COMPUTER SCIENCE TRIPOS Part IA – 2026 – Paper 1

1 Foundations of Computer Science (jjl25)

A Lindenmayer system is a type of formal grammar that can be used to generate self-similar fractals. It consists of three parts: an alphabet of symbols that can be assembled to form a 'sentence', an initial sentence S_0, and a set of rewrite rules. The rewrite rules are applied to sentence S_0 to produce the next sentence S_1, then they are applied to S_1 to produce S_2 and so on.

In this question we consider a Lindenmayer system with symbols $F, G, +, -$, and the following rewrite rules:

$$F \to F + G$$
$$G \to F - G$$

The symbols + and − are unchanged. The initial sentence is $S_0 = F$ so the first few sentences are therefore $S_1 = F + G$, $S_2 = F + G + F - G$.

(a) We will use a type `t` to represent the symbols and `t list` to represent a sentence. Define a suitable type `t` and a 'rewrite function' `rewrite : t -> t list` that applies the rewrite rules to a single symbol. [4 marks]

(b) Define a function `apply : ('a -> 'a list) -> 'a list -> 'a list` that takes a rewrite function and a sentence, and applies the rewrite function to obtain the next sentence. [4 marks]

(c) The symbols describe the movement of a robot on a grid. The state of the robot is completely defined by its current position and the direction of travel. It starts at the origin (0,0) and is facing in the positive y direction, or 'Up', such that moving forward one unit would leave it at position (0,1).

Symbol	Action
F, G	Move forward one unit in current direction
+	Turn right (90° clockwise)
-	Turn left (90° anticlockwise)

Using the types

```
type pos = int * int
type dir = Up | Left | Right | Down
type state = pos * dir
```

define `step : state -> t -> state` that performs one action. [6 marks]

(d) Define a function `run : t list -> (int * int) list` that, given a sentence, returns the coordinates visited executing the corresponding actions, including the start and end points. For example, the sentence $[F, +, G]$ should produce `[(0,0); (0,1); (1,1)]`. [6 marks]