## 5 Introduction to Probability (mj201+tms41)

Two persons have a meeting at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, where all delays are equally likely and independent of each other. Let $X_1, X_2 \in [0, 1]$ be the continuous random variables representing the delays of person 1 and 2, respectively.

(*a*) Calculate $\mathbf{E}[X_1 + X_2]$ and $\mathbf{V}[X_1 + X_2]$, justifying each step in your derivation carefully. [4 marks]

(*b*) What is $\mathbf{P}\left[X_1 \geq 0.5 \text{ hours} \;\middle|\; X_1 \geq 0.2 \text{ hours}\right]$? [2 marks]

(*c*) Assume now that the first person to arrive will wait for 15 minutes and will leave if the other has not yet arrived. We are interested in the event that the two persons meet.

(*i*) Formalise this event using mathematical notation. [2 marks]

(*ii*) Compute the probability of this event. [3 marks]

Consider now an arbitrary pair of random variables $(X_1, X_2)$.

(*d*) State two different equivalent definitions for $\mathbf{Cov}[X_1, X_2]$. [2 marks]

(*e*) What is the relationship between $\mathbf{Cov}[X_1, X_2] = 0$ and $X_1, X_2$ being independent? [2 marks]

Assume now that $X_1, X_2$ are independent samples from the same continuous distribution (not necessarily uniform) over non-negative real numbers with finite mean $\mu$. Let $U = \min(X_1, X_2)$ and $V = \max(X_1, X_2)$.

(*f*) Prove that $U$ and $V$ cannot be negatively correlated.
*Hint:* You may want to use the following identity among sets: $\{U, V\} = \{X_1, X_2\}$. [5 marks]