4 Introduction to Graphics (rkm38)

An ellipsoid centered at the origin and with semi-axis lengths a, b, and c that are aligned with the axes of the coordinate system has an implicit equation:

$$(P\,E) \cdot (P\,E) = 1 \tag{1}$$

where $P = \begin{bmatrix} x & y & z \end{bmatrix}$ is a point in 3D space, E is a diagonal matrix with the elements $\frac{1}{a}$, $\frac{1}{b}$, and $\frac{1}{c}$, and $\cdot$ is the dot product.

(*a*) Derive an equation for the intersection of the ellipsoid from Eq. 1 with a ray, given by the origin O and direction D. [6 marks]

(*b*) Use transformation matrices in homogeneous coordinates to find an implicit equation of an ellipsoid centered at point M, and with semi-axes given by line segments $\overline{MA}$, $\overline{MB}$, $\overline{MC}$. All semi-axies are orthogonal to each other, i.e., $\overline{MA} \cdot \overline{MB} = 0$, $\overline{MB} \cdot \overline{MC} = 0$, $\overline{MA} \cdot \overline{MC} = 0$. You can use operator $T(x)$ to denote the translation (translation by vector x), and $R_x(\omega)$, $R_z(\theta)$, $R_y(\phi)$ to denote the rotation by each axis. There is no need to write full matrices. [10 marks]

(*c*) Derive an equation for the intersection of the ellipsoid from Part (*b*), given by M, $\overline{MA}$, $\overline{MB}$ and $\overline{MA}$, with the ray $\overrightarrow{OD}$. [4 marks]