COMPUTER SCIENCE TRIPOS Part II – 2026 – Paper 8

13 Types (nk480)

(*a*) Suppose that $\Gamma; \Delta \vdash t : \neg A$ true and $\Gamma; \Delta, u : A \vdash t' : B$ true are derivable in the proof term assignment for classical logic. Show that there is a term t'' such that $\Gamma; \Delta \vdash t'' : B$ true is derivable. [6 marks]

(*b*) Suppose that $\Gamma; \Delta \vdash t : A \vee B$ true and $\Gamma, x : A; \Delta \vdash t_1 : C$ true and $\Gamma, x : B; \Delta \vdash t_2 : C$ true are derivable in the proof term assignment for classical logic. Show that there is a term t'' such that $\Gamma; \Delta \vdash t'' : C$ true is derivable. [8 marks]

(*c*) (*i*) Give typing rules that extend the simply-typed lambda calculus with introduction and elimination forms for the natural numbers, yielding Gödel's T. [2 marks]

(*ii*) Using the rules defined in Part *(c)(i)*, define the predecessor function `pred` : $\mathbb{N} \to \mathbb{N}$. In your answer, you may use all the constructs of the simply-typed lambda calculus, including functions, products and sum types. [4 marks]