

## COMPUTER SCIENCE TRIPOS Part IB – 2026 – Paper 6

### 1 Complexity Theory (tg508)

Let  $A \leq_p B$  denote polynomial-time many-one reducibility.

- (a) Define  $A \leq_p B$ . What does it mean for a language to be NP-complete? [2 marks]
- (b) Define K-SAT and show that  $2\text{SAT} \in \text{P}$ . [4 marks]
- (c) VERTEX-COVER is defined as follows: given a graph  $G = (V, E)$  and integer  $k$ , decide whether there exists  $S \subseteq V$  with  $|S| \leq k$  such that every edge has at least one endpoint in  $S$ . Prove that  $\text{VERTEX-COVER} \in \text{NP}$ . [2 marks]
- (d) Give a polynomial-time reduction from  $3\text{SAT}$  to  $\text{VERTEX-COVER}$ , and prove its correctness. [8 marks]
- (e) Define  $\text{INDEPENDENT-SET}$  and prove that it is NP-complete by giving a polynomial-time reduction from  $\text{VERTEX-COVER}$ . [4 marks]