CST0
COMPUTER SCIENCE TRIPOS Part IA

Wednesday 10 June 2026 14:00 to 17:00

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

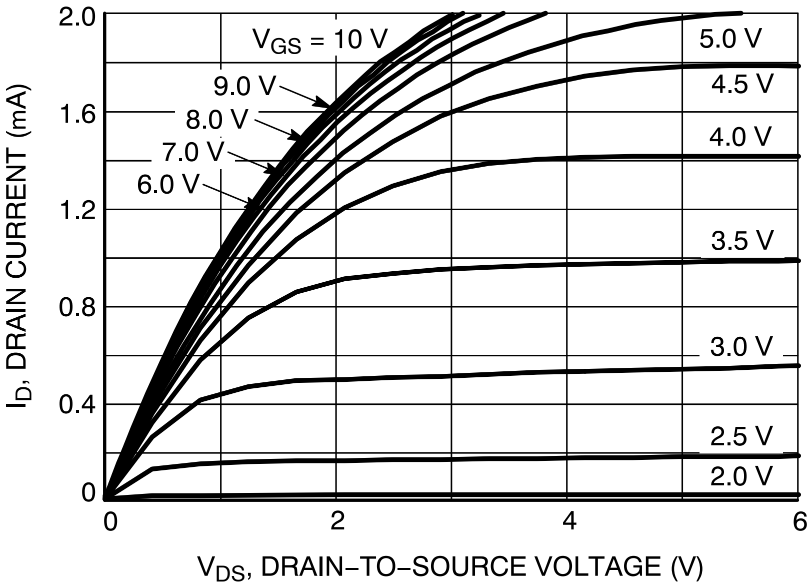
(*a*) Show that the following Boolean function can be simplified to a two term sum-of-products form where each term has two variables either in complemented or uncomplemented form.

$$F(X,Y,Z) = \overline{\overline{\overline{X.\overline{X.\overline{Y}}}.\overline{\overline{X.\overline{Y}}.\overline{Y}}.Z}.\overline{\overline{\overline{X.\overline{Y}}.\overline{Y}}.Z}}$$

[6 marks]

(*b*) A four variable Boolean function $G(A,B,C,D)$ has the following minterms $\{1,2,4,7,8,11,13,14\}$ (decimal), where A is the most significant bit of the equivalent binary reprsentation. Show G can be expressed using only exclusive OR operations. [4 marks]

(*c*) A NOT gate comprises an n-channel MOSFET and a resistor R. Assume the power supply voltage (V_{DD}) is 5 V and $R = 5100\ \Omega$. The MOSFET has the I_{D} - V_{DS} characteristic given in the following figure.



(*i*) Draw the circuit diagram of the NOT gate. [3 marks]

An extra copy of the figure above is attached to the back of this paper. This should be detached, amended as requested below, and handed in with your answer.

(*ii*) Plot the curve for R on the MOSFET I_{D} - V_{DS} characteristic. [3 marks]

(*iii*) Determine the voltage levels at the output of the NOT gate for input voltage levels of 2.5 V and 4 V. Sketch the input voltage to output voltage characteristic for the NOT gate over an input voltage range from 0 V to 5 V. Reasonable variability in the answer will not be penalised. [4 marks]

2 Digital Electronics

(a) A synchronous finite state machine (FSM) using a single D-type flip-flop has its next state output given by

$$D(F, G, E, Q) = \overline{F}.\overline{G}.\overline{E}.\overline{Q} + \overline{F}.G.E + F.\overline{G}.E + F.G.(\overline{E} + Q)$$

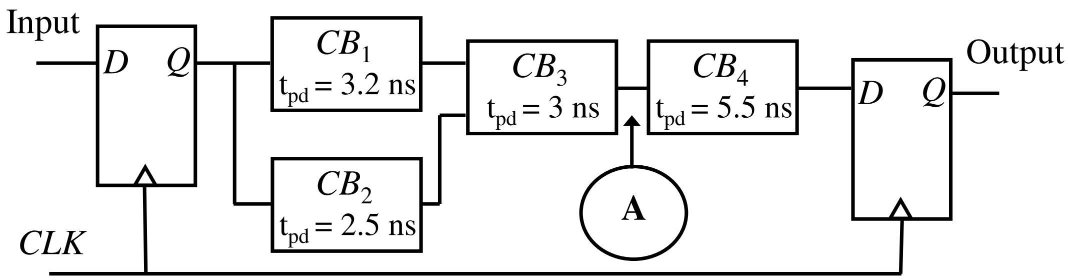
where F, G and E are inputs to the FSM, and the output of the FSM is the Q output of the flip-flop.

(i) Show how D could be implemented using a 4:1 multiplexer, a 2-input AND gate and a 2-input OR gate. Use F and G as the multiplexer control inputs. Assume that complemented variables are available for use. [4 marks]

(ii) Determine the state transition table for this FSM. [4 marks]

(iii) Determine the state diagram for this FSM. [5 marks]

(b) The following figure shows four blocks of combinational logic, CB_1 to CB_4, located between two D-type flip-flops that are clocked by the system clock, CLK. The maximum propagation delay, t_{pd}, for each combinational logic block is shown in the figure. The flip-flops have a maximum clock-to-Q propagation delay, t_{pc}= 0.2 ns, and a minimum set-up time, t_{su}= 0.1 ns.



(i) Determine the maximum clock frequency, i.e., the data throughput, and the time taken for one data item to pass from Input to Output, i.e., the system latency. [2 marks]

(ii) Now determine the data throughput and latency if an additional D-type flip-flop is inserted at position **A** in the figure. [2 marks]

(iii) Where might two further flip-flops be inserted to further increase throughput? What is the throughput and latency for this configuration? [3 marks]

SECTION B

3 Operating Systems

(*a*) What is the purpose of Inter-Process Communication (IPC)? What are the differences between pipes and shared memory for IPC, including their pros and cons? [6 marks]

(*b*) Suggest two ways how one Linux process can notify another process that something has happened. Explain briefly how they differ. [2 marks]

(*c*) What other reasons besides shared memory IPC are there for sharing memory pages between multiple processes? How does the operating system enforce memory protection for these various types of sharing? [6 marks]

(*d*) Your colleague wants to store a linked list within a shared memory segment. Suggest some issues that are likely to arise. [3 marks]

(*e*) Another colleague of yours wants to use a fixed-length file on a solid state drive (SSD) as a way to support IPC between processes. Like with shared memory, processes can read and write to arbitrary locations within this file. If the disk is shared between different machines, this could even support IPC across machines. Is this a good idea? Justify your answer. [3 marks]

4 Operating Systems

(*a*) What is a Process Control Block (PCB), and what information does it typically contain? [4 marks]

(*b*) A system is running four processes, two of which are I/O-bound, and two of which are CPU-bound. The I/O-bound processes alternate between using a 10 ms burst of CPU time and waiting for input. All processes run indefinitely on a single CPU core.

Using a Round Robin scheduling algorithm, describe one possible sequence of activiations of the four processes, i.e. in which order the processes are run and for how long. Show this for a quantum of 5 ms, 10 ms, and 20 ms respectively. On each context switch, indicate whether the running process is preempted or self-suspends. State any assumptions that you make. [6 marks]

(*c*) If you replace the Round Robin scheduler in Part (*b*) with a Shortest Job First scheduler, what behaviour would you expect? Briefly justify your answer. [3 marks]

(*d*) Now replace the Round Robin scheduler with a Shortest Remaining Time First scheduler. Explain the expected behaviour for the scenario in Part (*b*), and how the scheduler implements it. How fair is this scheduling policy? [4 marks]

(*e*) You are writing an app that allows users to make video calls and also record those calls as files on their hard drive. Discuss the scheduling issues that are likely to arise in this app, and how you would handle them. [3 marks]

SECTION C

5 Software and Security Engineering

(*a*) (*i*) Explain what is meant by IaaS, SaaS and PaaS, giving a real world example of each. [6 marks]

(*ii*) Explain how a startup would typically use these technologies as they grow. [3 marks]

(*b*) (*i*) Define the roles of a Software Engineer (SWE) and a Technical Programme Manager (TPM). Identify two similarities between these roles. [4 marks]

(*ii*) Explain how Requirements Modelling can help resolve conflicts between stakeholders in the early stages of a project. Identify the role that would lead this process. [3 marks]

(*c*) A team uses an AI-assisted coding tool to generate a high-performance data-processing module for their PaaS-hosted application. The generated code uses an undocumented API of the underlying platform to bypass certain latency bottlenecks. Critique this approach and discuss any precautions they should take. [4 marks]

SECTION D

6 Discrete Mathematics

For a natural number n, define the following sets:

$$[n] = \{\, i \in \mathbb{N} \,:\, 0 \leq i < n \,\} \quad , \quad [n]^{+} = \{\, j \in [n] \,:\, j \neq 0 \,\}$$

$$\mathrm{C}(n) = \{\, k \in [\,n\,]^{+} \,:\, \gcd(k, n) = 1 \,\} \ , \ \mathrm{I}(m, n) = \{\, \iota \in \big([m] \Rightarrow [n]\big) \,:\, \iota \text{ is injective} \,\}$$

(a) Describe the sets $[3] \times [3]^{+}$, $\mathrm{C}(3^2)$, and $\mathrm{I}(2, 3)$ by enumerating their elements. [3 marks]

(b) For a prime number p,

(i) give a mapping f_p from $\mathrm{C}(p^2)$ to $[p] \times [p]^{+}$ and prove that it defines a function $f_p : \mathrm{C}(p^2) \to [p] \times [p]^{+}$; [4 marks]

(ii) give a mapping g_p from $[p] \times [p]^{+}$ to $\mathrm{C}(p^2)$ and prove that it defines a function $g_p : [p] \times [p]^{+} \to \mathrm{C}(p^2)$; [4 marks]

(iii) prove that $g_p \circ f_p = \mathrm{id}_{\mathrm{C}(p^2)}$ and that $f_p \circ g_p = \mathrm{id}_{[p] \times [p]^{+}}$. [2 marks]

(c) For a prime number p, define a function $u_p : \mathrm{C}(p^2) \to \mathrm{I}(2, p)$ and a function $v_p : \mathrm{I}(2, p) \to \mathrm{C}(p^2)$ and prove that they determine bijections between $\mathrm{C}(p^2)$ and $\mathrm{I}(2, p)$. [5 marks]

(d) For a prime number p, determine the cardinality of $\mathrm{C}(p^2)$. [2 marks]

7 Discrete Mathematics

(*a*) For $\mathbb{N}^+$ the set of positive natural numbers, let

$$S = \left\{ m \in \mathbb{N}^+ \mid \sum_{i=1}^{m-1} i^m \equiv 0 \pmod{m} \right\}$$

where, by convention, $\sum_{i=1}^{0} i = 0$.

(*i*) State whether or not the set S is finite and prove your claim. [4 marks]

(*ii*) For O the set of odd integers, give an explicit description of the intersection set $O \cap S$ and prove it correct. [4 marks]

(*b*) Let $P(n)$ be the property of natural numbers n given by

$$\forall\, (a_1, \ldots, a_n) \in \mathbb{N}^n.\ n^n \cdot \prod_{i=1}^{n} a_i \leq \left(\sum_{i=1}^{n} a_i \right)^n$$

where, by convention, $\prod_{i=1}^{0} a_i = 1$ and $\sum_{i=1}^{0} a_i = 0$.

(*i*) Prove that $P(n)$ holds for all n that are even powers of 2. [6 marks]

(*ii*) Prove that $\forall\, n \in \mathbb{N}.\ P(n+1) \Rightarrow P(n)$. [4 marks]

[Hint: In proving $P(n)$ for the instance $(a_1, \ldots, a_n) \in \mathbb{N}^n$ consider $P(n+1)$ for the instance $\left(n \cdot a_1, \ldots, n \cdot a_n, \sum_{i=1}^{n} a_i\right) \in \mathbb{N}^{n+1}$.]

(*iii*) Argue whether or not $P(n)$ holds for all natural numbers n. [2 marks]

8 Discrete Mathematics

(*a*) We define the ***kernel*** of a function $f\colon A \to B$ to be the binary relation

$$\ker(f) := \{(x, y) \in A \times A \mid f(x) = f(y)\}.$$

(*i*) **Prove** that $\ker(f)$ is an equivalence relation. [3 marks]

(*ii*) **Prove** that f is an injective function iff $\ker(f)$ is the diagonal relation $\mathbf{1}_A = \{(a, a) \mid a \in A\}$. [1 mark]

(*b*) We define the ***coimage*** of $f\colon A \to B$ to be the quotient

$$\operatorname{coim}(f) := A/\ker(f)$$

of A by the kernel of f. We write $e_f\colon A \twoheadrightarrow \operatorname{coim}(f)$ for the corresponding quotient map. In what follows, we write $i_f\colon \operatorname{im}(f) \hookrightarrow B$ for the inclusion of the *image* of f into B.

(*i*) **Define** a function $h_f\colon \operatorname{coim}(f) \to \operatorname{im}(f)$ and **verify** that the composition

$$A \xrightarrow{e_f} \operatorname{coim}(f) \xrightarrow{h_f} \operatorname{im}(f) \xrightarrow{i_f} B$$

is equal to $f\colon A \to B$. [2 marks]

(*ii*) **Prove** that h_f from Part (*b*)(*i*) is surjective. [2 marks]

(*iii*) **Prove** that h_f from Part (*b*)(*i*) is injective. [2 marks]

(*iv*) **Prove** that f is injective iff $e_f\colon A \twoheadrightarrow \operatorname{coim}(f)$ is injective. [2 marks]

(*c*) Say whether or not the following are true, with proof.

(*i*) If A is finite and B is enumerable, then the function space $A \to B$ is enumerable. [2 marks]

(*ii*) If A and B are countable, then so is the function space $A \to B$. [2 marks]

(*iii*) If A is enumerable, then the quotient A/E of A by any equivalence relation $E \subseteq A \times A$ is enumerable. [2 marks]

(*iv*) If $B(a)$ is an enumerable set for each $a \in A$ and A is enumerable, then the indexed disjoint union $\biguplus_{a \in A} B(a)$ is enumerable. [2 marks]

9 Discrete Mathematics

(*a*) Let A be a set and let $R \subseteq A \times A$ be a binary relation on A.

(*i*) **Give** a system of *rules* generating the smallest reflexive relation containing R, so that there exists a derivation of (x, y) if and only if $(x, y) \in \mathbf{1}_A \cup R$. [1 mark]

(*ii*) Add **just one** rule to those of Part (*a*)(*i*) so that the combined system generates the *equivalence closure* of R, *i.e.* the smallest equivalence relation containing R. Argue for the correctness of your rule. [2 marks]

(*b*) Let Σ_0 and Σ_1 be two alphabets, and let $\rho\colon \Sigma_0 \to \Sigma_1$ be a function between them, writing $\rho\colon \Sigma_0^\star \to \Sigma_1^\star$ also for the induced function on strings. Given a formal language $\mathcal{L}$ over Σ_1, we define a formal language $\rho^*\mathcal{L}$ over Σ_0 as follows:

$$\rho^*\mathcal{L} := \{u \in \Sigma_0^\star \mid \rho(u) \in \mathcal{L}\}$$

(*i*) **Show** that if $\mathcal{L}$ is regular, then $\rho^*\mathcal{L}$ is regular too. [4 marks]

(*ii*) Assuming $\rho^*\mathcal{L}$ is regular, **propose and prove** a sufficient condition on ρ for $\mathcal{L}$ to be regular. [4 marks]

(*c*) (*i*) Let $\mathcal{K}$ and $\mathcal{L}$ be regular languages over an alphabet Σ. Say whether or not the formal language

$$\mathcal{M} = \{uv \mid u \in \mathcal{K} \implies v \in \mathcal{L}\}$$

is regular, and provide **proof**. [4 marks]

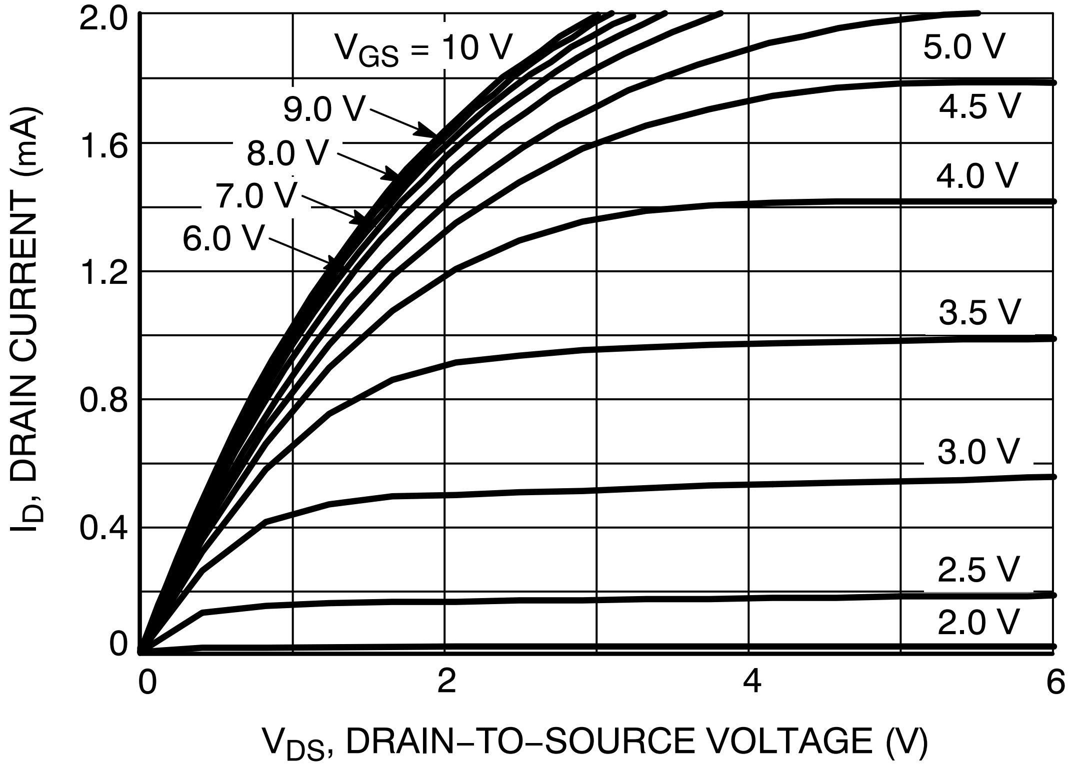
(*ii*) Let Σ be an alphabet. Give a **necessary and sufficient** condition for the formal language

$$\mathcal{L} = \{a^n b a^n \mid n \text{ even}, a \in \Sigma, b \in \Sigma\}$$

to be regular, and **provide proof**. [5 marks]

END OF PAPER

Extra copy of figure for Question 1 (*c*)



Detach this sheet, fill in your candidate number, and amend the figure above as requested in Question 1. Then attach it to and hand it in with your answer.