6 Introduction to Probability (mj201+tms41)

An internet service provider has installed c modems to serve the needs of a population of n users ($n \geq c$). It is estimated that at a given time, each user will need a connection with probability p, independently of the others. Let X be the total number of users needing a connection at a given time.

(a) What is the distribution of X, and what is its variance? [2 marks]

(b) If there are more than c users needing a connection, only c can be served while the rest of them will be blocked. Derive an expression for the expected number of blocked users. [2 marks]

Remark: For the following two questions, you do *not* need to compute an explicit, i.e., numerical value.

(c) Assuming $n = 200$ and $p = 1/100$, use a suitable approximation to estimate the probability that at a given time exactly 15 users need a connection. [3 marks]

(d) Assuming $n = 200$ and $p = 1/2$, use a suitable approximation to estimate the probability that at a given time at least 80 users need a connection. [5 marks]

Assume now that $p \in (0, 1]$ is an unknown parameter and that the provider records the number of required connections at m randomly chosen times. Let $X_1, X_2, \ldots, X_m$ denote these samples.

(e) Can we find an unbiased estimator for p? Justify your answer. [2 marks]

(f) Can we find an unbiased estimator for $\mathbf{P}[\,\text{at most } c \text{ users need a connection}\,]$? Justify your answer. [3 marks]

(g) Can we find an unbiased estimator for $1/p^2$? Justify your answer. [3 marks]