

2 Algebraic Techniques for Programming (nk480)

- (a) Recall that Datafun is the typed lambda calculus introduced to formulate fixed point computations, in which every type denotes a partially ordered set.

Suppose we want to extend Datafun with a new type constructor X_{\perp} , which adjoins a new element to X , with the property that the new element is below every element of X .

- (i) Give an interpretation of the X_{\perp} type constructor as a functor. [2 marks]
- (ii) Give typing rules to introduce and eliminate this type. [4 marks]
- (iii) Give a denotational interpretation of these rules, and explain why they are sound. [4 marks]
- (b) A graph G consists of a finite set of nodes N , a finite set of edges E , and a transition function $T : E \rightarrow N \times N$. Furthermore, suppose there is also a cost function $c : E \rightarrow \mathbb{N}^+$ which returns a positive integer for the cost of traversing each edge.
- (i) Which semiring lets us compute the length of the shortest path between any two nodes? [2 marks]
- (ii) Augment the star-semiring to additionally compute the set of shortest paths between any two nodes. It is not necessary to prove that the semiring properties are satisfied. [5 marks]
- (iii) How should the matrix whose star gives us the shortest paths be initialised? [3 marks]