COMPUTER SCIENCE TRIPOS Part IB – 2026 – Paper 7

7 Further Graphics (aco41)

(*a*) (*i*) You are given a function $f(t) : \mathbb{R} \to \mathbb{R}$. Give the x and y coordinates of the corresponding planar curve $\mathbf{p}$ without re-parameterisation. [1 mark]

(*ii*) Is there any $\mathbf{p}$ as defined above that is arc-length parameterised? If so, what are the possible functions f? [3 marks]

(*iii*) Given an arc-length parameterised planar curve $[t^2, g(t)]$, give the expressions for the tangent and absolute curvature in as simple a form as possible. [5 marks]

(*b*) A signed distance field (SDF) f is an implicit function for a surface with the property that $f(\mathbf{x})$ is the distance to the closest point to $\mathbf{x}$ on the surface and the gradient of f has unit norm.

(*i*) If f is an SDF of a sphere of radius r and d is a scalar, what surface does the SDF $f - d$ represent? [2 marks]

(*ii*) Assume the sphere was reconstructed from sample points on the surface to get the SDF f. Derive the expression of the SDF of the sphere if the sample points were rotated with the matrix $\mathbf{R}$ and translated with the vector $\mathbf{t}$ before reconstruction. [5 marks]

(*iii*) If $(f+g)/2$ is a valid SDF for general surfaces, what is the relation between the surfaces represented by the SDFs f and g? [4 marks]