

7 Algorithms 1 (j kf21)

(a) Let $T(n)$ be the recurrence relation defined by

$$T(n) = \begin{cases} 1 & n \leq 10 \\ 4T(n/4 - 2) + kn & \text{otherwise} \end{cases}$$

Is $T(n) \in O(n)$? Justify your answer. [4 marks]

(b) Find asymptotically tight lower and upper bounds for the recurrence relation

$$T(n) = \begin{cases} 1 & n = 1 \\ 8T(n/2) + n^2 & \text{otherwise} \end{cases}$$

[6 marks]

(c) Find an asymptotically tight upper bound for the recurrence relation

$$T(n) = \begin{cases} 1 & n \leq 5 \\ T(n - 5) + n^2 & \text{otherwise} \end{cases}$$

and justify that it is asymptotically tight. [3 marks]

(d) Show that the recurrence relation

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/4) + \frac{n}{\sqrt{n}} & \text{otherwise} \end{cases}$$

is in $\omega(\sqrt{n})$ and $O(n^2)$. Is $T(n) \in o(n \lg n)$? [7 marks]