9 Algorithms 2 (jkf21)

(*a*) Let T be a minimum spanning tree of an undirected graph G, and let T' be any other minimum spanning tree of G. Prove that T and T' must have the same sets of distinct edge weights. [5 marks]

(*b*) Say whether or not it is the case that, if the minimum spanning tree of an undirected graph G is unique then every cut of G has a unique light edge crossing the cut. Provide a proof or counterexample. Is the converse true? Again, provide a proof or counterexample. [5 marks]

(*c*) The Bellman–Ford algorithm is executed on a weighted, directed graph that contains a negative weight cycle. The cycle goes undetected by the algorithm. How can this occur? Provide an example graph for which this occurs. [4 marks]

(*d*) Bellman–Ford relaxes every edge in the input graph $|V| - 1$ times.

(*i*) For what purpose do we perform one extra relaxation of every edge? [2 marks]

(*ii*) Explain whether it is necessary to perform the extra relaxation if no changes were made in one of the first $|V| - 1$ scans. [2 marks]

(*iii*) What optimisation might you suggest if no changes were made in one of the first $|V| - 1$ scans? [2 marks]