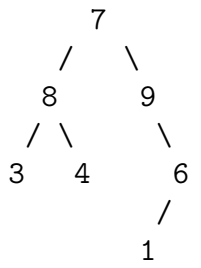
4 Prolog (ijl20)

Consider a binary tree whose nodes are either atom `lf` for a leaf or compound term `n(Value,Left,Right)` where `Value` is an integer and `Left` and `Right` are trees. For example `n(7,n(8,n(3,lf,lf),n(4,lf,lf)),n(9,lf,n(6,n(1,lf,lf),lf)))`. represents the tree



In your answers ensure each relation has a comment giving a declarative reading of its behaviour. Avoid unnecessary use of *cut* or other extra-logical relations. The library relations `\=` and `is` may be used. Other library relations should not be assumed.

(*a*) Write a relation `sum(T,Sum)` which, when given an input tree `T`, will succeed with `Sum` set to the integer sum of all values in the tree. [2 marks]

(*b*) Write a relation `summed_tree(Tree,SummedTree)` which will succeed with `SummedTree` having the same structure as `Tree`, but with each value being the sum of the sub-tree at the corresponding position in the input `Tree`. [3 marks]

(*c*) Write a relation `breadth_first(T,V)` which, given an input tree `T` and variable `V`, will succeed on each redo with `V` being each value in the tree found in breadth-first left-right order. In the example tree, this means visiting the nodes in the following order: 7,8,9,3,4,6,1. [4 marks]

(*d*) Write a relation `root_path(T,V,Path)` which, given a tree `T` and a value `V` unique within that tree, will succeed with `Path` being the list of values from the root of `T` to the node containing value `V`, inclusive of the end-points. For example, if `T` is set to the example tree then `root_path(T,4,Path)` succeeds with `Path = [7,8,4]`. [4 marks]

(*e*) Assume a tree including unique values `V1` and `V2` from which root paths `P1` and `P2` respectively have been derived as in Part (*d*). Write a relation `route(P1,P2,Route)` which will succeed with `Route` set to the shortest list of values from `V1` to `V2` including those end-points. For example, `route([7,9,6],[7,8,4],Route)` succeeds with `Route = [6,9,7,8,4]`. [7 marks]