

7 Logic and Proof (mj201)

(a) Consider the following statement:

$$\forall x \forall y \forall z (R(x, y) \wedge R(y, z) \rightarrow R(x, z)) \wedge \forall x \exists y R(x, y) \rightarrow \forall x R(x, x).$$

Prove that it is valid using resolution or find a counterexample. Show all steps: negation, skolemisation, conversion to clauses, and the resolution proof. If a counterexample exists, explain how resolution is blocked from producing a spurious proof.

[9 marks]

(b) Use the DPLL method to find a model satisfying the following set of clauses, or to prove that no such model exists. Show each step of the method clearly.

$$\{P, Q, \neg R\} \quad \{\neg P, R\} \quad \{R, \neg Q, \neg R\} \quad \{\neg Q, R\} \quad \{P, \neg Q\} \quad \{\neg P, \neg R\}$$

[6 marks]

(c) Apply the Fourier-Motzkin variable elimination method to determine the satisfiability of the following system of linear inequalities over real numbers.

(i) $x + y + z \leq 1$

(ii) $x + y - z \leq -1$

(iii) $x \geq 2$

If the system is satisfiable, give an assignment to the variables that satisfies all inequalities (a model). If it is not satisfiable, derive a contradiction.

[5 marks]