

3 Computation Theory (ad260)

A *Gödel numbering* of register machines is a bijection G between the natural numbers \mathbb{N} and the collection of register machines.

- (a) Give an example of a Gödel numbering of register machines. [5 marks]

The *Halting Problem* for register machines (with respect to a Gödel numbering G) is a set of pairs of numbers $H \subseteq \mathbb{N} \times \mathbb{N}$.

- (b) Give a precise statement of what pairs of numbers constitute H , given the Gödel numbering from Part (a). [3 marks]

- (c) Give a precise statement of what it means to say that H is *undecidable*. [2 marks]

Consider the following set of numbers

$$Z = \{n \in \mathbb{N} \mid G(n) \text{ halts when started with } 0 \text{ in all registers}\}$$

- (d) Describe a computable function $r : \mathbb{N}^2 \rightarrow \mathbb{N}$ such that for all $m, n \in \mathbb{N}$ we have $(m, n) \in H$ if, and only if, $r(m, n) \in Z$ and show that it has these properties. [7 marks]
- (e) What can you conclude about the decidability or otherwise of Z ? Give justification for your answer. [3 marks]