in which musical events are conventionally placed at roughly half their normal spacing. This method of calculating event durations was used for the automatic layout of the examples shown in this paper. The layout of each example was calculated automatically, and was not retouched manually.

Conclusion

Explicit polymetric notation has many potential uses, but its undeserved obscurity may stem from an inherent ambiguity of interpretation. The composer might have a polymetric scheme in mind, but the performer must rely on the placement of musical events and barlines to disambiguate the notation. Such information is not available if it is the aim to determine the placement of events and barlines automatically given only a time signature and a sequence of note values. By making explicit the definitions of two distinct interpretations, it is possible to calculate event durations precisely, and thereby enable the automatic processing of polymetric notation with a minimum of additional specification.

Acknowledgements

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