8 Logic and Proof (mj201)

(*a*) Consider an automated traffic management system for a narrow bridge with one-way traffic only. Cars may enter the bridge only from one side, which is controlled by a traffic light. Let G mean the light is green, R mean the light is red, and M mean a car is moving on the bridge. When the light is red, cars do not enter the bridge. Formalise the following requirements in S4 modal logic:

(*i*) It is necessarily true that if the light is green, it is possible for a car to be moving.

(*ii*) It is necessarily true that if a car is moving, then it is not possible for the light to be red.

(*iii*) Henceforth, eventually the light becomes green.

[3 marks]

(*b*) Using your formalisation from Part (*a*), show that the following formula holds:

$$\Box(M \to \neg R).$$

[3 marks]

(*c*) Provide a formal proof in the S4 sequent calculus for the following sequent:

$$\Box(P \to \Diamond Q) \Rightarrow \Diamond P \to \Diamond\Diamond Q$$

[7 marks]

(*d*) In S4 modal logic, the accessibility relation is reflexive and transitive. Define a new operator $\blacksquare$ such that $\blacksquare A \equiv A \land \Box A$.

(*i*) Derive the left and right sequent rules for $\blacksquare$ based on the rules for $\land$ and $\Box$.

(*ii*) In the context of S4, explain whether $\blacksquare A \simeq \Box A$ holds.

[7 marks]