COMPUTER SCIENCE TRIPOS Part II – 2026 – Paper 9

9 Machine Learning and Bayesian Inference (sbh11)

Consider the following learning problem. Examples are pairs (x, c) with single, real-valued feature $x \in [0, 1]$ and class label $c \in \{0, 1\}$. A hypothesis h_g is specified by a parameter $g \in [0, 1]$ and has likelihood

$$\Pr(C = 1|h_g; x) = gx.$$

We fix two parameter values g_1 and g_2 and only consider the two corresponding hypotheses. The prior on the hypothesis is specified by a hyperparameter $p = \Pr(h_{g_1})$.

(*a*) Give at least two advantages and two disadvantages of the Bayesian approach to supervised machine learning. [5 marks]

(*b*) For a training set with examples $\mathbf{x}^T = [x_1, x_2, \ldots, x_m]$ and labels $\mathbf{c}^T = [c_1, c_2, \ldots, c_m]$, show that the Bayes prediction can be expressed as

$$\Pr(C = 1|\mathbf{c}; x, \mathbf{x}) = \frac{x}{Z}\left(g_1 p \Phi(h_{g_1}) + g_2(1 - p)\Phi(h_{g_2})\right)$$

where the function Φ and quantity Z should be defined in your answer. Start by stating the formula for the Bayes prediction $\Pr(C|\mathbf{c}; x, \mathbf{x})$ in this instance. [10 marks]

(*c*) Explain how the *evidence* can be employed as a means of estimating hyperparameters. Illustrate your answer by identifying the evidence in your answer to Part (*b*) and explaining how it can be used to estimate p when the latter is treated as a hyperparameter. [5 marks]