4 Computation Theory (ad260)

(*a*) What is the class of *primitive recursive functions*? [2 marks]

(*b*) Show that the following function $\mathrm{nz} : \mathbb{N} \to \mathbb{N}$ is primitive recursive:

$$\mathrm{nz}(x) \triangleq \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{otherwise.} \end{cases}$$

[3 marks]

For any function $f : \mathbb{N}^n \to \mathbb{N}$, let $\mathrm{nz}_f : \mathbb{N}^n \to \mathbb{N}$ be the function:

$$\mathrm{nz}_f(\bar{x}) \triangleq \begin{cases} 0 & \text{if } f(\bar{x}) = 0 \\ 1 & \text{otherwise.} \end{cases}$$

(*c*) Show that if f is primitive recursive, then so is nz_f. [2 marks]

For any function $g : \mathbb{N}^{n+1} \to \mathbb{N}$, let $\mathrm{prod}_g : \mathbb{N}^{n+1} \to \mathbb{N}$ be the function:

$$\mathrm{prod}_g(\bar{x}, y) \triangleq \prod_{i=0}^{y} g(\bar{x}, i).$$

(*d*) Show that if g is primitive recursive, then so is prod_g. [5 marks]

For any function $h : \mathbb{N}^{n+1} \to \mathbb{N}$, let $\mathrm{bex}_h : \mathbb{N}^{n+1} \to \mathbb{N}$ be the function where $\mathrm{bex}_h(\bar{x}, y) = 0$ if there is no $i \leq y$ with $h(\bar{x}, i) = 0$ and $\mathrm{bex}_h(\bar{x}, y) = 1$ if there is such an i.

(*e*) Show that if h is primitive recursive, then so is bex_h. [4 marks]

(*f*) Now let $\mathrm{ex}_h : \mathbb{N}^n \rightharpoonup \mathbb{N}$ be the partial function where $\mathrm{ex}_h(\bar{x}) = 1$ if there is an x such that $h(\bar{x}, x) = 0$ and $\mathrm{ex}_h(\bar{x})$ is undefined otherwise. Give an example of a primitive recursive h for which ex_h is not primitive recursive. [4 marks]